## APPENDIX II

The basic equations ( 5.1 ) - ( 5.8 ), before normalization are

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_{ez}) = 0 \qquad (5.i)$$

$$\frac{\partial v_{ez}}{\partial t} + v_{ez}\frac{\partial v_{ez}}{\partial z} = \frac{e}{m_e}\left(\frac{\partial \psi}{\partial z} - \frac{T_e/e}{n_e}\frac{\partial n_e}{\partial z}\right) \qquad (5.ii)$$

for the electrons,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) + \frac{\partial}{\partial z}(n_i v_{iz}) = 0 \qquad (5.iii)$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial v_{ix}}{\partial x} + v_{iz}\frac{\partial v_{ix}}{\partial z} = -\frac{e}{m_i}\left(\frac{\partial \phi}{\partial x} + \frac{T_i}{e n_i}\frac{\partial n_i}{\partial x}\right) + \frac{eB_0}{m_i}v_{iy} \qquad (5.iv)$$

$$\frac{\partial v_{iy}}{\partial t} + v_{ix}\frac{\partial v_{iy}}{\partial x} + v_{iz}\frac{\partial v_{iy}}{\partial z} = -\frac{eB_0}{m_i}v_{ix} \qquad (5.v)$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial v_{iz}}{\partial x} + v_{iz}\frac{\partial v_{iz}}{\partial z} = -\frac{e}{m_i}\left(\frac{\partial \psi}{\partial z} + \frac{T_i}{e n_i}\frac{\partial n_i}{\partial z}\right) \qquad (5.vi)$$

for the ions,

$$\frac{\partial^4(\phi - \psi)}{\partial x^2 \partial z^2} = \mu_0 e\left[\frac{\partial^2}{\partial t\,\partial z}\left(n_i v_{iz}\right) + \frac{\partial^2 n_e}{\partial t^2}\right], \mu_0 = \frac{4\pi}{c^2} \qquad (5.vii)$$

for the integrated Maxwell's equations (derived in chapter 4) and

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial z^2} = -4\pi e\,(n_i - n_e) \qquad (5.viii)$$

the Poisson equation.

Linearizing equations ( 5.i) –( 5.viii ) and assuming disturbances to vary as $e^{i(k.r-\omega t)}$ as in Appendix I, we get

$$v_{e_1 z} = \frac{\omega - v'_e k_z}{n_o k_z} n_{e1}$$

$$n_{e1} = \frac{e n_o k_z^2}{T_e k_z^2 - m_e(\omega - v'_{ez} k_z)^2}\psi_1 \qquad (Ap.A')$$

$$v_{i1x} = \frac{e k_z m_i \omega}{(m_i^2 \omega^2 - e^2 B_0^2)} \phi_1 + \frac{T_i k_x m_i \omega}{n_0 (m_i^2 \omega^2 - e^2 B_0^2)} n_{i1}$$

$$v_{i1z} = \frac{e k_z}{m_i \omega} \psi_1 + \frac{T_i k_z}{n_0 \omega m_i} n_{i1} \qquad (\text{Ap.B}')$$

$$n_{i1} = \frac{e n_0 m_i^2 \omega^2 k_x^2}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} \phi_1$$

$$+ \frac{e n_0 k_z^2 (m_i^2 \omega^2 - e^2 B_0^2)}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} \psi_1 \qquad (\text{Ap.C}')$$

Using ( Ap.A$'$ ) and ( Ap.B$'$ ) for $n_{e1}$ and $v_{i1z}$ in the linearized Maxwell's equations, we have

$$n_{i1} = \frac{k_x^2 c^2 m_i}{4\pi e T_i} \phi_1 - \frac{k_z^2 \left[ k_x^2 + \frac{4\pi e^2 n_0}{c^2 m_i} - \frac{4\pi e^2 n_0 \omega^2 / c^2}{T_e k_z^2 - m_e (\omega - v'_e k_z)^2} \right]}{4\pi e k_z^2 T_i / c^2 m_i} \psi_1 \qquad (\text{Ap.D}')$$

Using ( Ap.A$'$) and ( Ap.C$'$ ) for values of $n_{e1}$ and $n_{i1}$ respectively, in the linearized Poisson's equation, we have

$$k_x^2 \left[ 1 - \frac{4\pi e^2 n_0 m_i^2 \omega^2}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} \right] \phi_1 =$$

$$k_z^2 \left[ \frac{4\pi e^2 n_0 (m_i^2 \omega^2 - e^2 B_0^2)}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} - \frac{4\pi e^2 n_0}{T_e k_z^2 - m_e (\omega - v'_e k_z)^2} - 1 \right] \psi_1$$

( Ap.X )

From ( Ap.C$'$) and ( Ap.D$'$ )

$$\left[ \frac{k_x^2 c^2 m_i}{T_i} - \frac{4\pi e^2 n_0 m_i^2 \omega^2 k_x^2}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} \right] \phi_1 =$$

$$k_z^2 \left[ \frac{4\pi e^2 n_0 (m_i^2 \omega^2 - e^2 B_0^2)}{(m_i \omega^2 - T_i k_z^2)(m_i^2 \omega^2 - e^2 B_0^2) - T_i m_i^2 \omega^2 k_x^2} \right.$$

$$\left. + \frac{k_z^2 \; k_x^2 + \frac{4\pi e^2 n_0}{c^2 m_i} - \frac{4\pi e^2 n_0 \omega^2 / c^2}{T_e k_z^2 - m_e (\omega - v'_e k_z)^2}}{k_z^2 T_i / c^2 m_i} \right] \psi_1 \qquad (\text{Ap.Y})$$

( Ap.X ) Can be written as

$$k^2_x \left[ 1 - \frac{(4\pi e^2 n_o/m_i)\omega^2}{\{\omega^2 - (T_i/T_e)(T_e/m_i)k^2_z\}(\omega^2 - e^2B^2_o/m^2_i) - (T_i/T_e)(T_e/m_i)\omega^2 k^2_x} \right]\phi_1 =$$

$$k_z^2 \left[ \frac{4\pi e^2 (\omega^2 - e^2B^2_o/m^2_i)}{\{\omega^2 - (T_i/T_e)(T_e/m_i)k^2_z\}(\omega^2 - e^2B^2_o/m^2_i) - (T_i/T_e)(T_e/m_i)\omega^2 k^2_x} - \frac{4\pi e^2 n_o/m_i}{(T_e/m_i)(m_i/m_e)k^2_z - (\omega - v_e k_z)^2} \right]\psi_1$$

$$k^2_x \left[ 1 - \frac{\omega^2_{pi}\omega^2}{(\omega^2 - \alpha c^2_s k^2_z)(\omega^2 - \Omega^2_{ci}) - \alpha c^2_s k^2_x \omega^2} \right]\phi_1 =$$

$$k^2_z \left[ \frac{\omega^2_{pi}(\omega^2 - \Omega^2_{ci})}{(\omega^2 - \alpha c^2_s k^2_z)(\omega^2 - \Omega^2_{ci}) - \alpha c^2_s k^2_x \omega^2} + \frac{\omega^2_{pe}}{\lambda^2} - 1 \right]\psi_1 \qquad \text{( Ap.X )}$$

Where $\lambda^2 = (\omega - v_e k_z)^2 - c^2_s k^2_z / Q$,

$Q = m_e/m_i$ (electron to ion mass ratio), $\alpha = T_i/T_e$ (ion to electron temperature ratio)

( Ap.Y ) can be written as

$$k^2_x \left[ \frac{c^2}{(T_i/T_e)(T_e/m_i)} - \frac{(4\pi e^2 n_o/m_i)\omega^2}{\{\omega^2 - (T_i/T_e)(T_e/m_i)k^2_z\}(\omega^2 - e^2B_o^2/m^2_i) - (T_i/T_e)(T_e/m_i)k^2_x\omega^2} \right]\phi_1$$

$$= \left[ \frac{k_z^2(4\pi e^2 n_o/m_i)(\omega^2 - e^2B^2_0/m^2_i)}{\{\omega^2 - (T_i/T_e)(T_e/m_i)k_z^2\}(\omega^2 - e^2B_o^2/m_i^2) - (T_i/T_e)(T_e/m_i)k^2_x\omega^2} + \frac{k^2_x + \frac{4\pi e^2 n_o}{m_i c^2} - \frac{(4\pi e^2 n_o/m_e)\omega^2/c^2}{(T_e/m_i)(m_i/m_e)k^2_z - (\omega - v_e k_z)^2}}{(T_i/T_e)(T_e/m_i)/c^2} \right]\psi_1$$

or,

$$k^2_x \left[ \frac{c^2}{\alpha c^2_s} - \frac{\omega^2_{pi}\omega^2}{(\omega^2 - \alpha c^2_s k^2_z)(\omega^2 - \Omega^2_{ci}) - \alpha c^2_s k^2_x \omega^2} \right]\phi_1 =$$

$$\left[\frac{k_z^2\omega_{pi}^2(\omega^2-\Omega_{ci}^2)}{(\omega^2-\alpha c_s^2k_z^2)(\omega^2-\Omega_{ci}^2)-\alpha c_s^2k_x^2\omega^2}+\frac{k_x^2+\frac{\omega_{pi}^2}{c^2}+\frac{\omega_{pe}^2\omega^2}{c^2\lambda^2}}{\alpha c_s^2/c^2}\right]\psi_1.$$

( Ap.Y′)

Eliminating $\phi_1$ and $\psi_1$ between ( Ap.X′) and ( Ap.Y′) we have

$$k_x^2\left[1-\frac{\omega^2c^2}{v_A^2(\omega^2-\alpha c_s^2k_z^2)(\omega^2/\Omega_{ci}^2-1)}\right]\times\left[1+\frac{\omega_{pi}^2}{k_x^2c^2}+\frac{\omega_{pe}^2\omega^2}{k_x^2c^2\lambda^2}\right]=$$

$$k_z^2\left[\left(1-\frac{\alpha\beta v_A^2}{2c^2}\right)\frac{\omega_{pi}^2(\omega^2/\Omega_{ci}^2-1)}{(\omega^2-\alpha c_s^2k_z^2)(\omega^2/\Omega_{ci}^2-1)}+(\omega_{pe}^2/\lambda^2-1)\left\{1-(\alpha\beta v_A^2/2c^2)\frac{\omega^2c^2}{v_A^2(\omega^2-\alpha c_s^2k_z^2)(\omega^2/\Omega_{ci}^2-1)}\right\}\right]$$

We have used the results ,

$$\frac{\omega_{pi}^2}{\Omega_{ci}^2}=\frac{c^2}{v_A^2}\quad,\quad\frac{\alpha c_s^2}{c^2}=\frac{\alpha\beta v_A^2}{2c^2}\quad,\quad\beta=8\pi n_oT/c^2B_o^2.$$

Or, $$k_x^2\left[c^2-v_A^2\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)\left(1-\frac{\alpha c_c^2k_z^2}{\omega^2}\right)\right]\times\left[1+\frac{\omega_{pi}^2}{k_x^2c^2}+\frac{\omega_{pe}^2\omega^2}{k_x^2c^2\lambda^2}\right]=$$

$$\frac{k_z^2}{\omega^2}\left[\left(1-\frac{\alpha\beta v_A^2}{2c^2}\right)\frac{\omega_{pi}^2\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)}{\left(\omega^2-\alpha c_s^2k_z^2\right)\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)}\right.$$

$$\left.+\left(\frac{\omega_{pe}^2}{\lambda^2}-1\right)\left\{1-\frac{\alpha\beta v_A^2}{2c^2}\times\frac{\omega^2c^2}{v_A^2\left(\omega^2-\alpha c_s^2k_z^2\right)\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)}\right\}\right]v_A^2$$

$$\times\left(\omega^2-\alpha c_s^2k_z^2\right)\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)$$

$$\text{or, } k_x^2\left[v_A^2\left(\omega^2-\alpha c_s^2 k_z^2\right)\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)-\omega^2 c^2\right]\times\left[1+\frac{\omega_{pi}^2}{k_x^2 c^2}+\frac{\omega_{pe}^2\omega^2}{k_x^2 c^2\lambda^2}\right]$$

$$= k_z^2 v_A^2\left[\left(1-\frac{\alpha\beta v_A^2}{2c^2}\right)\omega_{pi}^2\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)\right.$$

$$\left.+\left(\frac{\omega_{pe}^2}{\lambda^2}-1\right)\left\{\left(\frac{\omega^2}{\Omega_{ci}^2}-1\right)\left(\omega^2-\alpha c_s^2 k_z^2\right)-\frac{\alpha\beta}{2c^2}\omega^2 c^2\right\}\right]$$

Neglecting $\frac{\omega^2}{\Omega_{ci}^2}, \frac{Q}{c^2}, \frac{Q}{\omega^2}$

$$\left(\frac{\lambda^2 c^2}{\omega_{pe}^2}\left[\frac{k_z^2 v_A^2}{c^2}\left(1-\frac{\alpha\beta}{2}\right)+k_x^2\left(1+\frac{v_A^2}{c^2}\right)\right]\right.$$

$$\left.-\left[k_z^2 v_A^2\left(1-\frac{\alpha\beta}{2}\right)+k_z^2 v_A^2\frac{\alpha\beta v_A^2}{2c^2}\right]\right)\omega^2 =$$

$$\frac{\lambda^2 c^2}{\omega_{pe}^2}\left[\frac{k_z^2 v_A^2\,\alpha\, c_s^2 k^2}{c^2}\right]-k_z^4 v_A^2\alpha c_s^2-\omega^4\left(1+\frac{v_A^2}{c^2}\right)$$

$$\text{or, }\left(1+\frac{v_A^2}{c^2}\right)\omega^4+\left[\left(1+\frac{v_A^2}{c^2}\right)\frac{\lambda^2 c^2}{\omega_{pe}^2}k_x^2-k_z^2 v_A^2\left\{\left(1-\frac{\alpha\beta}{2}\right)\left(1-\frac{\lambda^2}{\omega_{pe}^2}\right)\right.\right.$$

$$\left.\left.+\frac{\alpha\beta v_A^2}{2c^2}\right\}\right]\omega^2+\frac{\alpha\beta}{2}k_z^2 v_A^4\left(k_z^2-\frac{\lambda^2 k^2}{\omega_{pe}^2}\right)=0$$

Where $\lambda^2=(\omega-v_e' k_z)^2-k_z^2 c_s^2/Q$

This is the linear dispersion relation for the solitary kinetic Alfvén waves. For the cold plasma ($\alpha=0$), this dispersion relation takes the form

$$k_z^2 v_A^2-\omega^2(1+v_A^2/c^2)=(\lambda^2 c^2/\omega_{pe}^2)[k_z^2 v_A^2/c^2+k_x^2(1+v_A^2/c^2)].$$

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