Introduction

The algebraic system $(G, +, \circ)$ where $(G, +)$ is a group and $\circ$ is a binary operation in $G$ defined by $a \circ b = a$, for all $a, b \in G$, satisfies all the ring axioms apart from the left distributive law and the commutativity of addition. Such an algebraic system with binary operations addition and multiplication satisfying all the ring axioms except, possibly, one of the distributive laws and commutativity of addition, is a near-ring. As a natural existence of a near-ring, we have the collection of all the mappings in a group $(G, +)$ (not necessarily abelian) into itself together with the operations of pointwise addition and composition of the mappings.

In 1905, Dickson [25] did an axiomatic research which was, historically, first step towards near-rings. The beginning of 30's saw the first proper near-ring considerations. Since then the theory of near-rings has been developed much and at present it becomes a sophisticated theory with numerous applications in various areas, namely, topology, geometries, interpolation theory, group theory, polynomials and matrices. Designs are an important application of near-ring. The use of planar near-rings to get excellent balanced incomplete block designs and experimental designs is probably the best known application of near-rings to the "outside world". In recent years its connection with computer science, automata, dynamical systems, rooted trees, coding theory, cryptography etc. have also been dealt with.

The review of research and development in the subject is stated as follows:
In contrast to the instance [35] of important classes of linear dynamical systems which forms rings w.r.t. parallel and series connections, for non-linear systems one gets near-rings. In this way one can expect results on the stabilization of non-stable general system. It is important to note that the working in this direction is just on the way. Another recent application is the affirmative solution of the rigidity problem for free nilpotent groups [56]. One surprising and down-to-earth application is on what may be called as fertilizer component problems [59]. One of the forerunners along this line is the famous near-ringer G. Pilz. This has some reference to balanced incomplete block design, which has many things to discuss with planar near-rings. Graph theoretic inference on near-rings or some associated concepts is another important topic in the study of non-deterministic theory of automata, which relates with near-ring with automata and thereby naturally to computer science. Natural existence of near-rings from a certain class of semi automata came into forefront since 1993 [22]. And its emergence related with so-called syntactic near-ring. Even dihedral group plays important role in this aspect. Credit goes to near-ringer like J. R. Clay and Y. Fong et al. The role of near-rings played in Automata Theory is seen in what a group semi automata said to be reachable or connected in so-called finite state cases. The relevance of the density theorem, a powerful structure theorem for near-ring is worthy to mention in case of such study.

Near-rings are generalizations of rings. In 1988 [7] Chowdhury has introduced the notion of a new algebraic structure—the so-called Goldie module and has obtained some
interesting results analogous to those of a Noetherian module, viz. the decomposition of zero of such a module and the Artin–Rees theorem for a Goldie module in some special cases. Chowdhury et al. discussed the characterization of group structures of modules and rings with acc on annihilators in [23]. It is natural to simplify various concepts of rings to near–rings. Betsch, Beidleman, Ramakotaiah, Ligh, Clay, Satyanarayana, Chowdhury and other had generalized various concepts to near–rings. Due to non–ring character of a near–ring the results have their own beauty.

Extensive research work is being carried out on near–rings and near–ring groups in which structure theory is one area of importance. Oswald, Beidleman, Ligh, Chowdhury and other have done considerable work on various aspects of near–rings with chain conditions on annihilators in [57, 2, 36, 21].

In 70’s Oswald [58] has obtained the structure theory of near–rings in which each near–ring subgroups is principal. In recent year Pilz, Meldrum and other have obtained elegantly the relations between near–rings and automata, near–ring and dynamical system, seminear–rings rooted trees [55, 59]. W, Blackett [4] studied simple and semi–simple near–rings around 1950. S. C. Choudhury [5]. Mason [52], Mason et al. [53] and others have generalized that concept to strictly semi–simple near–rings. The development of near–rings and near–fields has matured to the point where the theory is significant, the applications are numerous and both are suitably familiar.
Survey of the works of the group of researchers already carried out in North Eastern region of India on near-ring theory:

Tamuli and Chowdhury [64] introduced the notion of a Goldie near-ring and observed visible difference between a right Goldie near-ring and a left Goldie near-ring. Due to non-ring character of near-ring it is of obvious importance to study right Goldie near-rings and left Goldie near-rings separately. In [6] Chowdhury has established some results on near-rings of right quotients of a right Goldie near-ring and on some radical characters. Among these, Goldie theorem analogue of a right Goldie near-ring [8] and on radical Goldie near-rings [9] are worthy to be mentioned. Chowdhury and Masum dealt with right Goldie near-rings extending some results of A. Oswald. The above authors presented an elegant new proof to the results in [19] what Oswald obtained in [57, 58] by considering the strongly semi-prime character of a strictly left Goldie near-ring leading to the descending chain condition on left annihilators.

Another important and interesting work of Chowdhury and Masum was on near-rings with finite spanning dimension [20] which may be considered as some extensions of what Fleury [27] and Satyanarayana obtained in [63]. S. P. decomposition of zero of an \( N \)-group with finite spanning dimension 1 is another result worthy to be mentioned. Goldie structure on weakly regular near-rings was
dealt by the above authors leading to some interesting results to his credit.

Barua [1] studied the principal subgroup near-rings and defined cancellative elements in place of inverses. He extended the unique factorization theorem of rings to near-rings with the help of g.c.d. and l.c.m. Another important work of Barua was the introduction of wreath sum in a near-ring which is the generalization of wreath product of groups in some sense. He discussed near-rings of right quotients and established some important results which are the particular cases of Graves [30] and Tewari and Seth [65] in some cases. Chowdhury, Saikia studied in [21] near-rings with the ascending chain condition on annihilators having no infinite direct sum of ideals (subgroups) with parts satisfying the ascending chain condition or the descending chain condition on its substructures. The authors introduced the notion of what is termed as strictly Artinian radical [21] in such a near-ring. $\hat{d}$-near-rings and $\hat{d}$-near-ring groups are another important notion to be mentioned. The study of $\hat{d}$ near-rings [62] helps to obtain some elegant structure theorems to see the effect of projectivity, strictly-1 semi-simple character and chain conditions. It is worthwhile to mention that chain conditions of the near-rings with the help of $\hat{d}$-character give rise to some important features on near-rings and possibly, this is another interesting aspect of such near-rings and near-ring groups with which a lot of work may be carried out. Some interesting contributions so-called rank, $s$-rank of an $N$-group and left serial near-rings are carried out by Chowdhury, B. De and Kataki in [15,16,17,18].
A brief survey of topological near-rings and topological near-ring groups:

One may come across many interesting problems and fruitful results by studying the relationship between the topological and algebraic structures of various spaces. Besides semi-groups and semi-rings, topological rings also have deep implication in analysis. Among the researchers working along this line we may cite J. C. Beidleman, R. H. Cox, G. Betsch, K. D. Magill, Jr. etc. Here we would like to present what may be called some short of survey regarding definitions and examples of topological near-rings and topological near-ring groups so far the work of Beidleman, Cox and Magill—in particular, are concerned.

At the very outset, we present the definition of Beidleman and Cox [3] as follows:

In case of the left near-ring \((N, +, \cdot)\), a topological near-ring is a quadruple \((A^+, o, T)\) where \(T\) is a Hausdorff topology in \(A^+\) and both addition (+) and multiplication (\(\cdot\)) are co-ordinatewise continuous; i.e., for each \(n \in N\), the four functions, defined for each \(x \in A\) as

(a) \(f(x) = x + n\)
(b) \(g(x) = n + x\)
(c) \(h(x) = xn\)
(d) \(k(x) = nx\)

are continuous on \(N\).
Here it is observed that although in case of a topological ring, addition and multiplication are continuous on the product space, yet the co-ordinate wise continuity is all that is necessary in many cases. In this sense we find the structure of a topological near-ring in case of \( N \), the set of all continuous functions from \( G \) to \( G \) with \((0)f = 0\) where \((G, +)\) is a topological group. Each of the quadruples \((N, +, \circ, T_1)\) and \((N, +, \circ, T_2)\) where \( \circ \) is the composition of mappings, \( T_1 \) is compact open topology and \( T_2 \) is point-wise convergence on \( G \). The next one is an example, which becomes actually a ring. This happens when \( B \) is a Banach space and \( N \) is a set of continuous linear functions from \( B \) into \( B \). Here \((N, +, \circ, \text{topology of pointwise convergence on } B)\) is a topological near-ring where the identity function on \( B \) is identity of \( N \). An invariant metric group \((G, +)\) may give us the quadruple \((N, +, \circ, \text{uniform convergence on countable subsets of } G)\) as a topological near-ring structure when we consider \( N \) as the set of all uniformly continuous functions \( f \) from \( G \) to \( G \) with \((0)f = 0\).

Sometimes it is observed that a right near-ring structure is more familiar one than left one, when we deal with topological near-ring structure as given by K. D. Magill, Jr. as follows: By a topological near-ring we mean a triple \((N, +, \circ)\) where \((N, +)\) is a topological group and \((N, \circ)\) is a topological semi-group with the obvious condition \((a + b) c = ac + bc\), for all \( a, b, c \in N \). In the above definition if a triple \([39]\) consists of a topological near-ring \( N \), a topological group \( G \) and a continuous map \( \mu : N \times G \to G \) such that \( \mu (a + b, x) = \mu (a, x) + \mu (b, x) \) and \( \mu (ab, x) = \mu (a, \mu (b, x)) \), for all \( a, b \in N \) and \( x \in G \), then the pair
$(G, \mu)$ may be referred as a topological $N$-group. As seen [38], in case of the topological group $\mathbb{R}$ of real numbers under addition and $\mathbb{Z}$, the discrete group of integers under addition, $T = \mathbb{R} / \mathbb{Z}$ is the one-dimensional torus. On the other hand, $\mathbb{R}^n$ denotes the Euclidean $n$-group and $T^n$ – the $n$-dimensional torus. Also it is interesting to note that in case of any topological group $E$ and a locally compact Hausdorff group $H$ with compact open topology on $N(H)$ (the topological near-ring of all continuous self maps of $H$ under pointwise addition and composition), $E$ can be made a topological $N(H)$-group, where $\mu(f, x) = 0$, for all $f \in N(H)$ and $x \in E$. And Magill shows existence of such $\mu$ in case of $E$ to be $\mathbb{R}^n$ or $T^n$.

In what follows $\mathbb{R}^n$ will denote the additive topological Euclidean group of dimension $n$ and $\mathbb{R}$ will denote the additive group of real numbers. The range of a function $\lambda$ will be denoted by $\text{Ran}(\lambda)$.

Let $\lambda$ be any continuous map from $\mathbb{R}^n$ to $\mathbb{R}$ and define a multiplication on $\mathbb{R}^n$ by $vw = \lambda(w)v$, for all $v, w \in \mathbb{R}^n$. We have in [37] that $(\mathbb{R}^n, +, \circ)$ is a topological near-ring where $+$ denotes the usual addition if and only if $\lambda(\alpha v) = \alpha \lambda(v)$, for each $v \in \mathbb{R}^n$ and each $\alpha \in \text{Ran}(\lambda)$ and we refer to such maps as semilinear maps. We will denote by $\mathcal{N}(\mathbb{R}^n)$ (termed as \textit{semilinear near-ring}), the topological
near-ring \((\mathbb{R}^n, +, \circ)\) where the multiplication is induced by the semilinear map. Of course, every linear map from \(\mathbb{R}^n\) to \(\mathbb{R}\) is a semilinear map but semilinear maps which are not linear are in abundance. Let \(L\) be any non-zero linear map from \(\mathbb{R}^n\) to \(\mathbb{R}\) and define \(\lambda(v) = |L(v)|\). Then \(\lambda\) is semilinear but not linear. For example, a polynomial \(p(x_1, x_2, \ldots, x_n)\) of degree \(m\) in \(n\) indeterminates is homogeneous if

\[ p(tx_1, tx_2, \ldots, tx_n) = t^m p(x_1, x_2, \ldots, x_n), \text{ for all } t \in \mathbb{R}. \]

Choose any homogeneous polynomial \(p\) of degree \(m\) and defined

\[ \lambda(x_1, x_2, \ldots, x_n) = |p(x_1, x_2, \ldots, x_n)|^{1/m}. \]

One easily verifies that the map \(\lambda\) is semilinear and it is certainly not linear. If \(p\) is a homogeneous polynomial of odd degree \(m > 1\) we can define

\[ \lambda(x_1, x_2, \ldots, x_n) = (p(x_1, x_2, \ldots, x_n))^{1/m} \]

and \(\lambda\) is a nonlinear semilinear map in this case also. Moreover in near-ring \(\mathcal{N}(\mathbb{R}^n)\) the ideals and the multiplicative semigroups of it can be studied. Also we determined all the continuous maps \(\mu\) from \(\mathbb{R}^n\) to \(\mathbb{R}\) such that

\[ (v + w)x = vx + wx \]

and

\[ v(wx) = (vw)x, \text{ for all } v, w \in \mathcal{N}(\mathbb{R}^n) \text{ and } x \in \mathbb{R}, \text{where } vx = \mu(v, x). \]

The \(N\)-group \((G, \mu)\) is referred to as nontrivial \(N\)-group if \(\mu\) does not map everything to zero. As one might expect, there are many topological near-rings \(\mathcal{N}(\mathbb{R}^n)\) for which the only \(\mathcal{N}(\mathbb{R}^n)\)-group \((\mathbb{R}, \mu)\) is the trivial one. Interestingly we get those near-rings \(\mathcal{N}(\mathbb{R}^n)\) for which nontrivial \(\mathcal{N}(\mathbb{R}^n)\)-group \((\mathbb{R}, \mu)\) do exist. In [45], the author
shows that every non-zero quotient near-ring of a semilinear near-ring is isomorphic to a semilinear near-ring and determine precisely two isomorphic quotient near-rings of $\mathcal{N}(\mathbb{R}^n)$. Also we see that, upto isomorphism $\mathcal{N}(\mathbb{R}^n)$ has finitely many quotient near-rings and, in fact, this number cannot exceed $n + 1$ as an instance up to isomorphism, the four quotient near-rings of $\mathcal{N}(\mathbb{R}^3)$ are the zero rings, $\mathcal{N}_1(\mathbb{R})$, $\mathcal{N}_2(\mathbb{R}^2)$ and $\mathcal{N}_3(\mathbb{R}^3)$ where $\lambda_1: \mathbb{R} \to \mathbb{R}$, defined by $\lambda_1(x) = |x|$, for all $x \in \mathbb{R}$ and $\lambda_2: \mathbb{R}^2 \to \mathbb{R}$, defined by $\lambda_2(v) = |v_2|$, for all $v = (v_1, v_2) \in \mathbb{R}^2$.

K.D. Magill [40] has determined all those multiplications $*$ on the two dimensional Euclidean group $\mathbb{R}^2$ such that $(\mathbb{R}^2, +, *)$ is a zero symmetric topological near-ring with identity $(1,0)$, $\{ v \in \mathbb{R}^2 | v_1 = 0 \}$ is a right ideal and $(0, 1) * v = (0, 1) * w$, whenever $v_1 = w_1$. The author also determines all ideals and investigates the algebraic structure of their multiplicative semi-groups. Also in [41] all the real near-rings in case of topological near-rings whose additive groups are the additive groups of real numbers have been determined. Upto isomorphism, all two-dimensional Euclidean near-rings which have a proper non-zero closed connected right ideal and a left zero which is not contained in that ideal are also determined [46] and all left, right and two sided ideals of these near-rings are also found. Moreover homomorphisms from one such near-ring to another and the automorphism groups of many of these near-rings are determined. The structure of the
multiplicative semigroups of these near-rings is examined and characterizes those which are regular and determining Green’s relations for these semigroups. Magill describes all the isomorphisms from one such semi-group to another and these results permit us to describe all the automorphism groups of these semigroups which, in contrast to the case for the near-rings are quite extensive. The near-ring of all continuous functions under the pointwise operations, from a compact Hausdorff space into that near-ring is investigated in [44]. Especially they determine all the homomorphisms from one such near-ring of functions to another and the endomorphism semi-group of the near-ring of functions completely determines the topological structure of the space. In some special cases we get the bicalculation near-rings of $\mathbb{R}$ [51].

But, upto isomorphism, [47] there is exactly one topological near-ring which is not zero symmetric and has an identity, whose additive group is the two-dimensional Euclidean group $\mathbb{R}^2$. But over such type of topological near-ring, we get all topological $N$-groups and the topological group is any one of the n-dimensional Euclidean groups $\mathbb{R}^n$ [48]. The solution of a functional equation involving two continuous selfmaps of $\mathbb{R}^n$ is a key ingredient in all this. Similarly we get [47] endomorphism semi-group and the automorphism group of the near-ring and its right, left and two-sided ideals. In particular it is shown that it has exactly one non-zero proper two-sided ideal and that the corresponding quotient near-ring is the field of real numbers. Moreover some characteristics regarding
all those topological $N$-groups $(\mathbb{R}^2, \mu)$ where $N$ is real near-ring have been established. An interesting algebraic structure so-called laminated near-ring $N_a$ [43] induced from a given near-ring $N$ (called base near-ring of $N_a$) which addition is same as the addition in $N$ and for a fixed element $a \in N$ (called laminator), the product $x \otimes y$ of two elements $x, y \in N_a$ is defined by $x \otimes y = xay$. A left zero covering homomorphism is a homomorphism $\phi$ from a near-ring $N_1$ into a near-ring $N_2$ provided for each left zero $y \in N_2$, $\phi(x) = y$, for some $x \in N_1$. These left zero covering homomorphisms from one laminated near-ring into another are investigated where the base near-ring is the near-ring of all continuous selfmaps of the Euclidean group $\mathbb{R}^2$ under pointwise addition and composition and the laminators are complex polynomials. By inspecting the coefficients of the two laminating polynomials one can determines whether or not two such laminated near-rings are isomorphic. The so-called sandwich near-ring (these are generalizations of laminated near-rings) with sandwich function $\alpha$ is another interesting juicy mouthful concept is investigated by Magill et al. [42]. These concepts deal with two topological spaces $X$ and $G$ where a continuous function $\alpha$ from $G$ into $X$ is kept as fixed. Choosing all topological spaces considered as Hausdorff and has more than one point. Here $S(X, G, \alpha)$ is used to denote the semi-group of all continuous functions from $X$ into $G$ where the product $fg$ of two functions is defined by $fg = f \circ \alpha \circ g$. If $G$ happens to be an additive topological group, the family of all continuous functions from $X$ into $G$ is a near-ring $\mathcal{N}(X, G, \alpha)$ and is known as the sandwich near-ring.
with sandwich function $\alpha$. The homomorphisms from the semi-group $\mathcal{S}(X, G, \alpha)$ into the semi-group $\mathcal{S}(Y, H, \beta)$ and from the near-ring $\mathcal{N}(X, G, \alpha)$ into the near-ring $\mathcal{N}(Y, H, \beta)$ are investigated. We can get [50] all those right topological near-rings $N$ where the additive group of $N$ is two-dimensional Euclidean group and $N$ contains a left identity $e$ and a non-zero nilpotent element $z$ such that $z(e + z) = z$. Another interesting concept so-called a Euclidean $(\mathbb{R}^n, +, \circ)$ near-ring [49] is a topological near-ring where $(\mathbb{R}^n, +)$ is the $n$-dimensional Euclidean group, multiplication is continuous with the right distributive law. When the multiplication is associative then the Euclidean near-ring $(\mathbb{R}^n, +, \circ)$ is an associative Euclidean near-ring. We get the existence of various types of elements in Euclidean near-ring which forces it to be a ring. Upto isomorphism, one particular ring within the class of all two-dimensional Euclidean near-ring can be characterized and this class is truly enormous. For example let $(\mathbb{R}^n, +)$ denote the two dimensional Euclidean group. Let $n > 1$ be an integer and define a multiplication on $\mathbb{R}^2$ by $vw = (v_1w_1, v_1w_2 + v_2w_1^n)$. One can verify that the multiplication is associative and right distributive over addition and hence $(\mathbb{R}^2, +, \circ)$ is a two dimensional associative Euclidean near-ring. Moreover $(1, 0)$ is a two sided identity of $(\mathbb{R}^2, +, \circ)$ and all elements of the form $(0, y)$ are the nilpotent. But $(\mathbb{R}^2, +, \circ)$ is a ring.
Outcome of our work has been described in four chapters

The first Chapter is an attempt to evoke some known definitions and results of near-rings and near-ring groups together with the results of their topological aspects which are used in developing the remaining chapters of the thesis for making our discussion a self contained one.

The second chapter is the outcome of [[13]] read in NSAA and [[14]] which has been accepted for publication in Jour. of GURSA. Work on semi-prime near-ring goes back to Blackett [4]. Recently different types of semi-prime near-ring were studied by Groenewald [31], Rao [61]. In this chapter a generalized semi-prime character, the strongly semi-prime near-rings and so-called pseudo-strongly semi-prime near-ring group play an important role.

Second chapter highlights the following description for strongly semi-prime left Goldie near-rings and ps-strongly semi-prime N-groups with some finiteness conditions.

Here we attempt some characterization of near-rings and near-ring groups with some sort of finiteness conditions. In particular, we first discuss near-rings with ascending conditions on left annihilators of subsets of it and having no infinite independent family of left ideals of it (the so-called left Goldie near-ring, as it first framed and called by Chowdhury [64]). Then we consider the analogous character in case of a near-ring group, such as the annihilators of subsets of the group in the near-ring will satisfy the so-called acc and the near-ring will have no infinite independent
family of left ideals of $N$.

In the third section of this chapter if $N$ is a strongly semi-prime (so-called) left Goldie near-ring such that essential left ideal is a strictly essential left $N$-subgroup of $N$ then $N$ satisfies the dcc on left annihilators. If such a near-ring $N$ is with distributively generated left annihilators and essential left ideal is a strictly essential left $N$-subgroup of $N$, then there exists an element $e$ in $N$ such that $\ell(e) = 0$.

In the last section we deal with near-ring groups of special types which may be called in some sense an extension of results what we have obtained in the previous section of strongly semi-prime left Goldie near-rings. The usual notions of nilpotency and strongly semi-prime characters have been replaced by so-called its pseudo character in case of the $N$-groups. It is so dealt with keeping in note the corresponding Goldie character of any $N$-group. The above mentioned so-called pseudo character helps us in establishing important results on non-singular $N$-group which plays an significant role in establishing the descending chain conditions on annihilators of subsets of it. Pseudo-strongly semi-prime duo acc $N$-group leads us with some restriction to get the trivial annihilator of an element of the $N$-group. All these explained and obtained are in reference to the $N$-group where its topological structure dealt with.

The third chapter is the outcome of our paper [[11]] published in *Mathematica Pannonica*. 

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Here we investigate some properties of topological structures on some generalization of what has been achieved on near-rings with acc on annihilators having no infinite independent family of right (or left) ideals as observed by A. Oswald [57], K. C. Chowdhury et al. [21, 54]. The so-called pseudo character on nilpotency and strongly semi-primeness, discussed in second chapter, lead us satisfactorily towards our goal giving well-designed results on $N$-groups rather than the near-ring structure. Moreover topological structure on such $N$-groups gives some fascinating prolific results. An $N$-group $E$ with so-called Goldie character is well behaved so far as pseudo quality on nilpotency as well as strongly semi-primeness are involved for the proper development of what we have attempted for. The generalization of the idea of boundedness of Beidleman and Cox on topological near-rings [3], $E$-boundedness etc. together with the notion of topologically nilpotent sets is playing important role on an $N$-group $E$ having finite number of elements (e's) belonging to it with zero annihilators [Ann (e) = 0], which occur as a necessity of $N$-group $E$ in above context. It is interesting to note the relevancy and elegancy of the result obtained, as the same may be determined with obliging explanation on such topological $N$-groups that their discrete character is in association with the $E$-boundedness with zero radical or open character of the radical with $E$-boundedness. Some interesting results regarding the discrete character of the $N$-group is observed in case of locally compact groups if the near-ring is without unity.

In this chapter we try to extend the results what Beidleman and Cox have established in their paper “Topological near-ring” [3] to
topological near-ring group. With a view to extend these results we have developed some necessary concepts with accommodating justifications such as pseudo nilpotency, pseudo-strongly semi-prime character etc. Moreover besides all these, some other notions are those of extensions of $E$-boundedness, topological nilpotent set etc.

The content of this chapter is briefly presented as follows: The openness of $Q$ (set of quasi-regular elements of $N$) and $Q_e$ as subsets of $N$ and $N_e$ w.r.t. their respective topologies and $\text{Ann}(e) = 0$ lead us to justify the closeness of the direct sum of the group sum of ideals related to quasi-regular left ideal of $N$. Also coincidence of the radical of the $N$-group $E (= \bigoplus_{i=1}^n E_i)$ with radical subgroup appears under the condition that such type of direct sum is dense in the radical. Also such type of conditions helps us to get the cyclic character of $E$. The closed character of each of the $Q_{e_i}$ in $N_{e_i}$ with respect to the given topology enables us to the closeness of what has been stated above regarding the direct sum of the group sum of ideals related to quasi-regular left ideal of $N$, where zero is the only element of $N_{e_i}$ that kills the $e_i$'s. Some results are based on the notion of so-called $N_u$-nilpotent elements in the $N$-group $N_{e_i}$ where some of which are analogous to those obtained above. Here we assume that each $Q_i$ (the set of $N_u$-nilpotent elements in the $N$-group $N_{e_i}$) is an open proper $N$-subset of $N_{e_i}$ with $\text{Ann}(e_i) = 0$. Dropping of the identity from $N$ and locally compactness of topology on $N$ lead us due to Ellis [26] to get $(N, +)$ as a topological group i.e. the function $f(x, y) = x - y$ is continuous on $N \times N$ to $N$. Moreover we find the discreteness character of $E$, particularly finiteness also
under the assumption that when $E$ is locally compact, disconnected, containing no proper non-zero closed ideals and satisfies the descending chain condition (dcc) on the closed subgroups of $(E, +)$.

The **fourth chapter** is the outcome of our paper [[12]] which has been published in April 2007 ‘s issue of *International Journal of Modern Mathematics*.

The continuity in a topological space is carried out by internal and external compositions of a group or a near–ring group respectively and in some cases restrictions related to boundedness, connectedness and the Hausdorff properties lead us to some effective results. It is observed that in case of a topological group, where the binary operation is continuous in the product space, the corresponding co–ordinate wise continuity is an obvious characteristic in so–called **one sheet space**! But, the converse needs a hard work in the real sense if it happens in so–called **broken two sheets space**. Of course as Beidleman and Cox [3] are of view that coordinates–wise continuity is all that is necessary in many cases.

We recall that in the definition of topological ring, Kaplansky [32] insisted that addition and multiplication be continuous on the product space; however, as defined by Beidleman et al. the authors found that co–ordinate wise continuity is all that is necessary in many cases.

Some careful observations have elegantly revealed what we have attempted and carried out with some sort of rare and alarming beauty, hitherto the so–called continuity of such structures are concerned.
Keeping aside the concrete so-called topological aspect of what has been explained above, we dare to review this aspect of above type of algebraic structure from more or less algebraic point of view in a broaden court-yard with a view to play the same game in a more sophisticated country of algebra. For the moment we leave available topological nomenclature, however instead, embrace some abstract familiar algebraic way of approach. Undoubtedly everything would be justified with sufficient examples if and when necessary. The supposed pseudo continuity in such a so-called algebraic space is carried out by internal and external compositions of a group or a near-ring group respectively to give some general view of some topological properties including boundedness, connectedness and Hausdorff character etc.

Extending the idea of boundedness already available with the help of the notion of (so-called) $S_k$-nilpotent set, we get some elegant results in case of such a space biased near-ring groups with acc on annihilators. It is interesting to note the relevancy and elegancy of the results obtained, as the same may be determined with accommodating justification on such a space biased near-ring groups that their so-called ps(pseudo)-discrete character is in association with the so-called algebraic boundedness with zero-radical or so-called ps-openness of the radical with same character.

At this juncture we want to review some results which may be termed, in some sense, an extension of what Chowdhury et al. has carried out [10, 54, 21]. The structure of semi-prime ring [28, 29] seems to be still relevant due to its elegancy. At the same time the
authors are with sweet remembrance of what Meldrum has remarked about the importance and the intricacy of what has already been carried out by this group along the line of acc on annihilators in case of near-ring groups.

It is observed that near-ring groups with acc on annihilators of subsets of the group in the attached near-ring found to be well behaved so far the so-called space biased algebraic structure is concerned with some so-called pseudo quality on nilpotency as well as strongly semi-prime character. This has involved for the proper development of what we have attempted for.

The notion of boundedness of Beidleman and Cox are playing some shaman character with the algebraic space as the authors are claiming for! Together with these, a near-ring group with so-called Goldie character has been playing an interesting and elegant worthwhile game where the group having finite number of elements (e's) belonging to the group with zero annihilator, which occurs as a necessity of such a near-ring group. The justification has properly been accommodated with a sufficient number of examples so as to congregate our endeavor.

The discrete character of such an algebraic biased space is playing a generously subjective role with a very deep insight, which seems to include so many aspects even in some cases, the orientable and non orientable space relating to Klein's example.

The content of this chapter is summarized as follows: Here we discuss the answer to the question that is at the beginning put in case of a two sided continuity and one sided continuity.
separately in terms of so-called \((S_x \times S_x - S_x)\) and \((S_x - S_x)\) maps. We assume that both \(Q\) and \(Q_e\) are \(S_N\)-open and \(S_{N_e}\)-open subsets of \(N\) and \(N_e\) respectively together with the zero annihilator of the corresponding \(e\) which occurs in the second chapter as a necessity of near-ring group with so-called Goldie character. In this space biased near-ring group with acc on annihilators we study \(ps\)-closeness of direct sum of maximal \(N\)-subgroups together with the \(ps\)-closeness of the direct sum of group sum of ideals related to quasi-regular left ideal of \(N\). Moreover the radical of the \(N\)-group \(E\) \(= \bigoplus_{i=1}^n N_{e_i}\) coincides with the radical subgroup if such type of direct sum is \(S_E\)-dense in the radical. Also such type of conditions helps us to get the cyclic character of \(E\). The \(ps\)-closeness of direct sum of the group sum of ideals related to quasi-regular left ideal of \(N\) can be viewed when zero is the only element of \(N_{e_i}\) that kills the \(e_i\)'s under the assumption that each of the \(Q_{e_i}\) in \(N_{e_i}\) with respect to the \(S_{N_{e_i}}\) is \(S_{N_{e_i}}\)-squeezed. The notion of \(N_0\)-nilpotent element in \(N\)-group \(E\) gives some results, some of which are analogous to those obtained above. The results obtained here are on the assumption that each \(Q_{e_i}\) is a \(S_{N_{e_i}}\)-open proper \(N\)-subset of \(N_{e_i}\) with \(\text{Ann}(e_i) = 0\).