INTRODUCTION
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This thesis deals with some problems in the geometric theory of gravity. As is well-known, ever since Einstein formulated his General Theory of Relativity the gravitational interactions have found a new meaning. The General Theory of Relativity is a purely geometric theory of gravity in which matter dynamics is prescribed by the geometry of space-time or conversely the geometry is determined by matter. The large scale structure of the universe which is governed by gravitation is thus determined by the geometry of the universe which in turn is the result of space-time geometries of the various objects present in it. Thus we have in General Relativity the problem of solutions of different types of field equations that determines the geometry of space-time created by various objects. The solutions obtained thus far are in excellent agreement with all the present day experimental data. Still attempts are being constantly made to generalise this theory or to enrich it with new ideas so as to give gravitational interactions a character similar to the other three types of interactions viz. strong, weak and electromagnetic interactions. The attempts are made in a number of ways. One way is to modify it suitably to make it work efficiently in the microscopic world of elementary particles. The Einstein-Cartan theory is the result of such an attempt. This theory is based on the realisation of the fact that at the microscopic level, the energy-momentum
tensor alone is no longer sufficient to characterise dynamically the matter sources but also the spin density tensor arising out of the spins of the randomly oriented microparticles is needed. In quantum gravity we write the quantum physics in a generally co-variant language and then analyse how the quantum phenomena operate in presence of gravitational field or we investigate the quantum equivalent of Einstein's equations just as quantum electro-dynamics deals with the quantum analogue of Maxwell-Lorentz equations. Super gravity is another discipline which stems out of it.

The dimensions of space-time with which we deal in General Relativity is four-three space-dimensions with one dimension of time. This is because the present day universe in which we live in is four-dimensional. But recently attempts are being made by a sections of workers to recast the theory in a higher dimensional space-time. One reason which inspires this motivation is the considerable success achieved in recent times in solving the long-standing problems concerning consistency of general relativity and quantum mechanics by considering the space-times having dimensions higher than four. The recent emergence of ten-dimensional anomaly-free superstring theory as the most promising theory having the potential to lead us a step closer towards unification of four forces of nature and the success of ten-dimensional N=1 Yang-Mills supergravity in its field theory limit have given the studies in higher dimensional physics a new degree of importance. Since the finest of the instruments we have developed so far cannot visualise a
space-time having dimensions more than four, the size of the remaining dimensions should be of the order of Planck length as in any Kaluza-Klein type of theories. A Calabi-Yan compactification is thus favoured from phenomenological considerations. Yoshimura, Koikawa, Myers and Perry have made important contributions to this field of study.

In this thesis we have presented solutions in four dimensional relativity as well as in higher dimensional relativity. Accordingly the thesis is divided into three parts. Part I contains four papers in which we present solutions in four-dimensional space-time. Part II contains seven papers all worked out in higher dimensional space-times. The part III contains one paper worked out in four-dimensional space-time but we keep it in a different category due to its different nature.

The first two papers in Part I deal with charged matter and a one-parameter group of conformal motions. Here we study the consequences of the existence of a one-parameter group of conformal motions for charged matter (both isotropic and anisotropic) in the context of general relativity. In the first paper we have considered two types of such conformal motions and obtained altogether five analytical solutions of Einstein-Maxwell equations for a spherically symmetric, static distribution of charged matter. Two of these solutions correspond to perfect fluid, another two to anisotropic matter and the rest to charged dust. Now as has been pointed out by
Shvartsman\textsuperscript{7} that stellar objects may acquire charge from the environment and may become charged and by Ruderman\textsuperscript{8} and Canuto\textsuperscript{9} that matter distribution in the interior of some stars may be anisotropic, some of the solutions presented by us may have some astrophysical significance. In the second paper we have presented some illustrative applications of our solutions and have shown that at least one of these solutions may have astrophysical implications. In the third paper we have taken up the problems of neutron stars. Inside a neutron star the Einstein and Yukawa fields operate and the constituent particles have spin. Hence it would be appropriate to apply the Einstein-Cartan theory to neutron stars taking into account the Yukawa field. This is the subject matter of that paper. We have presented there two solutions one Schwarzschild-like and the other Tolman IV-like. We have also considered the role played by torsion in halting the gravitational collapse of a massive body and have shown that the torsion plays a significant role in the process. In the last paper of Part I we have presented a new form of Reissner-Nordström metric for a charged particle. Using a simple transformation following Dirac we have shown that the metric has the only irremovable singularity at $r=0$. This form of the metric is found to be suitable to study the problem of quantum fluctuations near a charged blackhole singularity.

Part II of this thesis is devoted to the problems of higher dimensions. Here we have generalised a number of well-known solutions in four dimensional space-time to
N-dimensions. We have also presented a few new solutions. As for generalisation we have developed interior Schwarzschild-like solutions in higher dimension. We have also found the N-dimensional extension to Florides\textsuperscript{10} interior solution with a cosmological constant and Kottler's exterior solution\textsuperscript{11}.

N-dimensional extensions are also made to the Krori-Paul solution\textsuperscript{12} describing a charged sphere and to the metric found by Berger, Hojman and Santamarina for static spherically symmetric matter distributions. We have also presented exact solutions of N-dimensional vacuum Einstein field equations with a Bianchi type-I metric.

A Marder-like solution in higher dimension is also obtained by us. The physical validity of all these solutions has been considered in each case and it is found that our solutions revert to their respective four-dimensional forms once we put the number of dimensions N=4. As for new solutions we have found a few exact solutions for spherically symmetric matter distribution in higher dimensions. These solutions are singularity free and they satisfy the necessary physical condition to claim as realistic solutions.

In Part III we have taken up the problem of quantum interference effects in some fields. We have considered the effects for non-spinning particles in Einstein and Yang-Mills fields and for spinning particles in Einstein and Einstein-Cartan fields and have shown that Bohm-Aharnov-like quantum interference effects in Yang-Mills and Einstein fields follow immediately from a unified gauge-theoretic treatment.
References


3. P.Candelas, G.T.Horowitz, A.Strominger


9. V.Canuto, Neutron stars, general review, Solvay conference
   on Astrophysics and Gravitation, Brussels Belgium (1973).

