CHAPTER 2

SOLITARY WAVES IN A WARM PLASMA WITH NEGATIVE IONS AND DRIFTING EFFECT OF ELECTRONS

(Published in Phys. Fluids B 5 (2), February, 1993, 440 - 446)

2.1 Introduction

Nonlinear behavior of natural phenomena is of utmost importance and the most spectacular of such phenomena in a dispersive medium is exhibited by the "solitary structures" observed by Russel in 1834. The theory of nonlinear waves in dispersive media is based on the idea put forward by Korteweg-de Vries (1895) with exact soliton solution. The development in the studies of ion-acoustic solitary waves in a magnetized or unmagnetized plasma, either cold or warm, takes a new turn at the present time from both theoretical and experimental points of view. Washimi & Tanuiti (1966), Davidson (1972), Tappert (1972), and Tagare (1972) have used the versatile perturbation technique to study these waves in a two-component plasma . Karpman and Kadomtsev (1971) have discussed the low effects of nonlinearity and dispersion that give rise to the solitary waves in plasma. On the other hand, total nonlinearity of these waves were taken into account by Sagdeev (1966) in his derivation of energy integral instead of the perturbation coefficients to study solitary waves.

At present, the role of negative ions, whose presence can not be ignored in the formation of ion-acoustic solitons in a plasma, is of great interest. The composition of magnetized and unmagnetized plasmas, either with warm or cold, positive and negative ions together with the usual electrons is the recent domain to study ion-acoustic solitary waves. Das (1979), Watanabe (1984), Verheest (1988) and Baboolal *et at.* (1989) have studied solitary ion-acoustic waves in such a composition of plasma. Also, the modified K-dV solitons have been investigated by Tagare (1986) with isothermal electrons and by Tagare and Reddy (1987) with nonisothermal electrons in a plasma with negative ions. Kalita and Kalita (1990) have established the existence of modified KdV solitons only for $Q\ (= m_j / m_i) > 1$ in a different physical situation. Tajiri and Toda (1985) have studied fully nonlinear ion-acoustic solitons in a plasma with positive and negative ions and the electrons.

Experimentally, Ludwig *et at.* (1984) and Nakamura *et at.* (1985a) have observed rarefactive solitons with small amplitudes in a plasma with significant percentage of negative ions. The modified KdV soliton has been experimentally observed by Nakamura and Tsukabayashi (1984) and Nakamura (1985). Further, Nakamura (1987) has observed experimentally that there is a functional relation between the amplitude of solitary waves in a plasma and the density of negative ions where the ion temperature is finite.

Almost in all the above-mentioned cases, the motion of electrons is described by the usual Boltzmann distribution or taken as nonisothermal electrons. But the characters of solitons are found to be substantially changed with the introduction of the complete fluid equation for the electrons with initial drift motion, rather than the simple Boltzmann distribution. Leven and Steinmann (1979) and Kalita *et at.* (1986) studied ion-acoustic solitary waves with the assumption of drift motion of the electrons in a simple composition of plasma.

This chapter deals with the investigation of ion-acoustic solitons in a plasma with negative ions together with the drift motion of the highly mobile electrons in one dimension. In this case, we have considered the multispecies plasma consisting of warm positive and negative ions with equal temperature. Unlike the general assumption of Boltzmann distribution for the electrons, we consider the initial drift motion of the electrons in its complete fluid equations. Emphasis has been given to the subsequent changes in the soliton characters for this consideration.

2.2 Basic equations

We have considered a warm collisionless plasma consisting of positive ions of mass m_i, velocity v_i, temperature T_i, density n_i and negative ions of mass m_j, velocity v_j, temperature T_j and density n_j. The highly mobile electrons with finite mass m_e, density n_e, temperature T_e and velocity v_e are considerd to have initial drift motion v'_e. In cases of dc (direct current) discharges in an ionized gas, the drift motion of the electrons is not small and can produce characteristic changes (Leven and Steinmann, 1979) in the ion-acoustic solitons. Also, under the influence of strong electric field, the unidirectional electrons can acquire rather a high drift velocity relative to the ions. The implication of this assumption (Kalita *et al.*,1986) is shown to admit that the initial electron drift velocity (v'_e) must be in the range $v'_e - M/k_z < 44.72$, where M is the Mach number and k_z is the direction cosine. All variations are considerd in the *x*-direction for one-dimensional propagation. Basic equations, governing the motion for possible ion-acoustic waves after normalization of densities by the equilibrium electron density n_{e0} , space by a characteristic length l, velocities by the ion-acoustic speed $c_s = (\sqrt{T_e/m_i})$, time by l/c_s and the electric potential ϕ by T_e/e (e being the electron charge) , are as follows.

For the positive ions,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{2.1}$$

$$\frac{\partial v_i}{\partial t} + v_i \frac{\partial v_i}{\partial x} = -\frac{\partial \phi}{\partial x} - \frac{\alpha}{n_i}\frac{\partial n_i}{\partial x}. \tag{2.2}$$

For the electrons,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{2.3}$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = \frac{1}{Q}\left(\frac{\partial \phi}{\partial x} - \frac{1}{n_e}\frac{\partial n_e}{\partial x}\right). \tag{2.4}$$

For the negative ions,

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial x}(n_j v_j) = 0, \tag{2.5}$$

$$\frac{\partial v_j}{\partial t} + v_j \frac{\partial v_j}{\partial x} = \frac{1}{Q'}\left(\frac{\partial \phi}{\partial x} - \frac{\beta}{n_j}\frac{\partial n_j}{\partial x}\right). \tag{2.6}$$

and the Poisson equation is

$$\varepsilon \frac{\partial^2 \phi}{\partial x^2} = n_e + n_j - n_i \tag{2.7}$$

where $\varepsilon = \lambda_D^2 / l^2 << 1$, λ_D being the electron Debye length.

Here $Q = m_e / m_i$ and $Q' = m_j / m_i$ are, respectively, the electron to ion and negative ion to positive ion mass ratios corresponding to the lightest ion, $\alpha = T_i / T_e$, and $\beta = T_j / T_e$ are the temperature ratios. The small perturbative parameter ε is such that $\varepsilon \sim Q^n (n > 1 \text{ slightly})$ for some suitable n to ascertain that the terms retained in the consideration are of order higher than ε^2 slightly but less than ε^3.

From the original set of equations (2.1) – (2.7) (before normalization), we obtain the linear dispersion relation for the plasma components (with $T_i = T_j$) as

$$(\omega - v'_e k)^2 = \frac{c_s^2 k^2}{Q}(1+\lambda)$$

where $\lambda = Q \alpha \omega^2_{pe} / (\omega^2_{pi} + Q' \omega^2_{pj})$.

2.3 Derivation of the Korteweg-de Veries (KdV) equation

To deduce the equation for the representation of ion acoustic waves, the flow variables in (2.1) --- (2.7) are expanded in terms of the small parameter ε as

$$\begin{aligned}
n_i &= n_{i0} + \varepsilon n_{i1} + \varepsilon^2 n_{i2} + \cdots\cdots, \\
n_e &= 1 + \varepsilon n_{e1} + \varepsilon^2 n_{e2} + \cdots\cdots, \\
n_j &= n_{j0} + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \cdots\cdots, \\
v_i &= \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \cdots\cdots, \\
v_e &= v'_e + \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \cdots\cdots, \\
v_j &= \varepsilon v_{j1} + \varepsilon^2 v_{j2} + \cdots\cdots, \\
\phi &= \varepsilon \phi_1 + \varepsilon^2 \phi_2 + \cdots\cdots,
\end{aligned} \qquad (2.8)$$

with $n_{eo} + n_{jo} = n_{io}$ so that

$1 + n_{jo} = n_{io}$ (normalizing by n_{eo}).

Here v'_e is taken as the initial drift motion of the electrons. We introduce the new stretch variables ξ and τ so that

$$\xi = x - Vt \ , \quad \tau = \varepsilon V t , \quad \text{where } V \text{ is the phase velocity.} \tag{2.9}$$

Using the tramsformations (2.9) or

$$\frac{\partial}{\partial x} \equiv \frac{\partial}{\partial \xi} \ , \quad \frac{\partial}{\partial t} \equiv \varepsilon V \frac{\partial}{\partial \tau} - V \frac{\partial}{\partial \xi}$$

in the equations (2.1) --- (2.7), the following relations are obtained.

$$\varepsilon V \frac{\partial n_i}{\partial \tau} - V \frac{\partial n_i}{\partial \xi} + n_i \frac{\partial v_i}{\partial \xi} + v_i \frac{\partial n_i}{\partial \xi} = 0,$$

$$\varepsilon V \frac{\partial v_i}{\partial \tau} - V \frac{\partial v_i}{\partial \xi} + v_i \frac{\partial v_i}{\partial \xi} + \frac{\partial \phi}{\partial \xi} + \frac{\alpha}{n_i} \frac{\partial n_i}{\partial \xi} = 0,$$

$$\varepsilon V \frac{\partial n_e}{\partial \tau} - V \frac{\partial n_e}{\partial \xi} + n_e \frac{\partial v_e}{\partial \xi} + v_e \frac{\partial n_e}{\partial \xi} = 0,$$

$$Q n_e \varepsilon V \frac{\partial v_e}{\partial \tau} - Q n_e V \frac{\partial v_e}{\partial \xi} + Q n_e v_e \frac{\partial v_e}{\partial \xi} - n_e \frac{\partial \phi}{\partial \xi} + \frac{\partial n_e}{\partial \xi} = 0, \tag{2.10}$$

$$\varepsilon V \frac{\partial n_j}{\partial \tau} - V \frac{\partial n_j}{\partial \xi} + n_j \frac{\partial v_j}{\partial \xi} + v_j \frac{\partial n_j}{\partial \xi} = 0,$$

$$Q' \varepsilon V \frac{\partial v_j}{\partial \tau} - Q' V \frac{\partial v_j}{\partial \xi} + Q' v_j \frac{\partial v_j}{\partial \xi} - \frac{\partial \phi}{\partial \xi} + \frac{\beta}{n_j} \frac{\partial n_j}{\partial \xi} = 0,$$

$$\varepsilon \frac{\partial^2 \phi}{\partial \xi^2} = n_e + n_j - n_i .$$

Now the flow variables are replaced by the expansions in (2.8) to get the lowest order, that is the ε^1 –order equations as

$$-V \frac{\partial n_{i1}}{\partial \xi} + n_{io} \frac{\partial v_{i1}}{\partial \xi} = 0 ,$$

$$-n_{io}V\frac{\partial v_{i1}}{\partial\xi}+n_{io}\frac{\partial\phi_1}{\partial\xi}+\alpha\frac{\partial n_{i1}}{\partial\xi}=0\ ,$$

$$-V\frac{\partial n_{e1}}{\partial\xi}+\frac{\partial v_{e1}}{\partial\xi}+v'_e\frac{\partial n_{e1}}{\partial\xi}=0\ ,$$

$$-QV\frac{\partial v_{e1}}{\partial\xi}+Qv'_e\frac{\partial v_{e1}}{\partial\xi}-\frac{\partial\phi_1}{\partial\xi}+\frac{\partial n_{e1}}{\partial\xi}=0\ , \tag{2.11}$$

$$-V\frac{\partial n_{j1}}{\partial\xi}+n_{jo}\frac{\partial v_{j1}}{\partial\xi}=0\ ,$$

$$-Q'Vn_{jo}\frac{\partial v_{j1}}{\partial\xi}-n_{jo}\frac{\partial\phi_1}{\partial\xi}+\beta\frac{\partial n_{j1}}{\partial\xi}=0\ ,$$

$$n_{e1}+n_{j1}-n_{i1}=0$$

and the second - order equations of ε are obtained as

$$V\frac{\partial n_{i1}}{\partial\tau}-V\frac{\partial n_{i2}}{\partial\xi}+n_{io}\frac{\partial v_{i2}}{\partial\xi}+n_{i1}\frac{\partial v_{i1}}{\partial\xi}+v_{i1}\frac{\partial n_{i1}}{\partial\xi}=0\ ,$$

$$n_{io}V\frac{\partial v_{i1}}{\partial\tau}-n_{io}V\frac{\partial v_{i2}}{\partial\xi}-Vn_{i1}\frac{\partial v_{i1}}{\partial\xi}+n_{io}v_{i1}\frac{\partial v_{i1}}{\partial\xi}+n_{io}\frac{\partial\phi_2}{\partial\xi}+n_{i1}\frac{\partial\phi_1}{\partial\xi}+\alpha\frac{\partial n_{i2}}{\partial\xi}=0\ ,$$

$$V\frac{\partial n_{e1}}{\partial\tau}-V\frac{\partial n_{e2}}{\partial\xi}+\frac{\partial v_{e2}}{\partial\xi}+n_{e1}\frac{\partial v_{e1}}{\partial\xi}+v'_e\frac{\partial n_{e2}}{\partial\xi}+v_{e1}\frac{\partial n_{e1}}{\partial\xi}=0\ ,$$

$$QV\frac{\partial v_{e1}}{\partial\tau}-QV\frac{\partial v_{e2}}{\partial\xi}-QVn_{e1}\frac{\partial n_{e1}}{\partial\xi}+Qv'_e\frac{\partial v_{e2}}{\partial\xi}+Qv_{e1}\frac{\partial v_{e1}}{\partial\xi}+Qv'_en_{e1}\frac{\partial v_{e1}}{\partial\xi}-\frac{\partial\phi_2}{\partial\xi}-n_{e1}\frac{\partial\phi_1}{\partial\xi}+\frac{\partial n_{e2}}{\partial\xi}=0\ , \tag{2.12}$$

$$V\frac{\partial n_{j1}}{\partial\tau}-V\frac{\partial n_{j2}}{\partial\xi}+n_{jo}\frac{\partial v_{j2}}{\partial\xi}+n_{j1}\frac{\partial v_{j1}}{\partial\xi}+v_{j1}\frac{\partial n_{j1}}{\partial\xi}=0\ ,$$

$$Q'Vn_{jo}\frac{\partial v_{j1}}{\partial \tau}-Q'Vn_{jo}\frac{\partial v_{j2}}{\partial \xi}-Q'Vn_{j1}\frac{\partial v_{j1}}{\partial \xi}-Q'n_{jo}v_{j1}\frac{\partial v_{j1}}{\partial \xi}-n_{jo}\frac{\partial \phi_2}{\partial \xi}$$

$$-n_{j1}\frac{\partial \phi_1}{\partial \xi}+\beta\frac{\partial n_{j2}}{\partial \xi}=0\ ,$$

$$\frac{\partial^2 \phi_1}{\partial \xi^2}=n_{e2}+n_{j2}-n_{i2}:$$

The set of equations (2.11), subject to the boundary conditions

$n_{i1}=n_{e1}=n_{j1}=0$, $v_{i1}=v_{e1}=v_{j1}=0$, at $|\xi|\to\infty$, after integration yields the results

$$n_{i1}=\frac{\phi_1}{V^2-\alpha}n_{io}\ ,\quad v_{i1}=\frac{V}{V^2-\alpha}\phi_1\ ,$$

$$n_{e1}=\frac{\phi_1}{1-Q(V-v'_e)^2}\ ,\quad v_{el}=\frac{V-v'_e}{1-Q(V-v'_e)^2}\phi_1\ , \tag{2.13}$$

$$n_{j1}=\frac{\phi_1}{\beta-Q'V^2}n_{jo}\ ,\qquad v_{j1}=\frac{V}{\beta-Q'V^2}\phi_1\ ,\qquad n_{i1}-n_{el}-n_{j1}=0.$$

Using the values of n_{i1} , n_{e1} , n_{j1} in the last equation of the set of equation (2.13), the expression for the phase velocity V can be put to the form

$$\left[1-Q(V-v'_e)^2\right]\left[n_{io}(\beta-Q'V^2)-n_{jo}(V^2-\alpha)\right]=(V^2-\alpha)(\beta-Q'V^2). \tag{2.14}$$

Differentiating the last equation of the set of equations (2.12), we get

$$\frac{\partial^3\phi_1}{\partial \xi^3}=\frac{\partial n_{e2}}{\partial \xi}+\frac{\partial n_{j2}}{\partial \xi}-\frac{\partial n_{i2}}{\partial \xi}. \tag{2.15}$$

First two equations of (2.12) give

$$\frac{\partial n_{i2}}{\partial \xi}=\frac{n_{io}V}{V^2-\alpha}\frac{\partial v_{i1}}{\partial \tau}+\frac{V^2}{V^2-\alpha}\frac{\partial n_{i1}}{\partial \tau}+\frac{V}{V^2-\alpha}v_{i1}\frac{\partial n_{i1}}{\partial \xi}+\frac{n_{io}}{V^2-\alpha}v_{i1}\frac{\partial v_{i1}}{\partial \xi}$$

$$+\frac{n_{io}}{V^2-\alpha}\frac{\partial \phi_2}{\partial \xi}+\frac{1}{V^2-\alpha}n_{i1}\frac{\partial \phi_1}{\partial \xi}. \tag{2.16}$$

Next two equations of (2.12) give

$$\frac{\partial n_{e2}}{\partial \xi} = \left[QV \frac{\partial v_{e1}}{\partial \tau} + QV(V - v'_e) \frac{\partial n_{e1}}{\partial \tau} + Q (V - v'_e) v_{e1} \frac{\partial n_{e1}}{\partial \xi} + Q v_{e1} \frac{\partial v_{e1}}{\partial \xi} - \frac{\partial \phi_2}{\partial \xi} - n_{e1} \frac{\partial \phi_1}{\partial \xi} \right] \Big/ \left[Q(V - v'_e)^2 - 1 \right]. \tag{2.17}$$

Fifth and sixth equations of the set (2.12) give

$$\frac{\partial n_{j2}}{\partial \xi} = \left[Q' V n_{jo} \frac{\partial v_{j1}}{\partial \tau} + Q' V^2 \frac{\partial n_{j1}}{\partial \tau} + Q' V v_{j1} \frac{\partial n_{j1}}{\partial \xi} + Q' n_{jo} v_{j1} \frac{\partial v_{j1}}{\partial \xi} - n_{jo} \frac{\partial \phi_2}{\partial \xi} - n_{j1} \frac{\partial \phi_1}{\partial \xi} \right] \Big/ \left[Q' V^2 - \beta \right]. \tag{2.18}$$

Lastly, elimination of the quantities $\frac{\partial n_{i2}}{\partial \xi}$, $\frac{\partial n_{e2}}{\partial \xi}$, $\frac{\partial n_{j2}}{\partial \xi}$ from the equations (2.15) , (2.16) , (2.17) and (2.18) give rise to the desired KdV equation

$$\frac{\partial \phi}{\partial \tau} + p\phi \frac{\partial \phi}{\partial \xi} + q \frac{\partial^3 \phi}{\partial \xi} = 0 \qquad \text{(replacing } \phi_1 \text{ by } \phi \text{)} \tag{2.19}$$

where $p = N/D$, $q = N'/D$ with (2.20)

$$N = [1 - Q(V - v'_e)^2]^2 (\alpha - Q'V^2)^2 (1 - \frac{2V^2}{\alpha - V^2}) - r(\alpha - V^2)(1 - \frac{2Q'V^2}{\alpha - Q'V^2})$$
$$- (1 - r) \left[1 - \frac{2Q(V - v'_e)^2}{1 - Q(V - v'_e)^2} \right] (\alpha - Q'V^2)^2 (\alpha - V^2)^2 ,$$

$$N' = (1 - r) (\alpha - V^2)^2 (\alpha - Q'V^2)^2 [1 - Q(V - v'_e)^2]^2 ,$$

$$D = 2\{ (1 - r) (\alpha - V^2)^2 (\alpha - Q'V^2)^2 QV(V - v'_e) + [1 - Q(V - v'_e)^2]^2$$
$$[rQ'V^2(\alpha - V^2)^2 + V^2(\alpha - Q'V^2)^2] \}.$$

Here $r = n_{jo} / n_{io}$ so that $n_{jo} = r/(1-r)$ and $n_{io} = 1/(1-r)$. Also, we have considered $T_i = T_j$ for the two ion species.

2.4 Solitary wave solution

Using the transformation $\eta = \xi - c\tau$, equation (2.19) may be integrated to give

$$-c\phi + \frac{1}{2}p\phi^2 + q\frac{\partial^2\phi}{\partial\eta^2} = 0,$$

under the boundary conditions

$$\phi = 0 \text{ and } \frac{\partial^2\phi}{\partial\eta^2} = 0 \quad \text{as } \eta \to \pm \infty.$$

Here, c is the velocity with which the wave travels towards the right.

Simplifying this result, we obtain

$$2\frac{\partial^2\phi}{\partial\eta^2}\frac{\partial\phi}{\partial\eta} = \frac{2}{q}\left(c\phi\frac{\partial\phi}{\partial\eta} - \frac{p}{2}\phi^2\frac{\partial\phi}{\partial\eta}\right).$$

On integration under the conditions

$$\phi = 0 \text{ as } \eta \to \pm \infty ,$$

we obtain

$$\phi = \frac{3c}{p}\,sech^2\left(\frac{1}{2}\sqrt{\frac{c}{q}}\,\eta\right). \tag{2.21}$$

Thus the wave amplitude is given by

$$\phi_0 = 3c/p.$$

The expression for the width of the soliton is found as

$$\Delta = 2\sqrt{q/c}.$$

2.5 Discussion

When the electrons are subjected to an initial drift motion, solitary waves with well accounted amplitude of compressive and rarefactive characters are found to exist in a plasma with negative ions for both $Q' < 1$ and $Q' > 1$. For $Q' < 1$, the amplitude of the compressive soliton rapidly decreases to zero [Fig. 1 (a)], at a critical value r_c of r, for each assigned value of v'_e chosen from its upper regime, and sharply increases with r after r_c. On the other hand, the corresponding widths [Fig.1 (b)], are found to change completely in opposite sense to those of amplitudes. But for $Q' > 1$, the amplitude (or width) of the compressive soliton for higher values of v'_e, decreases (or increases) as r increases to r_c and then changes the character [Figs. 2 (a) & 2 (b)], but not as rapidly as in the case of the former.

The value of r yielding minimum amplitude for $Q' > 1$ is greater than that for $Q' < 1$, at the same value of v'_e. But from figure 3 (b) and figure 4 (b), it is found for each α that the amplitude of the compressive soliton varies in wavy nature attaining a relative minimum and two maximums at definite values of v'_e for both $Q' > 1$ and $Q' < 1$ before being zero. The variations in widths are also depicted in figure 3 (a) and figure 4 (a). For $Q'(=2.6) > 1$, the amplitude of the compressive soliton that changes its character mildly to rarefactive one in the upper regime of v'_e [Fig.5], is greatest near $v'_e = 0$ when $r < 0.5$. But the greatest amplitude of the compressive soliton is unaccountable in the middle range of v'_e for $r \geq 0.5$. On the other hand, for $Q'(=0.2) < 1$ but small, the amplitude of the rarefactive soliton, which becomes a compressive one, [Fig. 6 (a)], in the upper regime of v'_e is also greatest near $v'_e = 0$. These amplitudes increase with r for a fixed value of v'_e. But the unaccountable high amplitudes of compressive or rarefactive solitons are observed [Fig. 6(a) and Fig. 6 (b)] only in the vicinity of the middle range of v'_e. Besides, solitons of such amplitudes are found for comparatively smaller values of r [Fig. 6 (a) and Fig. 6 (b)] also.

Again, for a negligible concentration of negative ions ($r = 0.01$), the greatest amplitude of the compressive soliton near $v'_e = 0$ is attained for $Q'(=0.8) < 1$ [Fig. 3 (b)] and $Q'(=2.6) > 1$ [Fig. 4 (b)] for higher values of ion temperature α only but in contrast, for a small α they are found in the higher regime of v'_e in both the cases.

It is to be mentioned that the amplitude of the rarefactive soliton decreases towards a critical v'_e for small $Q'(<1)$ [Fig. 6 (a)], but for higher $Q'(<1)$, the amplitude of the compressive soliton decreases and those of rarefative soliton increases and decreases depending upon the values of r [Fig. 6 (b)].

References

Baboolal, S., Bharathram, R. and Hellberg, M. A. (1989) J. Plasma Phys. **41**, 341

Das, G. C. (1979) Plasma Phys., **21**, 257

Davidson, R.C. (1972) Methods in nonlinear plasma theory (Academic Press, New York, London), Chapter 2.

Kalita, B. C. and Kalita, M. K. (1990) Phys. Fluids **B2**, 674

Kalita , B. C., Kalita, M. K. and Chutia, J. (1986) J. Phys. A: Math. Gen., **19**, 3559

Karpman, V. I. and Kadomtsev, B. B. (1971) Sov. Phys. Usp. **14,** 40

Korteweg, D. J. and de - Vries, H. (1895) Philos. Mag. **39**, 422

Leven, R. and Steimann, W. (1979) Plasma Phys. , **3/ 80**, 195

Ludwig , G. O. Ferreira, J. L. Nakamura, Y. (1984) Phys. Rev. Lett., **52** , 275

Nakamura, Y. (1985) Nonlinear and environmental electromagnetics (ed. H. Kikuchi), Elsevier, 401

Nakamura, Y. (1987) J. Plasma Phys, **38,** 461

Nakamura, Y. and Tsukabayashi, I. (1984) Phys. Rev. Lett., **52,** 2356

Nakamura, Y.J., Ferreira, L. and Ludwig, G.O. (1985a) J. Plasma Phys., **33,** 237

Sagdeev , R. Z. (1966) Reviews of Plasma Physics, **3**, p 23 (NEW YORK , CONSULTANT BUREAU),

Tagare, S.G. (1972) Plasma Phys., **15**, 1247

Tagare, S.G. (1986) J. Plasma Phys., **3**, part 2, 301

Tagare, S.G. and Reddy, R.V. (1987) Plasma Phys., Countr. Fusion, **29**, 671

Tajiri, M. and Toda, M. (1985) J. Phys. Soc. Japan, **54**, 19

Tappert, F. D. (1972) Phys. Fluids, **15**, 2446

Verheest, F. (1988) J. Plasma Phys. **39**, 71

Washimi, H and Tanuiti, T. (1966) Phys. Rev. Lett. **17,** 996

Watanabe, S. (1984) J. Phys. Soc. Japan, **53 (3),** 950

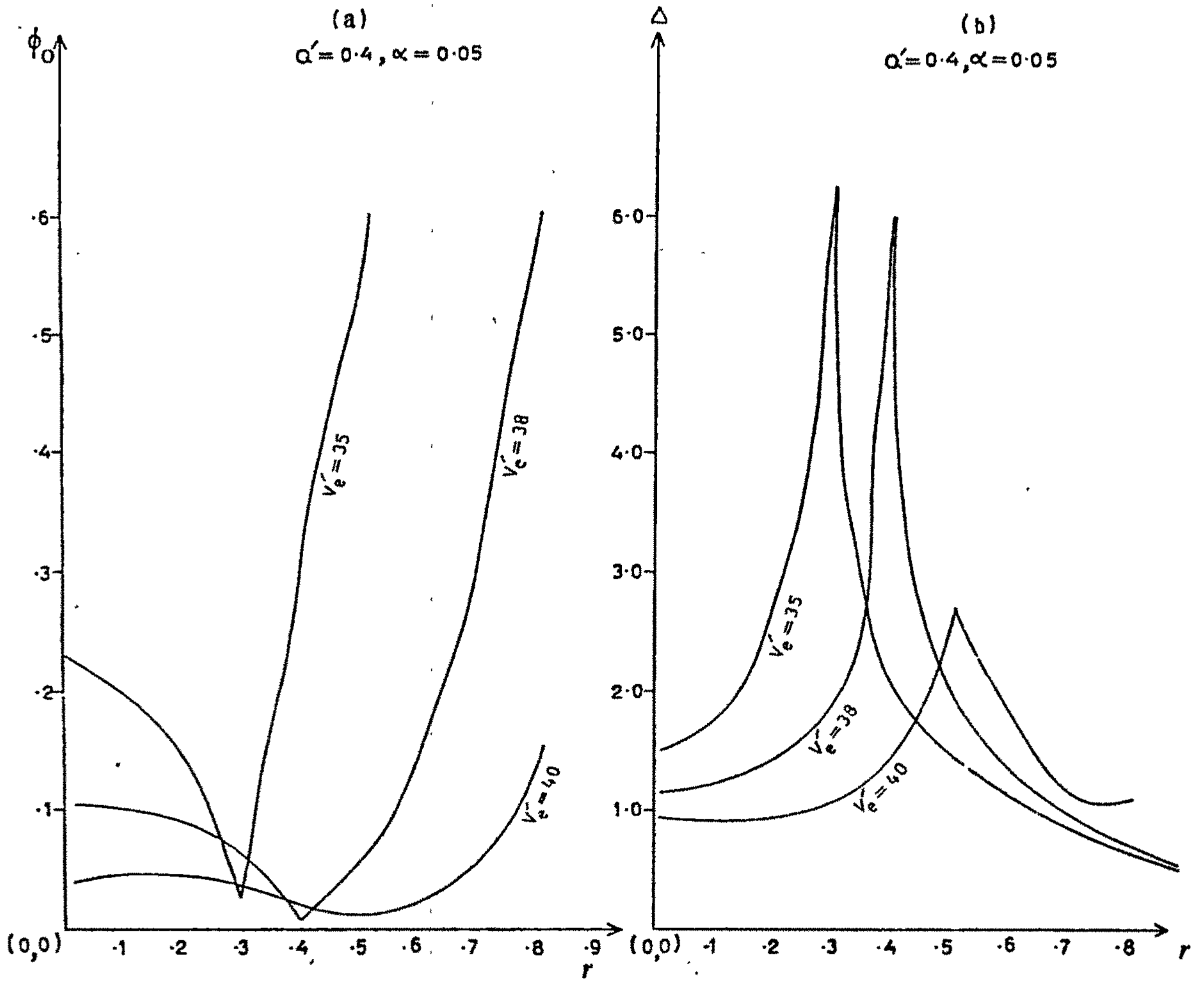

Fig. 1. Amplitude (ϕ_0) versus density ratio r (a) and width (Δ) (b) of solitons for fixed

$Q' = 0.4$ (< 1) and $\alpha = 0.05$.

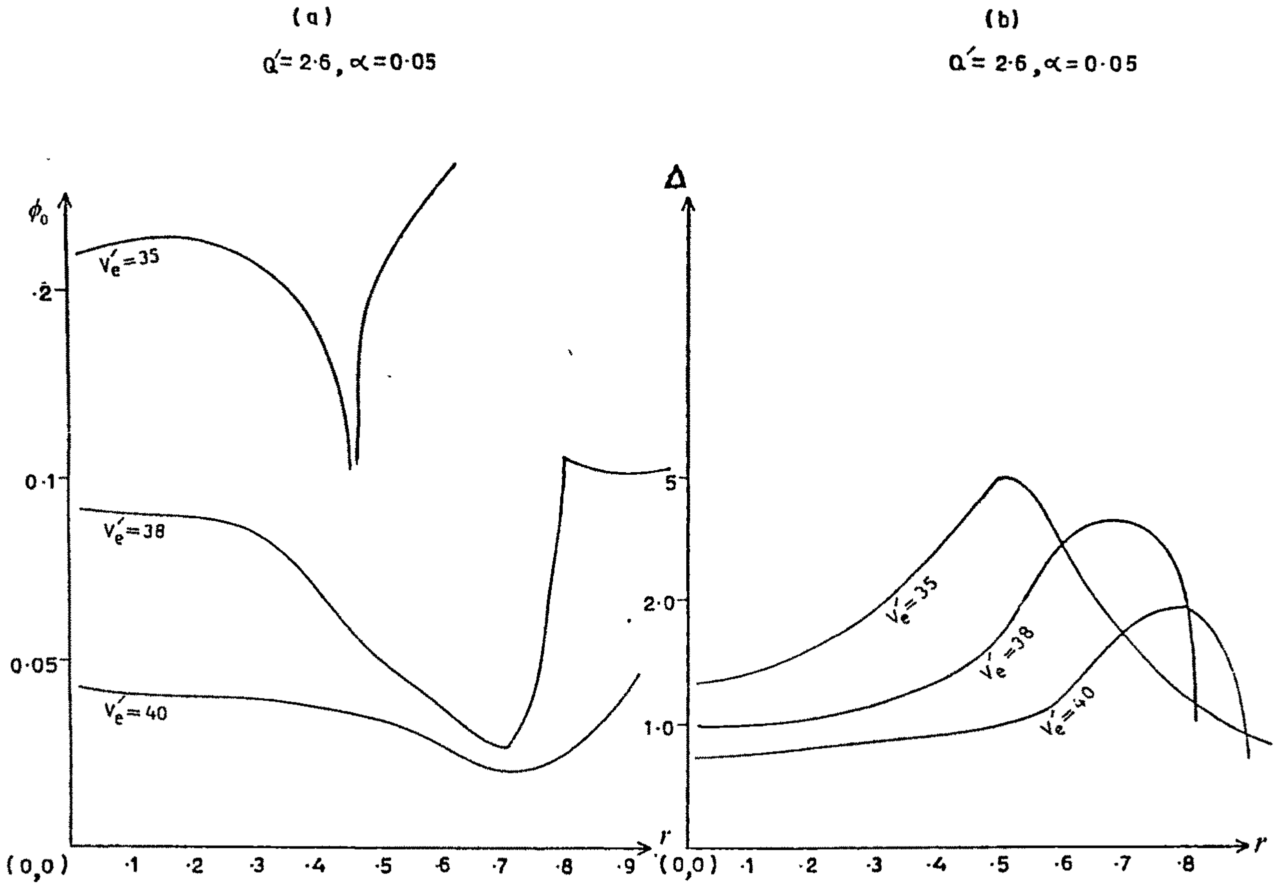

Fig 2. Amplitude (ϕ_0) versus density ratio r (a) and width (Δ) (b) of solitons for fixed

$Q' = 2\,6$ (> 1) and $\alpha = 0.05$

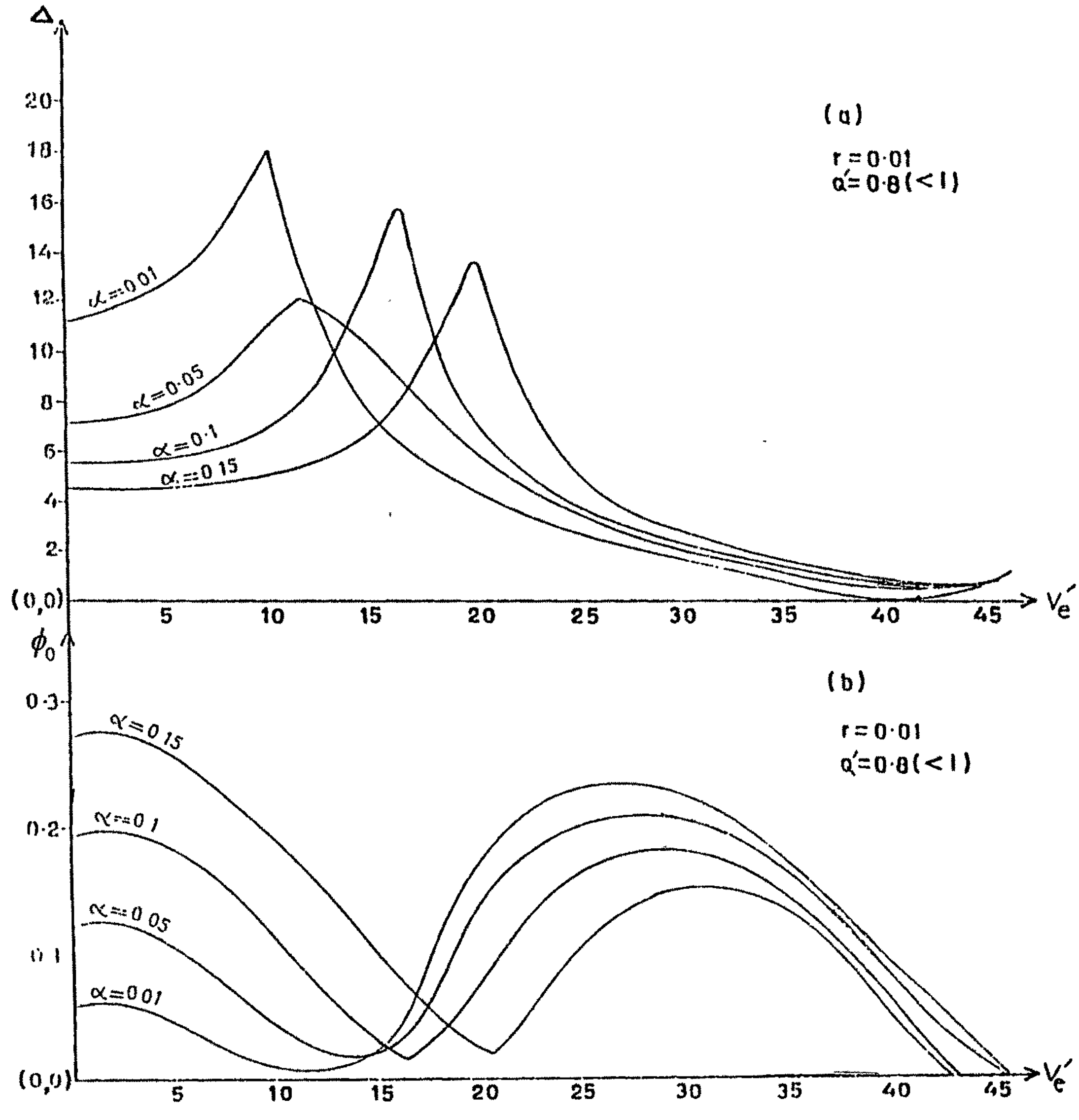

Fig.3. WidthΔ (a) and amplitude ϕ_0(b) of solitons versus drift velocity v'_o of electrons for different values of α shown against the curve when $Q' = 0.8$ (< 1) and $r = 0.01$.

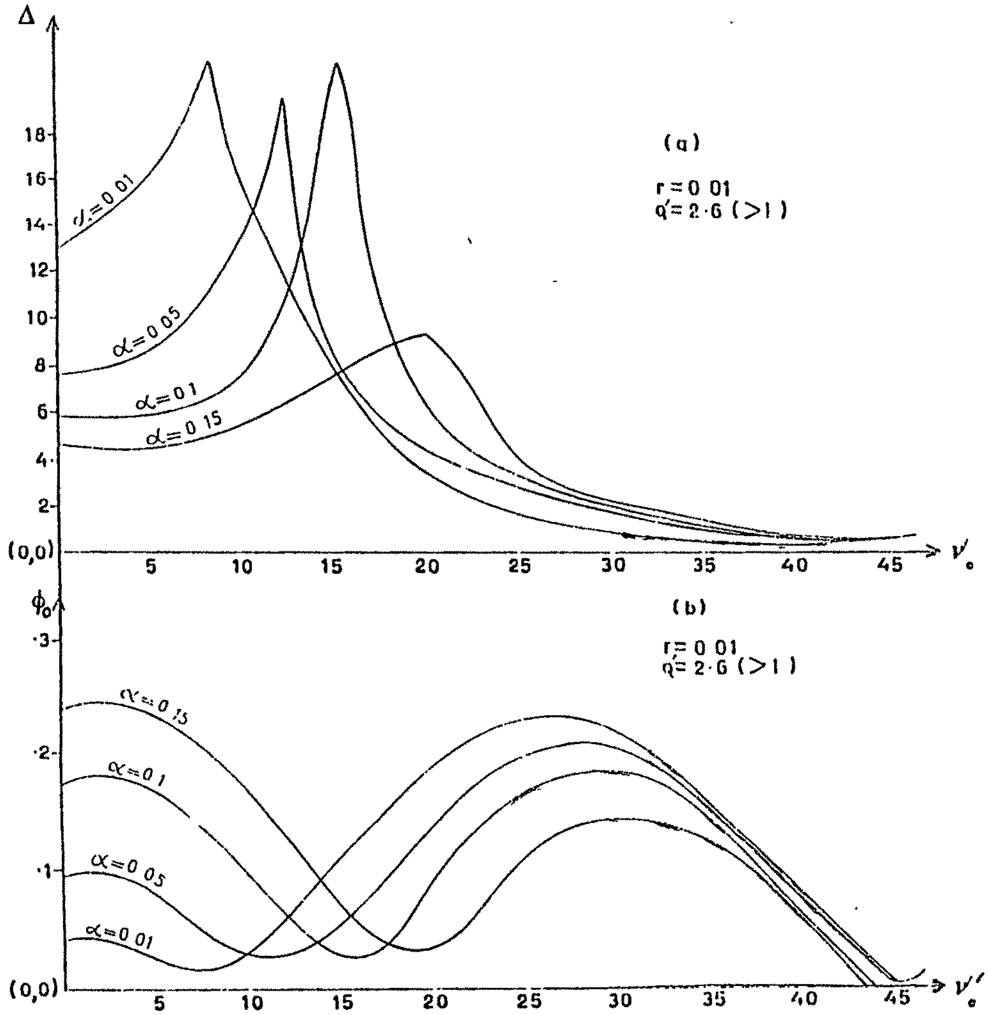

Fig.4. Width Δ (a) and amplitude ϕ_0(b) of solitons versus drift velocity v'_0 of electrons for different values of α shown against the curve when $Q' = 2.6$ (> 1) and $r = 0.01$.

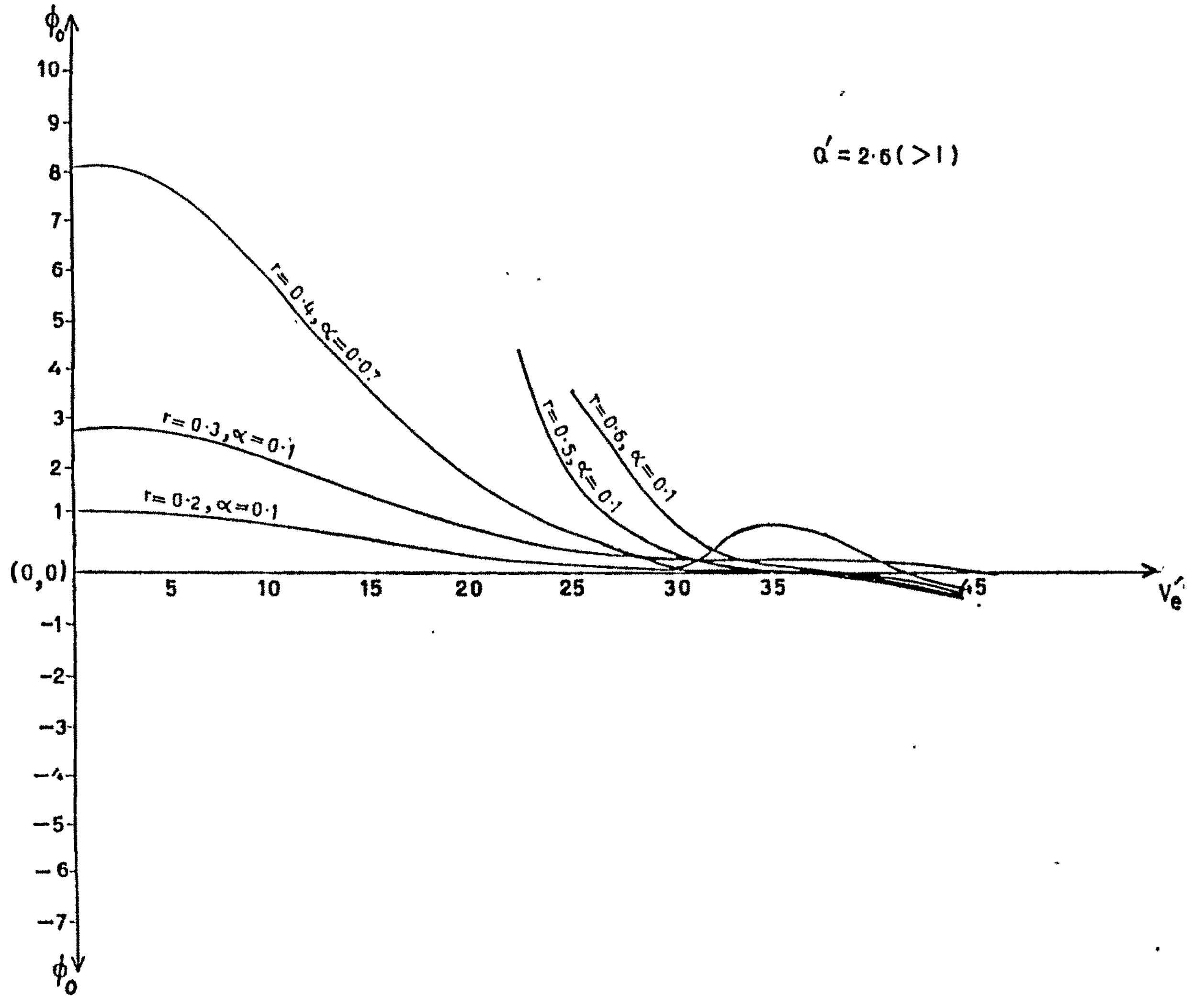

Fig.5. Ampiltude (ϕ_0) of compressive and rarefactive solitons versus drift velocity v'_e of electrons for fixed values of α and r shown against the curves when $Q' = 2.6$ (> 1).

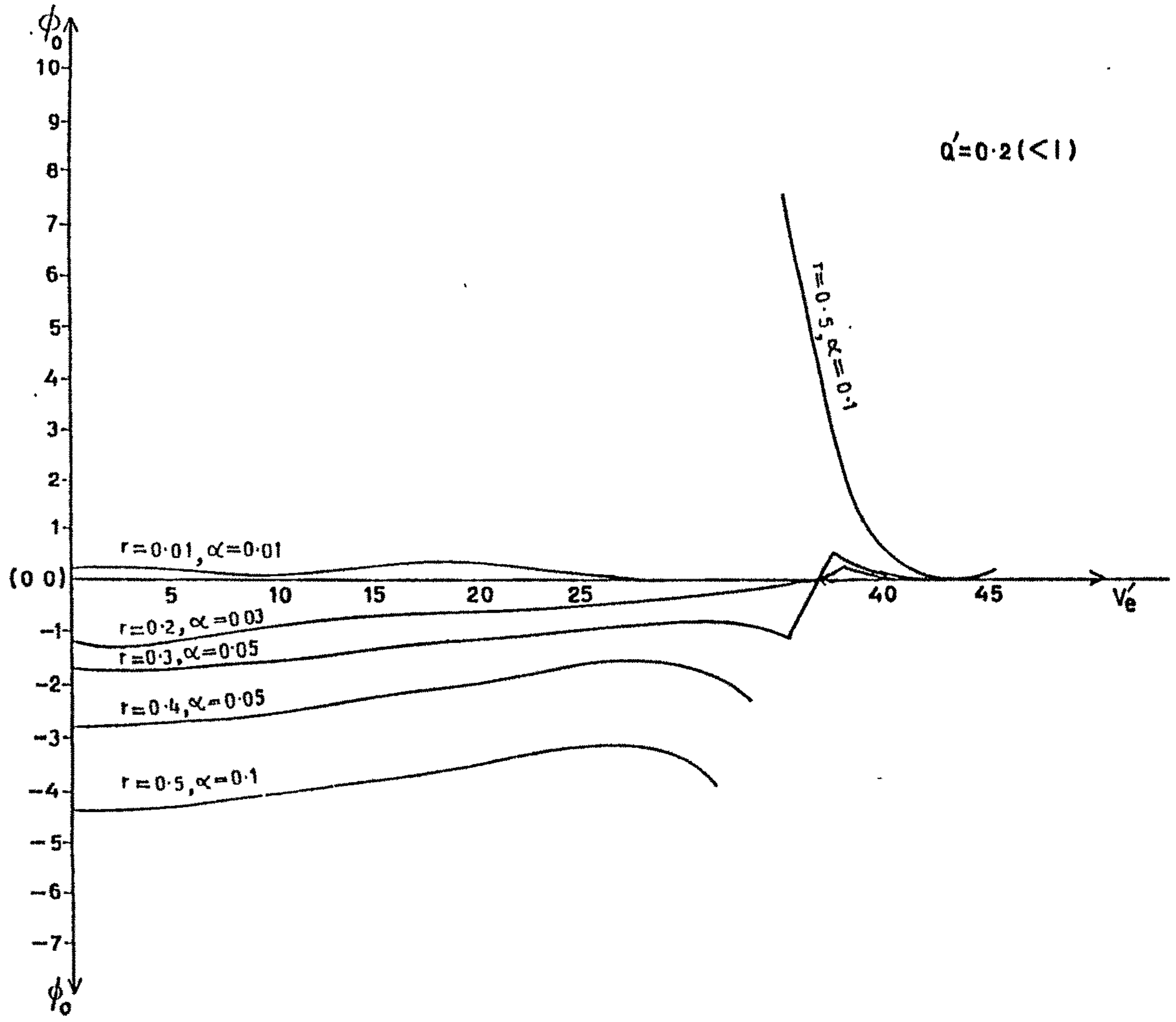

Fig.6(a). Amplitude (ϕ_0)of compressive and rarefactive solitons versus drift velocity v'_e of electrons for fixed values of α and r shown against the curves when$Q' = 0.2$ (< 1).

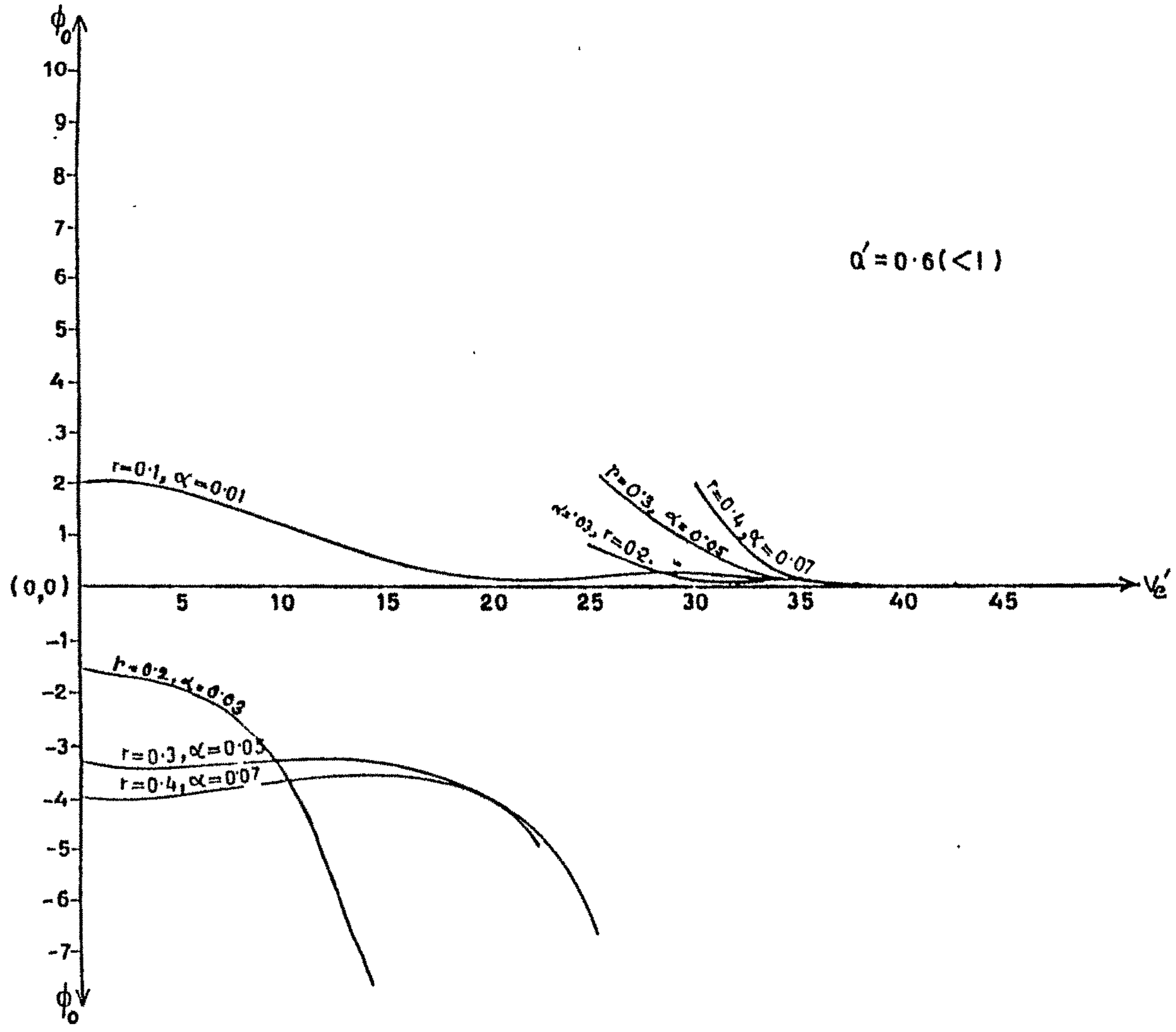

Fig.6(b). Amplitude (ϕ_0) of compressive and rarefactive solitons versus drift velocity v'_0 of electron for fixed values of α and r shown against the curves when $Q' = 0.6$ (< 1).