SPREAD OF GONORRHEA IN
ANANTAPUR DISTRICT
(A CASE STUDY)
Gonorrhea ranks first today among reportable communicable diseases in the India. There are more reported cases of gonorrhea every year than the combined totals for syphilis, measles, mumps, and infectious hepatitis. This painful and dangerous disease, which is caused by the gonococcus, is spread from person to person by sexual contact. A few days after the infection there is usually itching and burning of the genital area, particularly while urinating. About the same time a discharge develops which males will notice, but which females may not notice. Infected women may have no easily recognizable symptoms, even while the disease does substantial internal damage. Gonorrhea can only be cured by antibiotics (usually penicillin). However, treatment must be given early if the disease is to be stopped from doing serious damage to the body. If untreated, gonorrhea can result in blindness, sterility, arthritis, heart failure, and ultimately, death.

In this section we construct a mathematical model of the spread of gonorrhea. Our work is greatly simplified by the fact that the incubation period of gonorrhea is very short (3-7 days) compared to the often quite long period of active infectiousness. Thus, we will assume in our model that an individual becomes infective immediately after contracting gonorrhea. In addition, gonorrhea does not confer even partial immunity to those individuals who have recovered from it. Immediately after recovery, an individual is gain susceptible. Thus, we can split the sexually active and promiscuous portion of the population into two
groups, susceptibles and infectives. Let $c_1(t)$ be the total number of promiscuous males, $c_2(t)$ the total number of promiscuous females, $x(t)$ the total number of infective males, and $y(t)$ the total number of infective females, at time $t$. Then, the total numbers of susceptible males and susceptible females are $c_1(t) - x(t)$ and $c_2(t) - y(t)$ respectively. The spread of gonorrhea is presumed to be governed by the following rules:

I. Males infectives are cured at a rate $a_1$ proportional to their total number, and female infectives are cured at a rate $a_2$ proportional to their total number. The constant $a_1$ is larger than $a_2$ since infective males quickly develop painful symptoms and therefore seek prompt medical attention. Female infectives, on the other hand, are usually asymptomatic, and therefore are infectious for much longer periods.

II. New infectives are added to the male population at a rate $b_1$ proportional to the total number of male susceptibles and female infectives. Similarly, new infectives are added to the female population at a rate $b_2$ proportional to the total number of female susceptibles and male infectives.

III. The total number of promiscuous males and promiscuous females remain at constant levels $c_1$ and $c_2$, respectively.

It follows immediately from rules I-III that

\[
\begin{align*}
\frac{dx}{dt} &= -a_1 x + b_1 (c_1 - x) y \\
\frac{dy}{dt} &= -a_2 y + b_2 (c_2 - y) x
\end{align*}
\]

(4.1)
If \( x(t_0) \) and \( y(t_0) \) are positive, then \( x(t) \) and \( y(t) \) are positive for all \( t \geq t_0 \).

If \( x(t_0) \) is less than \( c_1 \) and \( y(t_0) \) is less than \( c_2 \), then \( x(t) \) is less than \( c_1 \) and \( y(t) \) is less than \( c_2 \) for all \( t \geq t_0 \).

We can show that equation (4.1)

(a) Suppose that \( a_1 a_2 \) is less than \( b_1 b_2 c_2 \). Then, every solution \( x(t), y(t) \), of (4.1) with \( 0 < x(t) < c_1 \) and \( 0 < y(t) < c_2 \), approaches the equilibrium solution

\[
x = \frac{b_1 b_2 c_1 c_2 - a_1 a_2}{a_1 b_2 + b_1 b_2 c_2}, \quad y = \frac{b_1 b_2 c_1 c_2 - a_1 a_2}{a_2 b_1 + b_1 b_2 c_2}
\]

as \( t \) approaches infinity. In other words, the total numbers of infective males and infective females will ultimately level off.

Proof: The result can be established by splitting the rectangle \( 0 < x < c_1, 0 < y < c_2 \) into regions in which both \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) have fixed signs. This is accomplished in the following manner. Setting \( \frac{dx}{dt} = 0 \) in (4.1), and solving for \( y \) as a function of \( x \) gives

\[
-a_1 x + b_1 (c_1 - x) y = 0
\]

\[
b_1 y(c_1 - x) = a_1 x
\]

\[
y = \frac{a_1 x}{b_1 (c_1 - x)} = \phi_1 x
\]

Similarly, setting \( \frac{dy}{dt} = 0 \) in (4.1)

\[
-a_2 y + b_2 (c_2 - y) x = 0
\]

\[
b_2 x(c_2 - y) = a_2 y
\]
\[
x = \frac{a_2 y}{b_2 (c_2 - y)}
\]

\[
x b_2 (c_2 - y) = a_2 y
\]

\[
x b_2 c_2 - x b_2 y = a_2 y
\]

\[
x b_2 c_2 = a_2 y + x b_2 y
\]

\[
x b_2 c_2 = y(a_2 + x b_2)
\]

\[
\frac{x b_2 c_2}{a_2 + x b_2} = y
\]

\[
y = \frac{x b_2 c_2}{a_2 + x b_2} = \phi_2 x
\]

Observe first that \(\phi_1 x\) and \(\phi_2 x\) are monotonic increasing functions of \(x\); \(\phi_1 x\) approaches infinity as \(x\) approaches \(c_1\) and \(\phi_2 x\) approaches \(c_2\) as \(x\) approaches infinity. Second, observe that the curves \(y=\phi_1 x, y=\phi_2 x\) intersect at \((0,0)\) and at \((x_0, y_0)\) where

\[
x_0 = \frac{b_1 b_2 c_1 c_2 - a_1 a_2}{a_1 b_2 + b_1 b_2 c_2}
\]

\[
y_0 = \frac{b_1 b_2 c_1 c_2 - a_1 a_2}{a_2 b_1 + b_1 b_2 c_1}
\]

Third, observe that \(\phi_2 x\) is increasing faster than \(\phi_1 x\) at \(x = 0\), since

\[
\phi_2'(0) = \frac{b_2 c_2}{a_2} > \frac{a_1}{b_1 c_1}.
\]

Hence, \(\phi_2 x\) lies above for \(0 < x < x_0\) and lies below \(\phi_1 x\) for \(x_0 < x < c_1\). The point \((x_0, y_0)\) is an equilibrium point of (1) since both \(\frac{dx}{dt}\) and \(\frac{dy}{dt}\) are zero when \(x = x_0\) and \(y = y_0\).
Finally, observe that \( \frac{dx}{dt} \) is positive at any point \((x, y)\) above the curve \( y = \phi_1 x \), and negative at any point \((x, y)\) below this curve. Similarly, \( \frac{dy}{dt} \) is positive at any point \((x, y)\) below the curve \( y = \phi_2 x \), and negative at any point \((x, y)\) above this curve. Thus, the curves \( y = \phi_1 x \) and \( y = \phi_2 x \) split the rectangle \( 0 < x < c_1, 0 < y < c_2 \) into four regions in which \( \frac{dx}{dt} \) and \( \frac{dy}{dt} \) have fixed signs.

It can be established that

Any solution \( x(t), y(t) \) of (1) which starts in region I at time \( t = t_0 \) will remain in this region for all future time \( t \geq t_0 \) and approach the equilibrium solution \( x = x_0, y = y_0 \) as \( t \) approaches infinity.

Any solution \( x(t), y(t) \) of (1) which starts in region II at time \( t = t_0 \), will remain in region II for all future time, must approach the equilibrium solution \( x = x_0, y = y_0 \) as \( t \) approaches infinity.

Any solution \( x(t), y(t) \) of (1) which starts in region IV at time \( t = t_0 \), and remains in region IV for all future time, must approach the equilibrium solution \( x = x_0, y = y_0 \) as \( t \) approaches infinity.

We make use of the above mentioned deterministic model to study of severity of Gonorrhea diseased in Anantapur district during the period of 1995-2003 based on the data collection from the Head Quarters of Hospital Anantapur during this period.
Year wise Male population in Anantapur district and case study of Gonorrheal disease from the recorded data of Government Head Quarters Hospital, Anantapur, Andhra Pradesh.

<table>
<thead>
<tr>
<th>Years</th>
<th>Total Male population</th>
<th>Total number of promiscuous Males</th>
<th>Total number of Infective Males</th>
<th>Total number of Males Cured</th>
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Table 1
Year wise Female population in Anantapur district and case study of Gonorrheal disease from the recorded data of Government Head Quarters Hospital, Anantapur, Andhra Pradesh.

<table>
<thead>
<tr>
<th>Years</th>
<th>Total Female population</th>
<th>Total number of promiscuous Females</th>
<th>Total number of Infective Females</th>
<th>Total number of Females Cured</th>
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Table 2
Figure 1: Profile of Infective Males with Gonorrheal Disease during the period of 1995-2003
Figure 2: Profile of Infective Females with Gonorrheal Disease during the period of 1995-2003

Gonoria cases in Anantapur District
Year wise

Figure 2: Profile of Infective Females with Gonorrheal Disease during the period of 1995-2003
Figure 3: Number of Infective Males Vs Number of Infective Females During the Period 1995-2003