CHAPTER VI

EVALUATION OF Q

6.1 Data

Since good seismograms covering the entire epicentral distance range considered here, were not available for the explosions of different sites, only two explosions at Eastern Kazakh has been considered here for computation of Q. Vertical short period seismograms of the two explosions marked (with *) in Table 1 are digitized using a two-dimensional microscope at an interval of 0.01 sec. Tremblay et al. (1968) while studying seismic source characteristics from explosion generated P-waves at teleseismic distances found that one and half cycles of the recorded first arrival P-waves gave most reliable results about source characteristics. Long (1968) also has utilized only one and half cycle of the seismic trace. But here first two cycles of the recorded P-arrivals have been utilized in the attenuation studies with the assumption that contamination due to arrivals of other phases is very small in the first two cycles. The velocity gradients in deeper mantle are believed to be not very sharp and hence the modulation effect due to reflection by layer-interface or other
interference is also assumed to be very small. Nevertheless contamination from local noises etc. are not ruled out. This may result in small maxima or minima, and upon forming spectral ratios may appear as magnified in the form of spikes in either direction (Teng, 1968). Considering these factors only 19 clearly recorded seismograms have been used out of 30 that have been digitized (Selection have been made, of course, after spectrum analysis). A few of the traces of the $P$-waves used are shown in fig. 8(a), while in fig. 8(b) two representatives of the traces of those which were rejected are reproduced.

Fourier transform of these digitized pulses were computed to get the spectrum $g(w)$ for the frequency range 0.1 to 5 cps only at 0.1 cps and 0.5 cps interval.

The fourier transform pair for continuous data of infinite length in this case is

$$g(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(t) \exp(-i\omega t) \, dt$$

$$F(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} g(\omega) \exp(+i\omega t) \, d\omega$$

where,

$F(t)$ = seismic trace in time domain

$g(\omega)$ = spectrum in frequency domain

$i = \sqrt{-1}$

For digital trace the integral can be reduced to (Huang, 1966),
Fig. 8 (a) Some examples of first arrival P wave forms utilized.
Some examples of P wave forms not used in the study.
\[
\begin{align*}
\mathbf{g}(\omega_j) &= X_j + i Y_j = G_j \exp\left(i \varphi_j \right) \\
F(t_k) &= \Delta \omega \sum_{j=1}^{N} G_j \cos \left( \frac{2\pi k J}{N} + \varphi_j \right) \\
&= \Delta \omega G_N + 2 \Delta \omega \sum_{j=1}^{N/2} G_j \cos \left( \frac{2\pi k J}{N} + \varphi_j \right) \\
&= \Delta \omega G_{N/2}
\end{align*}
\]

where,
\[
\begin{align*}
G_j &= \left( X_j^2 + Y_j^2 \right)^{1/2} \quad = \text{the modulus} \\
\varphi_j &= \tan^{-1} \left( \frac{Y_j}{X_j} \right) \quad = \text{phase angle} \\
X_j &= \frac{1}{N \Delta \omega} \sum_{k=1}^{N} F(t_k) \cos \left( \frac{2\pi k J}{N} \right) \\
Y_j &= \frac{1}{N \Delta \omega} \sum_{k=1}^{N} F(t_k) \sin \left( \frac{2\pi k J}{N} \right)
\end{align*}
\]

\(N = \text{Number of evenly spaced samples of the seismic trace}\)
\(\Delta \omega = \text{frequency interval}\)

Since in the time domain digitization contains \(N\) independent values, the number of independent values in the frequency domain is also \(N\), the number of real and imaginary values each being equal to \(N/2\) or \(N/2\) modulus values and phase in
equation (50). The folding frequency at which the Fourier spectrum is duplicated, is given by

\[ f_c = \frac{1}{2 \Delta t} \]

With the digitization interval \( \Delta t = 0.01 \text{ sec} \), this turns out to be 50 ops, which is much greater than the maximum frequency used in the analysis. So that the digitization interval is quite adequate.

The amplitude of the different frequencies were converted to ground amplitudes by dividing them by corresponding instrument amplification factors. Most of the seismometers used in the stations under consideration are short period Benioff seismometers with \( T_s \sim 1.0 \) and \( T_g \sim 1.0 \text{ sec} \) except those in the Canadian stations where Wilmore seismometers with about the same periods are used. Some of the ground amplitude spectra thus obtained are reproduced in fig. 9.

6.2 Spectral Ratios

For calculating the spectral ratio the station BAG has been selected as the reference station \((j)\). The ratios \([A_i(f)/A_j(f)]\) were calculated for each station and logarithms taken in accordance with the equation \((44)\). Plot of \( \ln [A_i(f)/A_j(f)] \) against frequency theoretically should show linear trend on the basis of the assumption of \( Q \) to be independent of frequency. Fig. 10 shows some of the plots. From these plots
Fig 9  Spectrum of some first arrival P-waves from explosions in Eastern Kazakh.
it can be seen that the values of \( \frac{A_i(t)}{A_j(t)} \) follow the linear relationship with frequency range 0.4 to 1.5 cps quite satisfactorily with only a small scatter. Beyond this frequency range, on both sides, they are highly scattered or tend to follow high degree curve. The small scatter can be attributed to noise, error for truncation and digitization. It can, therefore, safely be concluded that within the range of 0.5 to 1.5 cps, the attenuation parameter \( A \) is independent of frequency, while above 1.5 cps it may be a complicated function of frequency.

Least square straight lines are fitted through these points, the slope of which give the differential \( \delta A_j^i \) with respect to the station BAG. Table 8 gives the relevant data of the stations along with the differential attenuation in the form

\[
\alpha_j^i = \frac{1}{n} \sum A_j^i
\]

It can be seen that \( \delta A_j^i \), may be of both sign. \( \delta A_j^i < 0 \) implies that the ith P-wave has suffered stronger attenuation than the wave arriving at BAG (jth station), and \( \delta A_j^i > 0 \) means the opposite. BAG has the \( \delta A_j^i \) equal to zero.
Table 8

Differential attenuation values for stations

The reference station is BAG.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Station code</th>
<th>Epicentral distance $\triangle$ (deg)</th>
<th>Azimuth $\theta$ (deg)</th>
<th>$i \cdot \frac{1}{\pi} \cdot \angle A_j$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>KEV</td>
<td>31.26</td>
<td>329.2</td>
<td>0.419</td>
</tr>
<tr>
<td>2</td>
<td>IST</td>
<td>35.00</td>
<td>275.0</td>
<td>0.665</td>
</tr>
<tr>
<td>3</td>
<td>KBS</td>
<td>37.05</td>
<td>342.9</td>
<td>0.599</td>
</tr>
<tr>
<td>4</td>
<td>HKC</td>
<td>39.13</td>
<td>304.7</td>
<td>-0.019</td>
</tr>
<tr>
<td>5</td>
<td>SHK</td>
<td>41.80</td>
<td>90.7</td>
<td>0.678</td>
</tr>
<tr>
<td>6</td>
<td>STO</td>
<td>45.85</td>
<td>296.4</td>
<td>-0.073</td>
</tr>
<tr>
<td>7</td>
<td>BAG</td>
<td>47.23</td>
<td>119.8</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>AKU</td>
<td>48.48</td>
<td>326.7</td>
<td>-0.051</td>
</tr>
<tr>
<td>9</td>
<td>AAE</td>
<td>52.64</td>
<td>233.2</td>
<td>-0.435</td>
</tr>
<tr>
<td>10</td>
<td>GDH</td>
<td>56.00</td>
<td>341.5</td>
<td>0.044</td>
</tr>
<tr>
<td>11</td>
<td>PTO</td>
<td>58.52</td>
<td>297.8</td>
<td>0.266</td>
</tr>
<tr>
<td>12</td>
<td>COL</td>
<td>59.88</td>
<td>21.0</td>
<td>-0.181</td>
</tr>
<tr>
<td>13</td>
<td>FRB</td>
<td>63.76</td>
<td>344.5</td>
<td>-0.455</td>
</tr>
<tr>
<td>14</td>
<td>YKC</td>
<td>67.38</td>
<td>6.7</td>
<td>0.802</td>
</tr>
<tr>
<td>15</td>
<td>FCC</td>
<td>71.46</td>
<td>356.1</td>
<td>-0.329</td>
</tr>
<tr>
<td>16</td>
<td>FFC</td>
<td>75.66</td>
<td>0.5</td>
<td>-0.673</td>
</tr>
<tr>
<td>17</td>
<td>LON</td>
<td>81.91</td>
<td>14.2</td>
<td>-0.161</td>
</tr>
<tr>
<td>18</td>
<td>PMG</td>
<td>83.60</td>
<td>112.7</td>
<td>0.409</td>
</tr>
<tr>
<td>19</td>
<td>PRE</td>
<td>87.77</td>
<td>224.3</td>
<td>0.393</td>
</tr>
</tbody>
</table>
Plots of $\log_{e} \left( \frac{A_i(t)}{A_j(t)} \right)$ versus frequency. (Reference station (j) is BAG). Straight lines are fitted through the points, slopes of which give $\delta A_j^1$. 

**FIG. 10**
6.3 Evaluation of Travel Time Matrix

In order to calculate the travel time matrix $C$ the mantle is divided into 18 layers, one less than the number of stations. The thickness of these layers are fixed by the lowest depth reached by rays, obtained from the velocity-depth profile obtained earlier. The nearest station chosen is KEV with epicentral distance $31.26^\circ$. The ray corresponding to this attains a depth of 770 km, which is, therefore, is the thickness of the top layer. Similarly last layer is limited to a depth of 2650 km.

The average velocity ($\bar{v}_t$) of the first layer is obtained from the approximate relation (vide Appendix I)

$$\bar{v}_t = \frac{2r_c \sin(\Delta/2) C \sec \Theta}{T_1}$$

(53)

$\Delta$ = epicentral distance

$\Theta$ = angle of reflection (or incidence) as would have been made by a ray moving in a uniform medium and reflected from the interface of two layers

$T_1$ = travel time of the first ray

$\equiv \Delta t_1^i$ in the matrix $C$

Now assuming the seismic rays to form arcs of circle approximately, the radius of which are calculated as (vide Appendix IIA)
\[ \alpha_n = \frac{1}{2} \left[ \frac{2 r_n (r_n - Z_n) \left\{ \frac{1 - \cos (\Delta n/2)}{Z_n - r_n \left( 1 - \cos (\Delta n/2) \right)} \right\}} \right] \]  

(54)

where \( Z_n \) = depth reached by the \( n \)th ray.

The second ray has radius \( a_2 \), the segment of which enclosed in the first layer is given by

\[ L_1^2 = 2 a_2 \theta_2^2 \]

vide Appendix II(B) and (C).

The transit time of the second ray in the first layer is given by

\[ \Delta t_1^2 = \frac{L_1^2}{V_1} \]

The transit time in the second layer itself is then

\[ \Delta t_2^2 = T_2 - \Delta t_1^2 \]  

(55)

where \( T_2 \) = travel time of the ray.

The length of the arc in the second layer is

\[ L_2^2 = 2 a_2 \theta_2^2 \]  

(56)

Hence the average velocity in the second layer is given by

\[ \bar{V}_2 = \frac{L_2^2}{\Delta t_2^2} \]  

(57)

The definitions of the angles \( \theta_1^2, \theta_2^2 \) and method of calculations are given in Appendix II(B) and II(C).

Thus the transit times \( \Delta t_1^2 \) can be calculated.
The first average velocity \( \bar{V}_i \) can also be calculated as follows.

The length of the first ray, \( L_1^i = a_i \theta_1^i \)

\[
\bar{V}_i = \frac{a_i \theta_1^i}{T_i}
\]  

(58)

The values calculated by the two methods do not differ by more than 3 per cent. In actual calculations the latter method using equation (58) has been used, maintaining uniformity in computing the average velocity for all the layers.

6.4 Evaluation of Q values for the Layers

As already noted the station BAG has been chosen as the reference station which is passing through the 7th layer. The elements of the travel time matrix is obtained by subtracting the 'differential travel time' of the different rays from that corresponding to BAG in accordance with equation (47). A computer programme was made using Jordan's method of matrix inversion and the travel time matrix \( \mathcal{C} \) was inverted to obtain the Q values in accordance with (49). In the table 9 the third column gives the values of Q for the different layers obtained by this method.

It can be seen from the third column of the table that one of the layers has high negative value of Q which seem
Table 9

\( Q \)-values for different layers

<table>
<thead>
<tr>
<th>Layer No.</th>
<th>Depth range (km)</th>
<th>( Q_x ) (First method)</th>
<th>( Q_x ) (Second method)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>1</td>
<td>-770</td>
<td>118.3</td>
<td>120.0</td>
</tr>
<tr>
<td>2</td>
<td>770-830</td>
<td>369.8</td>
<td>292.3</td>
</tr>
<tr>
<td>3</td>
<td>830-880</td>
<td>258.9</td>
<td>276.4</td>
</tr>
<tr>
<td>4</td>
<td>880-950</td>
<td>100.4</td>
<td>141.5</td>
</tr>
<tr>
<td>5</td>
<td>950-1020</td>
<td>1061.2</td>
<td>2386.1</td>
</tr>
<tr>
<td>6</td>
<td>1020-1090</td>
<td>110.6</td>
<td>120.0</td>
</tr>
<tr>
<td>7</td>
<td>1090-1190</td>
<td>179.0</td>
<td>311.3</td>
</tr>
<tr>
<td>8</td>
<td>1190-1250</td>
<td>131.9</td>
<td>178.5</td>
</tr>
<tr>
<td>9</td>
<td>1250-1350</td>
<td>501.3</td>
<td>(-)137.2*</td>
</tr>
<tr>
<td>10</td>
<td>1350-1410</td>
<td>942.2</td>
<td>127.0</td>
</tr>
<tr>
<td>11</td>
<td>1410-1470</td>
<td>146.3</td>
<td>106.6</td>
</tr>
<tr>
<td>12</td>
<td>1470-1580</td>
<td>247.3</td>
<td>196.5</td>
</tr>
<tr>
<td>13</td>
<td>1580-1730</td>
<td>(-)549.5*</td>
<td>(-)1178.8*</td>
</tr>
<tr>
<td>14</td>
<td>1730-1870</td>
<td>136.3</td>
<td>131.7</td>
</tr>
<tr>
<td>15</td>
<td>1870-2070</td>
<td>146.2</td>
<td>100.4</td>
</tr>
<tr>
<td>16</td>
<td>2070-2400</td>
<td>390.0</td>
<td>414.5</td>
</tr>
<tr>
<td>17</td>
<td>2400-2490</td>
<td>519.1</td>
<td>162.2</td>
</tr>
<tr>
<td>18</td>
<td>2490-2650</td>
<td>218.2</td>
<td>(-)172.3*</td>
</tr>
</tbody>
</table>

* regions giving high -ve values. Actually they are regions of high +ve values (Vide text).
physically unrealistic at first sight. But after analysing the process of calculation we find that the negative values only indicate regions of high $Q$ values. For, the final step for calculation of $Q$ involves taking reciprocal of $Q$ after multiplication of the matrix $A^{-1}$ with $A$. The elements of $A^{-1}$ being very small quantities with both signs, at large depths, for slight error in the value of the elements may result in small negative value for $Q$, instead of small positive ones, corresponding to large positive value of $Q$. Precisely this is the reason for high negative values of $Q$. The method is, therefore, very sensitive for regions with high $Q$. Consequently appearance of greater number of negative values would indicate slight error in the original measurement of elements of either the matrix $A$ or $Q$. In this case, there being only one negative value, such error is presumably small, so that the $Q$ values might be accepted as fairly reliable.

A plot of $Q$ versus depth is shown in fig. 11. The $Q$-depth distribution model (F) of Teng is also reproduced alongside for comparison.

6.5 A Second Method for Determination of $Q$-Depth Profile

The $Q$ values presented in the third column of table 9 seem to differ from the generally accepted view that its value increases with depth. The values show that the deeper mantle
Fig 11. (a) Derived by the first method in this study. (b) P (or G) model of Tensol below 700 km.
also may have layers with comparatively lower values of Q. Also the value is not as high as 2000-3000 as reported by Asada and Takano (1963) and others (See table 7). In order to check our results the Q values have been recalculated with another scheme, based on the spectral ratio method which closely resembles to that of Long (1968).

The amplitude equations for the waves, suffering attenuation in the different layers, can be written as,

\[ A_1(f) = A_0(f) \exp\left(-\frac{\pi f t_1}{Q_1}\right) \]  
\[ A_2(f) = A_0(f) \exp\left(-\frac{\pi f t_2}{Q_2}\right) \]  
\[ = A_0(f) \exp\left[-\pi f\left(\frac{\Delta t^2_1}{Q_1} + \frac{\Delta t^2_2}{Q_2}\right)\right] \]  

suppressing the other factors, \( Q_1, Q_2 \) are the attenuation parameters of the first and second layers respectively and \( \Delta t^2_1, \Delta t^2_2 \) are the transit times in them.

Taking spectral ratio with respect to the first station, we have

\[ \frac{1}{\pi} \frac{\partial}{\partial f} \left[ \ln \frac{A_2(f)}{A_1(f)} \right] = \frac{t_1}{Q_1} - \left(\frac{\Delta t^2_1}{Q_1} + \frac{\Delta t^2_2}{Q_2}\right) \]

or

\[ \frac{1}{\pi} \frac{\partial}{\partial f} A_2 = \left(\frac{\Delta t^2_1}{Q_1} - \frac{\Delta t^2_2}{Q_2}\right) - \frac{\Delta t^2_2}{Q_2} \]
replacing \( \Delta t_1^{t_1} = t_1 \)

The general term for the \( j \)th ray is

\[
\frac{1}{\Pi} \delta A_j = \left( \frac{\Delta t_1^{t_{1j}}}{\delta^1} - \frac{\Delta t_{2j}}{\delta^2} \right) - \frac{\Delta t_{2j}}{\delta^2} \cdots \frac{\Delta t_{kj}}{\delta^k} \quad (62)
\]

denoting a set of equations with \( j = 2 \ldots, N \) where \( N = \) number of layers (or stations). Now \( \delta A_{1j} \), may be defined for symmetry

\[
\frac{1}{\Pi} \delta A_{1j} = \frac{\Delta t_1^{t_{1j}}}{\delta^1}
\]

that is

\[
\frac{1}{\Pi} \delta A_{1j} = \frac{1}{\Pi} \frac{\partial}{\partial f} \left[ L_n \left( \frac{A_0(f)}{A_1(f)} \right) \right] = \frac{\Delta t_1^{t_{1j}}}{\delta^1} \quad (63)
\]

where \( A_0(f) \) is the source amplitude. Equations (62) together with (63) then can be written in matrix form

\[
A = \mathcal{C} Q \quad (64)
\]

where \( \mathcal{C} \), the travel time matrix defined by,

\[
\mathcal{C} = \begin{vmatrix}
\Delta t_1^{t_{11}} & 0 & 0 & \cdots & 0 \\
\Delta t_2^{t_{12}} - \Delta t_1^{t_{11}} & \Delta t_2^{t_{12}} & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\Delta t_N^{t_{1N}} - \Delta t_1^{t_{11}} & \Delta t_N^{t_{1N}} - \Delta t_2^{t_{12}} & \cdots & \cdots & \Delta t_N^{t_{1N}} - \Delta t_N^{t_{1N}}
\end{vmatrix} \quad (65)
\]
The matrix $C$ in this case is triangular which may be treated as square and can be inverted in the usual way. The solution for $Q$ is, therefore, given by

$$Q = C^{-1} A$$

(67)

The first station is KEV. All the other stations listed in table 3 except BAG are taken and the spectral ratios with respect to KEV is calculated. The elements of the travel time matrix are also calculated using the travel times used in the earlier section. Thus all the values of $\delta A_j$ are known from the data except the first element, $\delta A_1$. As is noted it is not directly measurable in our case, but can be calculated indirectly by taking a value of $Q_1$ from the other workers. In this regard some difficulties arise, as the values quoted in the literature, seem not to be very reliable as they vary by wide margins. Also they are actually referred to different parts of the mantle, obtained by using waves of different frequencies. From the table 7, it appears that only Kanamori's (1967) values are close to our frequency range and depth. So, the value of $Q = 200$ which lies between 180 and 240 for frequency range 0.04 to 1.2 cps, for the upper 870 km depth is
taken as \( Q_1 \) in the matrices (66) and calculations are made accordingly.

But in doing so, it is found that for a large number of layers, the \( Q \) values become negative. Keeping other parameters unchanged, changing of the value of \( Q \) also changes the number of such regions, which show that in this analysis the first chosen value \( Q_1 \) has a great influence in fixing the values of the subsequent layers. This is also evident from the nature of the matrices (64) and (67). Therefore the value \( Q_1 = 200 \) seems not to be reasonable enough at least for our analysis, as it leads to quite a few number of negative \( Q \) values. On the other hand the number of layers with negative \( Q \) values are greatly reduced and the values become comparable with those obtained by the first method if the initial \( Q_1 \) is taken to be 120 instead of 200. Assumption of other values for \( Q_1 \) much different from 118 (obtained by the first method) does not give result comparable to those by the first method in a better way. The \( Q \) values thus obtained are enlisted in the column 4 of the table 9.

It can be seen that, the three negative values occurring in column 4 correspond to the layer with high \( Q \) values of column 3, in general, themselves being fairly high. The explanations for the high negative values from the point of view of computational errors are given in the earlier section. The most important sources of error, are therefore lie in the values of \( Q_j \) and the transit times through the different layers, \( \Delta t_i^j \), which are difficult to eliminate. Assumption of
circular ray path and subsequent formulae used are approximate ones. Probably these inherent errors are responsible for the unrealistic negative $\alpha$ values for some layers.