8.1 INTRODUCTION

Studies on the influence of dust particles on viscous fluid flows are of importance in petroleum industry and in the purification of crude oils. Other important applications involving dust particles in boundary layers include soil solvation by natural winds, lunar surface erosion by the exhaust of a landing vehicles and dust entrainment in a cloud formed during a nuclear explosion. The unsteady motion of fluid resulting due to the pure rotation of a solid boundary or due to the application of a uniformly distributed shear stress along a solid boundary is of both theoretical and practical significance in fluid mechanics. A number of workers studied both steady and unsteady two dimensional axisymmetric rotational flow of a viscous fluid in view of its growing importance in viscous technical problems. Khamrui [1] analysed the slow steady motion of an infinite viscous fluid due to the rotation of a circular cylinder. Iben [2] considered the non stationary, plane circular-symmetric flow of a viscous fluid which forms itself within as well as outside a rotating infinitely long circular cylinder. Bhattacharyya [3] studied the rotational motion produced in an enclosed fluid, contained in a circular cylinder of infinite depth. The disturbance was generated on the surface
of the fluid by an impulsive couple. Later Mukherjee and
Bhattacharyya [4] studied the rotational flow of viscous fluid
due to the rotation of a circular cylinder or by the action of
shearing stress on the boundary. Saffman [5] studied the
stability of the laminar flow of a dusty gas with uniform
distribution of dust particles. Michael [6] considered the
Kelvin-Helmholtz instability of the dusty gas. Michael and
Miller [7] has discussed the motion of dusty gas enclosed in the
semi-infinite space above a rigid plane boundary. Using the
formulation of Saffman [5] many authors have studied a number of
dusty gas problems and the results are well documented in a
review by Marble [8]. Recently Mandal, Mukherjee and Mukherjee
[9] analysed the rotational motion of a dusty viscous fluid
contained in the semi-infinite circular cylinder due to an
initially applied impulse on the surface. In the present
investigation we have extended the problem studied by Mandal et.
al. [9] to the MHD case. The flow of the dusty gas is initiated
by an impulsive shearing force on the surface in presence of a
transverse magnetic field.

8.2 FORMULATION OF THE PROBLEM

Following Saffman [5], the basic equation to represent
the motion of a dusty fluid in presence of uniform magnetic
field are given by

\[
\rho \left[ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla) \vec{u} \right] = - \text{grad } p + \mu \nabla^2 \vec{u} + \kappa \text{N} (\vec{v} - \vec{u}) + \vec{J} \times \vec{B} \quad (8.2.1)
\]

\[
m \left[ \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = \kappa (\vec{u} - \vec{v}) \quad \quad \quad (8.2.2)
\]
\[ \text{div} \, \mathbf{u} = 0 \quad \text{(8.2.3)} \]
\[ \frac{\partial N}{\partial t} + \text{div} \, N \mathbf{v} = 0 \quad \text{(8.2.4)} \]

where the velocities of the fluid and dust particles are \( \mathbf{u} \) and \( \mathbf{v} \) respectively. \( N \) is the number density of dust particles each of mass \( m \). Here \( K \) is the stokes coefficient of resistance (for spherical particles of radius \( d \), it is \( 6\pi \mu d \)), \( t, p, \rho, \mu, \) the time, the pressure, the density and the viscosity of the fluid respectively. The last term on the right hand side of (8.2.1) represent the force on the fluid due to the interaction of magnetic induction \( \mathbf{B} \) and the electric current \( \mathbf{J} \) in the fluid.

In the present analysis, the following assumptions are made:

1. The dust particles are spherical in shape and are uniformly distributed.
2. The interaction between particles themselves is not considered.
3. The flow is fully developed.
4. The number density of dust particle is constant throughout the motion.
5. The applied magnetic field is considered to be along the axial direction of the cylinder.
The flow induced magnetic field is neglected.

There is no external applied electric field.

Only the electromagnetic body forces are present.

Fluid properties are invariable.

Initially the fluid and dust particles are at rest. Consider the flow of a dusty fluid in a long circular cylinder of radius 'a'. Disturbance is set up by an impulse of the shearing force on the surface. Referring the problem to cylindrical polar co-ordinate (r, θ, z) we take the z-axis along the axis of the cylinder and the origin on the surface of the fluid. The prescribed impulsive shearing force will obviously generate a rotational velocity in the dusty fluid. Thus the fluid as well as the dust particles will be moving in circular paths around the axis of the cylinder. Also a uniform magnetic field \( \vec{B} (0, 0, B_0) \) is applied along the z-axis so that \( B_0 \) will be transverse to the flow field, \( r \) denotes the radial direction measured outward from the z-axis.

The symmetry consideration gives

\[
\begin{align*}
    u_1 &= u_3 = 0, \quad v_1 = v_3 = 0, \quad u_2 = u_2 (r, z, t) \\
    v_2 &= v_2 (r, z, t) \quad \text{and} \quad \frac{\partial}{\partial \theta} = 0
\end{align*}
\]

\[\cdots \quad (8.2.5)\]

where \( u_2 \) and \( v_2 \) are circumferential velocities of liquid and dust particles respectively. Since the distribution of dust particles is uniform, the number density \( N \) of the particles
equals \( N_0 \), a constant throughout the motion.

Then with the help of (8.2.5), equations (8.2.1) to (8.2.4) become

\[
\frac{\partial u_2}{\partial t} = \mu \left[ \frac{\partial^2 u_2}{\partial r^2} + \frac{1}{r} \frac{\partial u_2}{\partial r} - \frac{u_2}{r^2} + \frac{\partial^2 u_2}{\partial z^2} \right] + KN_0 (v_2 - u_2) - \sigma B_0^2 u_2 \\
\frac{\partial v_2}{\partial t} = K (u_2 - v_2)
\]  \hspace{1cm} (8.2.6) \hspace{1cm} (8.2.7)

where \( \sigma \) is the electrical conductivity.

Initial boundary conditions are

\[
u_2 = 0 \text{ at } t < 0 \text{ for all } z, \quad v_2 = 0 \text{ at } t < 0 \text{ for all } z \\
P_{\theta z} \big|_{z=0} = \mu \left( \frac{\partial u_2}{\partial z} \right)_{z=0} \text{ is prescribed as function of } r \text{ and } t \text{ for } r < a, \ t > 0.
\]  \hspace{1cm} (8.2.8) \hspace{1cm} (8.2.9)

where \( P_{\theta z} \) is the shearing stress.

\[
u_2 \to 0 \text{ as } z \to \infty \\
u_2 \big|_{r=a} = 0, \ z > 0
\]  \hspace{1cm} (8.2.10) \hspace{1cm} (8.2.11)

Introducing the following non dimensional quantities

\[
u = (u_2a)/\nu, \ v = (v_2a)/\nu, \ t_1 = t/\tau, \ r_1 = r/(\nu \tau)^{\frac{1}{2}}
\]

\[
z_1 = z/(\nu \tau)^{\frac{1}{2}}, \ a_1 = a/(\nu \tau)^{\frac{1}{2}}, \ f = (mN_0)/\rho \ \text{(mass concentration of dust particles)}, \ \tau = m/K \ \text{(relaxation time of dust particles)} \text{ in (8.2.6) to (8.2.11), we get (dropping suffices)}
\]
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} + f. (v-u) - M^2 u \quad \text{...(8.2.12)}
\]

\[
\frac{\partial v}{\partial t} = u - v \quad \quad \quad \quad \text{...... (8.2.13)}
\]

\[u = 0 \text{ at } t < 0 \text{ for all } z, \ v = 0 \text{ at } t < 0 \text{ for all } z \quad \text{...... (8.2.14)}\]

\[p_{\theta z}|_{z=0} = \frac{\partial u}{\partial z}|_{z=0} = F(r). \delta(t) \text{ (prescribed)} \quad \text{...... (8.2.15)}\]

\[u \to 0 \text{ as } z \to \infty \quad \text{...... (8.2.16)}\]

\[u|_{r=a} = 0, \ z > 0 \quad \text{...... (8.2.17)}\]

where \(M = Bo \left( \frac{r \sigma}{\rho} \right)^{1/2} \) is the Hartmann number and \(\delta(t)\) is the Dirac's delta function.

### 8.3 Method of Solution

We solve the present problem by using technique of Laplace transform. Taking Laplace transform of equations (8.2.12), (8.2.13) and using (8.2.14), we get

\[
pu = \frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} - \frac{\bar{u}}{r^2} + \frac{\partial^2 \bar{u}}{\partial z^2} + f. (\bar{v}-\bar{u}) - M^2 \bar{u} \quad \text{......(8.3.1)}
\]

\[
p\bar{v} = \bar{u} - \bar{v} \quad \quad \quad \quad \quad \text{...... (8.3.2)}
\]

where \(\bar{u} = \int_0^\infty u \ e^{-pt} \ dt, \ \bar{v} = \int_0^\infty v \ e^{-pt} \ dt, \ \text{Re}(p) > 0\)

Eliminating \(\bar{v}\) from (8.3.1) and (8.3.2) we get

\[
\frac{\partial^2 \bar{u}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{u}}{\partial r} - \frac{\bar{u}}{r^2} + \frac{\partial^2 \bar{u}}{\partial z^2} - [p\left(\frac{f}{p+1}\right) + 1] + M^2] \bar{u} = 0 \quad \text{...... (8.3.3)}
\]
The solution of the above equation (8.3.3) can be expressed in the form

$$
\bar{u}(r, z, p) = \sum_{n=1}^{\infty} \bar{u}_n
$$

where

$$
\bar{u}_n = J_1(a_n r) \bar{\phi}_n(z, p)
$$

and $a_n$ can be determined from the equation (8.2.17) as the positive root of

$$
J_1(a_n a) = 0
$$

On substituting the value of $\bar{u}_n$ from (8.3.5) in (8.3.3) we get

$$
\frac{d^2\bar{\phi}_n}{dz^2} - \left[ a_n^2 + M^2 + p \left( \frac{f}{p+1} + 1 \right) \right] \bar{\phi}_n = 0
$$

Solution of (8.3.7) subject to the Laplace transform of the condition (8.2.16) is

$$
\bar{\phi}_n = A_n \exp \left[ -z \left( a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right)^{\frac{1}{2}} \right]
$$

where $A_n$ is independent of $z$ for all $n$.

From (8.2.15) transformed shearing stress on the surface is given by

$$
\bar{P}_{\theta z} \bigg|_{z=0} = F(r), \quad r \leq a
$$

$$
= \sum_{n=1}^{\infty} C_n J_1(a_n r)
$$

where

$$
C_n = 2a^{-2} [J_2(a_n a)]^{-2} \int_0^a r F(r) J_1(a_n r) dr
$$
Again from (8.3.4), (8.3.5) and (8.3.8) we get

\[ \overline{p}_z = - \frac{1}{n!} \int_1^n (\alpha_n r) A_n \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \ldots \ldots (8.3.11) \]

we now compare the expressions (8.3.9) and (8.3.11) to obtain

\[ A_n = - C_n \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \ldots \ldots (8.3.12) \]

The expression for velocity becomes

\[ u = - \frac{1}{n!} \int_1^n C_n J_1 (\alpha_n r) \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \cdot \exp \left\{ -z \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \right\} \ldots \ldots (8.3.13) \]

where \( L^{-1} \) is the inverse Laplace transformation operator.

Inverse Laplace transform of (8.3.13) presents some difficulties and we restrict ourselves to calculate the velocity expression for large values of time 't' only. For large time t, p > 1 and

\[ \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \approx \left[ a_n^2 + M^2 + p \left( 1+f \right) \right]^{\frac{1}{2}} \]

Then

\[ L^{-1} \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}}. \exp \left\{ -z \left[ a_n^2 + M^2 + p \left( 1 + \frac{f}{1+p} \right) \right]^{\frac{1}{2}} \right\} \]

\[ = \frac{1}{(1+f)^{\frac{1}{2}}} \exp \frac{-(a_n^2 + M^2)t}{1+f} \cdot \left\{ (\pi t)^{-\frac{1}{2}} e^{-z^2(1+f)/(4t)} \right\}, \]
Equation (8.3.13) is then given by

\[ u = - \frac{1}{[\pi t (1+f)]^{\frac{1}{2}}} \exp \left[ -z^2 \frac{(1+f)}{(4t)} \right] \]

\[ \times \sum_{n=1}^{\infty} C_n J_1 (a_n r) \exp \left[ - \frac{(a_n^2 + M^2)t}{1 + f} \right] \] .... (8.3.14)

This solution satisfies initial and boundary conditions given by (8.2.14) to (8.2.17)

8.4 PARTICULAR CASES

CASE-1 : Motion due to impulsive shearing force applied within a circular area on the surface :

We take

\[ F(r) = \varepsilon r \text{ for } 0 \leq r \leq b \]
\[ = 0 \text{ for } b < r < a \] ....(8.4.1)

Relation (8.4.1) corresponds to the situation where the applied force is acting within a circular area \( r = b \), the rest of the surfaces being kept free from the impulse.

From (8.3.10) we get

\[ C_n = 2\varepsilon b^2 J_2^{-2} (a_n a) J_2 (a_n b)/(a_n a^2) \]

on substituting the value of \( C_n \) in (8.3.14), we get the expression for velocity as
u = \frac{-2eb^2}{a^2 \left[ \pi t \left( 1+f \right) \right]^{\frac{1}{2}}} \cdot \exp \left[ - \frac{z^2 \left( 1+f \right)}{4t} \right] \cdot \exp \left[ \frac{-M_r t}{1+f} \right] \\
\times \sum_{n=1}^{\infty} J_2(a_n b) \frac{J_{n-2}(a_n a) J_1(a_n r) a_n^{-1}}{J_2(a_n a) J_1(a_n b) a_n^{-1}} \cdot \exp \left[ \frac{-a_n^2 t}{1+f} \right] \ldots \ldots (8.4.2)

In absence of magnetic field (M=0), the expression for velocity profile (equation (8.4.2)) becomes same as was deduced by Mandal [9] and in case when the fluid is clean (f=0) and M=0 the expression (8.4.2) becomes same as was deduced by Bhattacharyya [3].

CASE-2: Flow due to applied impulsive force distributed over the circumference of the circle r = b; b<a:

We take

F(r) = s \delta(r - b) \ldots \ldots (8.4.3)

where s is constant and \delta is Dirac delta function

From (8.3.10) we get

C_n = 2 a^{-2} s b J_1(a_n b) J_{n-2}(a_n a) \ldots \ldots (8.4.4)

On substituting the value of C_n in (8.3.14) we get the expression for velocity as

u = \frac{-2bs}{a^2 \left[ \pi t \left( 1+f \right) \right]^{\frac{1}{2}}} \cdot \exp \left[ - \frac{z^2 \left( 1+f \right)}{4t} \right] \exp \left( \frac{-M_r t}{1+f} \right) \\
\times \sum_{n=1}^{\infty} J_1(a_n r) J_1(a_n b) J_{n-2}(a_n a) \exp \left( \frac{-a_n^2 t}{1+f} \right) \ldots \ldots (8.4.5)
In absence of magnetic field \((M=0)\), the expression for velocity (equation (8.4.5)) is same as that deduced by Mandal [9]. Again, when the fluid is clean and there is no magnetic field, that is, \(f = 0\) and \(M = 0\), the expression (8.4.5) becomes same as was deduced by Bhattacharyya [3].

8.5 DISCUSSION OF THE RESULTS

In order to illustrate the effects of mass concentration of dust particles on the flow field in presence of magnetic field and the effects of the transverse magnetic field on the velocity, numerical calculations are carried out for suitable values of the parameters entering the problem and depicted in figures (1) to (4). From figures 1 and 2 it is observed that magnitude of velocity of dusty fluid increases with the increase of mass concentration of dust particles for a fixed value of the Hartmann number \(M\). From figures (3) and (4) it is concluded that magnitude of velocity of dusty fluid decreases as the strength of the magnetic field increases for a fixed value of \(f\).
FIG. 1: VELOCITY DISTRIBUTION OF DUSTY FLUID FOR DIFFERENT VALUES OF $f$ WHEN $t=1$, $\epsilon=1$, $z=1$, $a=1$, $b=0.5$, $M=1$
FIG. 2: VELOCITY DISTRIBUTION OF DUSTY FLUID FOR DIFFERENT VALUES OF $f$ WHEN $t = 1, s = 1, z = 1, a = 1, b = 0.5, M = 1$. 

$-u \times 10^5$ 

$r$
FIG. 3: VELOCITY DISTRIBUTION OF DUSTY FLUID FOR DIFFERENT VALUES OF M WHEN $f = 0.6, t = 1, \epsilon = 1, z = 1, a = 1, b = 0.5$
FIG. 4: VELOCITY DISTRIBUTION OF DUSTY FLUID FOR DIFFERENT VALUES OF M WHEN \( f = 0.6, t = 1, s = 1, z = 1, a = 1, b = 0.5 \)
REFERENCES


