Chapter 8

Steady Flow and Heat Transfer between Two Rotating Discs of different Transpiration at Constant Heat Flux

8.1 Introduction

The flow of an incompressible fluid over a single infinite rotating disc was first studied by Von Karman (98) and Bodewadt (147). Batchelor (100) applied the solution of Von Karman and Bodewadt for the case of two infinite rotating discs. Stewartson (99) obtained the approximate solutions for large and small values of Reynolds numbers. Stuart (101) investigated the flow of a single rotating disc of infinite radius with uniform suction at the disc and obtained numerical solution for both large and small values of suction parameter. Pearson (148), Lance and Rogers (102) and Mellor (149) et al. obtained the numerical solution of the problem of Stewartson. Rao and Gupta (104) extended the Stuart's problem by considering the effect of transverse magnetic field for large suction Reynolds number. Loper and Benton (113) studied the spin up of electrically conducting fluid. Gaur (150) discussed the problem of Stewartson by considering the effect of porosity. Narayan and Rudraih (103) studied the flow of a viscous incompressible fluid between two co-axial circular discs with uniform suction at the stationary disc and they obtained the solutions for large and small suction Reynolds number. Wilson (151) studied the Narayan and Rudraih's problem only by changing the application of suction in either one of the discs. Chawla (106,107) studied hydromagnetic spin up and flow induced by torsionally oscillating disc. Khare (105) studied the Narayan and Rudraih's problem for electrically conducting viscous fluid in presence of transverse magnetic field. Hossain and Rahman (152) studied the problem of Gaur by considering transverse magnetic field.
In all these above investigations heat transfer aspect has not been considered. Purohit and Patidar (153) studied the steady flow and heat transfer of a viscous incompressible fluid between two infinite rotating discs for small Reynolds number. They have considered the rate of suction to be different from the rate of injection. Dhanak (154) studied the effects of uniform suction on the stability of flow on a rotating disc. Recently Das and Aziz (155) extended the problem studied by Purohit and Patidar by introducing a transverse magnetic field.

The aim of the present paper is to investigate the effect of constant heat flux at the lower disc on the flow of a viscous incompressible fluid between the two parallel porous rotating discs of infinite extent. The governing equations have been solved with perturbation technique taking cross flow Reynolds number as the perturbation parameter.

8.2 Mathematical Analysis

Consider the flow of a viscous incompressible fluid between two co-axial parallel porous discs of infinite radius. The rate of suction at the upper disc is taken different from the rate of injection at the lower disc. The discs are rotating with different angular velocities. There is a constant heat flux at the lower disc while the upper disc is kept at a constant temperature $T_0$. We shall work with the cylindrical polar coordinate $(\bar{r}, \bar{\theta}, \bar{z})$. The surfaces of the discs are defined by planes $\bar{z} = 0$ and $\bar{z} = d$ respectively, the lower plate being taken as the $(r, \theta)$ plane. By symmetry, all the variables will be independent of $\bar{\theta}$. In view of above assumption the governing equations for velocity and temperature fields are

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial \bar{v}}{\partial \bar{z}} + \frac{\dot{u}}{\bar{r}} = 0$$

$$\frac{\bar{u} \frac{\partial \bar{u}}{\partial \bar{r}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{z}} - \bar{v}^2}{\bar{r}} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial \bar{r}} + \nu \left( \frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} - \frac{\bar{u}}{\bar{r}^2} \right)$$

(8.2.1)  

(8.2.2)
\[
\frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{u \partial \tilde{v}}{\partial \tilde{z}} + \frac{\tilde{u} \tilde{v}}{\tilde{r}} = \nu \left( \frac{\partial^2 \tilde{v}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} - \frac{\tilde{v}}{\tilde{r}^2} \right) \tag{8.2.3}
\]

\[
\frac{\partial \tilde{w}}{\partial \tilde{r}} + \frac{w \partial \tilde{w}}{\partial \tilde{z}} = -\frac{1}{\rho} \frac{\partial \tilde{p}}{\partial \tilde{z}} + \nu \left( \frac{\partial^2 \tilde{w}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial \tilde{w}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{w}}{\partial \tilde{z}^2} \right) \tag{8.2.4}
\]

\[
\tilde{u} \frac{\partial \tilde{T}}{\partial \tilde{r}} + \tilde{w} \frac{\partial \tilde{T}}{\partial \tilde{z}} = \alpha \left( \frac{\partial^2 \tilde{T}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial \tilde{T}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} \right) + \frac{\mu}{C_p \rho} \left[ 2 \left( \frac{\partial \tilde{u}}{\partial \tilde{r}} \right)^2 + \left( \frac{\partial \tilde{w}}{\partial \tilde{r}} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial \tilde{r}} \right)^2 + \left( \frac{\partial \tilde{u}}{\partial \tilde{z}} \right)^2 + \left( \frac{\partial \tilde{v}}{\partial \tilde{z}} \right)^2 \right] \tag{8.2.5}
\]

Boundary conditions are

\[
\tilde{z} = 0: \quad \tilde{u} = 0, \tilde{v} = m\tilde{r}\Omega, \tilde{w} = n\tilde{r}, \frac{\partial \tilde{T}}{\partial \tilde{z}} = -\frac{\tilde{Q}}{k}
\]

\[
\tilde{z} = \tilde{d}: \quad \tilde{u} = 0, \tilde{v} = \tilde{r}\Omega, \tilde{w} = \tilde{w}, \tilde{T} = \tilde{T}_o
\tag{8.2.6}
\]

Using the following non dimensional quantities

\[
r = \frac{\tilde{r}}{d}, \quad z = \frac{\tilde{z}}{d}, \quad u = \frac{\tilde{u} d}{\tilde{v}}, \quad w = \frac{\tilde{w} \tilde{d}}{\tilde{v}}, \quad Pr = \frac{\tilde{P}}{c \left( \frac{\nu}{d} \right)^2}, \quad Gr = \frac{c \rho d^3 \tilde{Q}}{c \nu^2 \tilde{v} k}
\]

\[
\tilde{T} = \frac{c (\tilde{T} - \tilde{T}_o)}{\tilde{Q} d / k}
\tag{8.2.7}
\]

in above equations we get

\[
\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z} + u = 0 \tag{8.2.8}
\]

\[
u \frac{\partial u}{\partial r} + \frac{u \partial \tilde{v}}{\partial \tilde{z}} + \frac{1}{\tilde{r}} \frac{\partial \tilde{u}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{u}}{\partial \tilde{z}^2} \frac{u}{\tilde{r}^2} \tag{8.2.9}
\]

\[
u \frac{\partial \tilde{v}}{\partial r} + \frac{w \partial \tilde{v}}{\partial \tilde{z}} + \frac{uv}{\tilde{r}} = \frac{\partial^2 \tilde{v}}{\partial \tilde{r}^2} + \frac{1}{\tilde{r}} \frac{\partial \tilde{v}}{\partial \tilde{r}} + \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} \frac{v}{\tilde{r}^2} \tag{8.2.10}
\]
\[
\frac{\partial w}{\partial r} + \frac{w}{\partial z} = -\frac{\partial p}{\partial z} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \tag{8.2.11}
\]

\[
\frac{\partial T}{\partial r} + \frac{w}{\partial z} = \frac{1}{Pr} \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{1}{Gr} \left[ 2 \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial u}{\partial r} - \frac{\partial u}{\partial z} \right)^2 \right] + \left( \frac{\partial v}{\partial r} - \frac{\partial v}{\partial z} \right)^2 \tag{8.2.12}
\]

where \( c \) is a constant, \( Pr \) is Prandtl number and \( Gr \) is a kind of heat flux parameter.

The corresponding boundary conditions (8.2.6) now reduce to the following:

\[
z = 0: ~ u = 0, \ v = m \lambda R, \ w = nR, \ \frac{\partial T}{\partial z} = -c
\]

\[
z = 1: ~ u = 0, \ v = \lambda R, \ w = R, \ T = 0
\tag{8.2.13}
\]

where \( \lambda = d \Omega / \nu \), dimensionless rotational parameter.

In order to solve the equations from (2.8) to (2.12), we introduce the following variables

\[
u = -\left( \frac{r R}{2} \right) F'(z), \ v = r \lambda G(z), \ w = RF(z),
\]

\[
p = p(z) + \frac{1}{2} \lambda^2 r^2, \ T = \phi(z) + r \psi(z) + r^2 H(z)
\tag{8.2.14}
\]

Here the expressions of \( \nu \) and \( w \) satisfy the equation (8.2.8). Also the equations (8.2.9) to (8.2.12) reduce to

\[
2RF'' - 2R^2 F F' + R^2 F' F'' - 4R^2 \lambda^2 G^2 + 4A \lambda^2 = 0
\tag{8.2.15}
\]

\[
G'' + RF'G - RG'F = 0
\tag{8.2.16}
\]

\[
RF'' - R^2 F' F'' - p' = 0
\tag{8.2.17}
\]

\[
\phi'' - R Pr \ F \phi' + 4H + \frac{3 Pr}{Gr} R^2 F'^2 = 0
\tag{8.2.18}
\]

\[
\psi'' - RF \psi' - \frac{1}{2} RF' \psi = 0
\tag{8.2.19}
\]

\[
H'' - R Pr FH' + R Pr HF'' + \frac{Pr}{Gr} \left( \lambda^2 G'^2 + \frac{F'^2}{4} \right) = 0
\tag{8.2.20}
\]

\[
\psi = 0
\tag{8.2.21}
\]
The corresponding boundary conditions now become

\begin{align*}
z = 0: & \quad F' = 0, G = m, F = n, \phi' = -c, H' = 0 \\
\text{and} \quad z = 1: & \quad F' = 0, G = 1, F = 1, \phi = 0, H = 0 \quad (8.2.22)
\end{align*}

Taking suction Reynolds number R as small parameter the unknown functions F, G, \phi, H and constant A are expressed in a power series of R as

\begin{align*}
F(z) &= \sum_{n=0}^{\infty} R^n F_n(z) \\
G(z) &= \sum_{n=0}^{\infty} R^n G_n(z) \\
\phi(z) &= \sum_{n=0}^{\infty} R^n \phi_n(z) \quad (8.2.23) \\
H(z) &= \sum_{n=0}^{\infty} R^n H_n(z) \\
A &= \sum_{n=0}^{\infty} R^n A_n
\end{align*}

Substituting equations (3.10) into equations (3.2) to (3.9) and comparing the coefficients of like powers of R we get

\begin{align*}
A_0 &= 0 \quad (8.2.24) \\
F_0'' + 2 \lambda^2 A_1 &= 0 \quad (8.2.25) \\
2F_1''' - 2F_0 F_0'' + F_0'^2 - 4 \lambda^2 G_0^2 + 4 \lambda^2 A_2 &= 0 \quad (8.2.26) \\
F_2''' - F_0 F_1'' - F_1 F_0'' + F_0' F_1' - 4 \lambda^2 G_0 G_1 + 2 \lambda^2 A_3 &= 0 \quad (8.2.27) \\
G_0'' &= 0 \quad (8.2.28) \\
G_1'' - F_0 G_0' + F_0 G_0 &= 0 \quad (8.2.29) \\
G_2'' - F_0 G_1' + F_0' G_1 + G_0 F_1' - G_0' F_1 &= 0 \quad (8.2.30) \\
\phi_0'' + 4H_0 &= 0 \quad (8.2.31) \\
\phi_1'' - \text{Pr} F_0 \phi_0' + 4H_1 &= 0 \quad (8.2.32)
\end{align*}
\[ \phi''_2 - \Pr F_0 \phi'_1 - \Pr \phi'_0 F_1 + 4H_2 + \frac{3\Pr}{Gr} F'_0 = 0 \]  
(8.2.33)

\[ H''_0 = 0 \]  
(8.2.34)

\[ H'_1 - \Pr F_0 H'_0 + \Pr F'_0 H_0 = 0 \]  
(8.2.35)

\[ H''_2 - \Pr F_0 H'_1 + \Pr F'_0 H'_1 + \Pr H_0 F'_1 - \Pr F_0 F'_0 \frac{1}{Gr} \left[ \frac{1}{4} F_0'' + \lambda^2 G_0'^2 \right] = 0 \]  
(8.2.36)

The boundary conditions now reduce to

For \( z = 0 \): \( F_0 = n, G_0 = m, F_1 = F_2 = 0, F'_0 = F'_1 = F'_2 = 0, G_1 = G_2 = 0, \)
\[ H'_0 = H'_1 = H'_2 = 0, \phi'_0 = -c, \phi'_1 = \phi'_2 = 0 \]

For \( z = 1 \): \( F'_0 = F'_1 = F'_2 = 0, G_0 = 1, G_1 = G_2 = 0, F_0 = 1, F_1 = F_2 = 0, \)
\[ \phi'_0 = \phi'_1 = \phi'_2 = 0, H_0 = H_1 = H_2 = 0 \]  
(8.2.37)

Using the above boundary conditions we obtain the following solutions for velocity and heat transfer components

\[ G_0(z) = m + (1 - m)z \]
\[ F_0(z) = n + (1 - n)(3 - 2z)z^2 \]
\[ H_0(z) = 0 \]
\[ \phi_0(z) = c(1 - z) \]
\[ G_1(z) = \frac{1}{20} (1 + mn + 9m - 11n)z + \frac{1}{2} n(1 - m)z^2 + (1 - n) \left( \frac{3m - 1}{4} z + \frac{1 - m}{5} z^2 - m \right) z^3 \]
\[ F_1(z) = \frac{1 - n}{2} \left( n(2 - z) + \frac{1 - n}{35} (2z - 7)z^3 \right) z^2 + \frac{\lambda^2}{30} \left[ -10m^2 + (1 - m)^2 z^2 + 5m(1 - m)z \right] z^3 - \frac{1}{3} A_2 \lambda^2 z^3 + \frac{1}{4} C_1 z^2 \]
\[ H_1(z) = 0 \]
\[ \phi_1(z) = c \frac{P}{20} \left[ 3 + 7n - 10nz^2 - (1 - n)(5z^4 - 2z^5) \right] \]

\[ G_2(z) = c_2 z + \frac{1}{2} B_1 z^2 + \frac{1}{6} B_2 z^3 + \frac{1}{12} B_4 z^4 + \frac{1}{20} B_4 z^5 + \frac{1}{30} B_4 z^6 + \frac{1}{42} B_5 z^7 + \frac{1}{56} B_5 z^8 + \frac{1}{72} B_6 z^9 \]

\[ \begin{align*}
F_2(z) &= \frac{1}{2} C_5 z + \frac{z^3}{6} \left( \frac{1}{2} nc_1 - 2 A_3 \lambda^2 \right) + \frac{K_1}{24} z^4 + \frac{K_2}{60} z^5 + \frac{K_3}{120} z^6 + \frac{K_4}{210} z^7 + \frac{K_5}{336} z^8 \\
&\quad + \frac{K_6}{504} z^9 + \frac{3}{175} z^{10} \left( \frac{1}{2} - \frac{z}{11} (1 - n) \right) \end{align*} \]

\[ H_2(z) = C_4 (1 - z^2) + 8 C_5 (1 - 3z^2 + 4z^3 - 2z^4) \]

\[ \phi_2(z) = C_6 + \alpha_1 z^2 + \frac{\alpha_2}{6} z^3 + \frac{\alpha_3}{12} z^4 + \frac{\alpha_4}{20} z^5 + \frac{\alpha_5}{30} z^6 - \frac{\alpha_6}{42} z^7 + \frac{\alpha_7}{4} z^8 \left( \frac{1}{2} - \frac{z}{9} \right) \]

8.3. Discussions on the results:

The wall temperature \( T(0) \) is given by

\[ T(0) = \phi(0) + r^2 H^2(0) = c \]

The coefficients of heat transfer ie Nusselt numbers at the two discs are defined as

\[ Nu = - \left| \frac{K}{Q} \frac{\partial T}{\partial z} \right| \bigg|_{z=0,1} = - \left| \frac{\partial T}{c} \right| \bigg|_{z=0,1} \]

\[ = 1, \quad \text{at} \ z = 0 \quad \text{and} \]
\[ Nu = 1 + \frac{1 + n}{2} R Pr - \frac{R^2}{c} \left( 2\alpha_1 - \frac{\alpha_2}{2} + \frac{\alpha_3}{3} + \frac{\alpha_4}{4} + \frac{\alpha_5}{5} - \frac{\alpha_6}{6} + \frac{3\alpha_7}{4} - (2C_4 + 16C_5)r^2 \right) \]

at \( z = 1 \)

The skin friction coefficient at the two discs are defined as

\[ C_f = \frac{\tau_z}{\mu V/d^2} \bigg|_{z=0,1} = \frac{d^2}{d\xi^2} \frac{\partial u}{\partial \xi} \bigg|_{z=0,1} = \frac{\partial u}{\partial \xi} \bigg|_{z=0,1} = \frac{-rR}{2} \left( R \left( \frac{c_1}{2} + Rec_3 \right) + 6(1 - n) \right) \] when \( z = 0 \)

and

\[ C_f = -\frac{rR}{2} \left[ 6(n-1) + R \left\{ -\frac{9}{5} (1-n)^2 + \frac{2\lambda^2}{3} \left( 1 + m + m^2 \right) - 2A_2\lambda^2 + \frac{c_1}{2} \right\} \right] + R^2 \left\{ c_3 + \frac{n}{2}c_1 - 2A_2\lambda^2 + \frac{1}{2}K_1 + \frac{1}{3}K_2 + \frac{1}{4}K_3 + \frac{1}{5}K_4 + \frac{1}{6}K_5 + \frac{1}{7}K_6 + \frac{3}{5}(1-n)^2 \right\} \] when \( z = 1 \).

8.3.1 First and second order axial velocities (\( F_1, F_2 \)):

It is clear from the equations (8.2.8) to (8.2.11) that in the present forced convection flow the velocity components do not depend on the temperature field or in other words, the parameters \( Pr \), Prandtl number and \( Gr \), the heat flux parameter. The first order axial velocity \( F_1(z) \) against \( z \) has been shown in Figure 1 for various values of the parameter \( m \) (the rotation ratio of the discs), \( n \) (injection- suction ratio at the discs) and \( \lambda \) (the rotational parameter). From the graphs (i, ii and iii), it is observed that \( F_1(z) \) decreases when \( \lambda \) or \( m \) increases. From the graphs (i and iv) it is seen that \( F_1(z) \) increases with \( n \). Further it is interesting to note that for \( n = 0.5, F_1(z) \) becomes negatives through out the fluid region between the discs. It signifies that when the suction velocity at the upper discs is far greater than the injection velocity at the lower disc, \( F_1(z) \) becomes negatives. Also from the graph
it is clear that when the lower disc is kept stationary while the upper disc continues to rotate, $F_1(z)$ decreases throughout the fluid region between the discs. It also shows that $F_1(z)$ is symmetrical about $z = 0.5$ plane and in this region it has the maximum distribution showing the maximum axial velocity.

Figure 5 shows that the second order axial velocity, $F_2(z)$ is negative below the $z = 0.5$ plane when suction rate at the upper disc is less than the injection rate at the lower disc for any rotational velocity of the lower disc. Again $F_2(z)$ decreases when rotational parameter $\lambda$ (graphs(i),(ii)) or rotational velocity of the lower disc decreases (graphs(i),(iii)). There is an opposite character of $F_2(z)$ in all cases above the plane $z = 0.5$ of the fluid region. On the other hand $F_2(z)$ is negative in the whole fluid region when the lower disc is stationary with equal suction-injection rate.

8.3.2 First and second order Radial velocities ($F_1', F_2'$):

The first and second order radial velocities are shown against $z$ in Fig.2 and Fig.6 for various values of parameters. It is seen from the Fig.2 that the fluid moves radially away from the axis of rotation near the lower disc, while near the upper disc it moves towards the axis when the suction rate at the upper disc is larger than the injection rate at the lower disc (iv). But opposite pattern is seen when the injection rate is larger than suction rate (i,ii,iii). Also it is observed that the first order radial velocity accelerate near both discs when the rotational parameter $\lambda$ increases (i,ii) or rotational velocity of the lower discs is less than that of upper disc (iii). It is also seen that in the middle part of the channel there is no radial flow. Again $F_1'(z)$ is very small (v) for all cases for no motion of the lower disc.

Fig.6 shows that the second order axial velocity $F_2'(z)$ is negative and positive (i,ii,iii,iv) for smaller and larger suction rate respectively. However it changes direction at some equal distances in the channel. It also shows that $F_2'(z)$ has small magnitude (v) when the lower disc is stationary at equal suction-injection. $F_2'(z)$ has the maximum distribution (iii) for smaller rotational and larger injection velocity at the lower disc.
8.3.3 First and second order transverse velocities (\(G_1, G_2\)):

Fig. 3 shows that the first order transverse velocity \(G_1(z)\) has maximum distribution at the middle of the channel. It also shows that \(G_1(z)\) is negative for all cases except when the suction rate at the upper disc is greater than the injection rate at the lower disc.

Fig. 7 shows that the second order transverse velocity \(G_2(z)\) increases graphs (ii, iii) when the rotational parameter \(\Lambda\) or rotational velocity of the lower disc decreases at the lower portion of the fluid region. From this middle part \(G_2(z)\) becomes negative. \(G_2(z)\) decreases graphs (i, v) when the lower disc is stationary with equal suction-injection rate. On the other hand \(G_2(z)\) decreases graphs (i, iv) and then increases when the suction rate is greater than the injection rate.

8.3.4 Temperature field:

The temperature field components are shown in figures 4, 8, 9. The first order temperature field \(\varphi_1(z)\) is shown in Fig. 4 against \(z\). From this figure we observe that \(\varphi_1(z)\) is independent of rotational velocity i.e. of \(m\) of the discs. It also shows that \(\varphi_1(z)\) decreases (graphs i, iii) with injection rate \((n)\) and increases (graphs i, ii) with Prandtl number \((Pr)\). Also from the expression of \(\varphi_0(z)\) we see that the zeroth order temperature field decreases linearly with the altitude of the upper disc. We observe that \(\varphi_1(z)\) and \(\varphi_0(z)\) are symmetrically of opposite signs for all cases with the opposite signs of \(c\).

In Fig. 8 the second order temperature field \(H_2(z)\) is shown. It shows that \(H_2(z)\) increases (graphs (i), (vi)) with \(Pr\) and decreases (graphs (i), (ii)) when \(Gr\) increases. Again \(H_2(z)\) decreases with rotational parameter \(\Lambda\) (graphs (i), (iii)), injection rate \((n)\) (graphs (i), (iv)) and also when the lower disc is fixed with same suction-injection rate \((n)\) (graphs (i), (v)). It may be mentioned here that zeroth order and first order temperature fields \(H_0(z)\) and \(H_1(z)\) vanishes identically.

Figure 9 shows that \(\varphi_2(z)\) decreases when \(c\) becomes negative for all cases in the fluid region and the differences diminishes gradually towards the upper disc. It also shows that \(\varphi_2(z)\) increases with Prandtl number \((Pr)\) (graphs (i), (vi)), rotational parameter.
8.3.4 The rate of heat transfer:

In Fig. 10 the rate of heat transfer coefficient in terms of Nusselt number (Nu) is shown at a unit radial distance i.e. r = 1 from the axis of rotation. It shows that when c = 1, the Nusselt number Nu increases with m, n or \( \lambda \) (graphs i, ii, iii, iv)). But Nu decreases when Gr increases. Also Nu is equal in amplitude but opposite in sign when c = 1 and c = -1. It also shows that the rate of heat transfer does not depend on the rotational velocity of the lower disc (graphs ii, iii) i.e. on m.

Appendix

\[
k_1 = 6n^2(1-n) + 2n\lambda^2 (m^2 - A_1) + \frac{m\lambda^2}{5}(1 + mn + 9m - 11n)
\]

\[
k_2 = \frac{\lambda^2}{5}(1-m)(21mn + 9m - 11n + 1) - 6(1-n)n^2
\]

\[
k_3 = (1-n)(6n(1-n) - 2(m^2 + A_1)\lambda^2 - C_1) + \frac{8}{3}n\lambda^2 (1-m)^2
\]

\[
k_4 = 2(n-1)[9n(1-n) + \lambda^2 (m(1-m) - A_1)]
\]
Fig. 1 Profile of $F_1$ vs $z$ for various values of $m, n$ and $\lambda$.

<table>
<thead>
<tr>
<th></th>
<th>$m$</th>
<th>$n$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>III</td>
<td>0.5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>IV</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Fig. 2 Profile of $F_1^r$ vs $z$ for different values of $m, n$ and $\lambda$. 

<table>
<thead>
<tr>
<th></th>
<th>$m$</th>
<th>$n$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>2</td>
<td>0.5</td>
</tr>
<tr>
<td>III</td>
<td>0.5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>IV</td>
<td>2</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>V</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Fig. 3 $G_1(z)$ vs. $z$
Fig. 4 $\phi_1(z)$ vs. $z$; $c = 1(--), c = -1(....)$
Fig. 6 $F_2(z)$ vs. $z$
Fig. 7 $G_2(z)$ vs. $z$
Fig. 8 $H_2(z)$ vs $z$
Fig. 9 $\phi_2$ vs $z$ for $c=1$ (---) and $c=-1$ (---)
Fig. 10 Nu vs R for P=0.7; c=1(-----), c = -1 (.........)
Appendix

\[ k_1 = 6n^2 (1 - n) + 2n \lambda^2 (m^2 - A_2) + \frac{m \lambda^2}{5} (1 + mn + 9m - 11n) \]

\[ k_2 = \frac{\lambda^2}{5} (l - m)(21mn + 9m - 11n + 1) - 6(1 - n)n^2 \]

\[ k_3 = (1 - n) \{ 6n(l - n) - 2(m^2 + A_2) \lambda^2 - C_1 \} + \frac{8}{3} n \lambda^2 (1 - m)^2 \]

\[ k_4 = 2(n - 1)[9n(1 - n) + \lambda^2 \{ m(1 - m) - A_2 \}] \]

\[ k_5 = \frac{1 - n}{5} [36n(1 - n) + (1 - m)(3m + 1) \lambda^2 \]

\[ k_6 = \frac{1 - n}{15} [(1 - m) \lambda^2 - 90(1 - n)^2] \]

\[ A_1 = \frac{6(1 - n)}{\lambda^2} \]

\[ A_2 = \frac{1}{10} (3m^2 + 4m + 3) - \frac{27}{35 \lambda^2} (1 - n)^2 \]

\[ A_3 = \frac{1}{\lambda^2} \left( \frac{nc_1}{4} + \frac{K_1}{4} + \frac{K_2}{20} + \frac{K_3}{14} + \frac{K_4}{56} + \frac{K_5}{24} + \frac{K_6}{55} \right) \]

\[ B_1 = \frac{n}{20} (1 + mn + 9m - 11n) \]

\[ B_2 = n^2 (1 - m) - \frac{1}{2} mc_1 \]

\[ B_3 = \frac{1}{5} mn \left( \frac{123}{4} n - 24 \right) - \frac{1}{20} (3 + 27n) + \frac{3n}{5} (3 - 11n) - \frac{1 - m}{4} c_1 + m \lambda^2 (A_2 - m^2) \]

\[ B_4 = \frac{1 - n}{5} (36 mn - 26n + 9m + 1) + \frac{2}{3} (1 - m)(A_2 - 2m^2) \lambda^2 \]

\[ B_5 = \frac{7n}{2} (1 - n)(1 - m) - 3m(1 - n)^2 - \frac{2}{3} m(1 - m)^2 \lambda^2 \]
\[ B_6 = \frac{(1-n)^2}{10} (51m-15) - \frac{2}{15} (1-m)^3 \lambda^2 \]

\[ B_7 = \frac{(1-n)^3}{10} (28-40m) \]

\[ B_8 = -\frac{34}{35} (1-m)(1-n)^3 \]

\[ C_1 = \frac{2\lambda^2}{15} (2+m-3m^2) - \frac{2}{35} (1-n)(22n+13) \]

\[ C_2 = -\left( \frac{1}{2} B_1 + \frac{1}{6} B_2 + \frac{1}{12} B_3 + \frac{1}{20} B_4 + \frac{1}{30} B_5 + \frac{1}{42} B_6 + \frac{1}{56} B_7 + \frac{1}{72} B_8 \right) \]

\[ C_3 = A_3 \lambda^2 - \frac{1}{4} nC_1 - \frac{1}{6} k_1 - \frac{1}{12} k_2 - \frac{1}{20} k_3 - \frac{1}{30} k_4 - \frac{1}{42} k_5 - \frac{1}{56} k_6 - \frac{12}{175} (1-n)^3 \]

\[ C_4 = \frac{Pr \lambda^2}{2Gr} (1-m)^2 \]

\[ C_5 = \frac{3Pr}{16Gr} (1-n)^2 \]

\[ C_6 = -\left( \alpha_1 - \frac{1}{6} \alpha_2 + \frac{1}{12} \alpha_3 + \frac{1}{20} \alpha_4 + \frac{1}{30} \alpha_5 - \frac{1}{42} \alpha_6 + \frac{7}{72} \alpha_7 \right) \]

\[ \alpha_1 = -2(C_4 + 8C_5) \]

\[ \alpha_2 = n^3 Pr^2 C \]

\[ \alpha_3 = 4C_4 - \frac{1}{4} CC_1 Pr - 480C_5 \]

\[ \alpha_4 = -CPr n(1-n)(4Pr+1) - \frac{C}{3} Pr \lambda^2 (m^2 - A_2) + 1024C_5 \]

\[ \alpha_5 = CPr \left\{ \frac{1}{2} n(1-n)(5Pr+1) - \frac{1}{6} m(1-m)\lambda^2 \right\} - 512C_5 \]

\[ \alpha_6 = \left\{ 3(1-n)^2 Pr + \frac{1}{30} (1-m)^2 \lambda^2 \right\} CPr \]

\[ \alpha_7 = \frac{CPr}{70} (1-n)^2 (35Pr+1) \]