CHAPTER 6

STUDIES OF INSTABILITIES UNDER THE DRIFTING EFFECT OF ELECTRONS OF KINETIC ALFVÉN WAVES IN PLASMA

6.1 Introduction

A medium goes unstable if any small deviation from the prescribed condition for an exact solution renders significant deviation from that solution. Hence, one can employ a linearized analysis based on small perturbation about the exact solution state in order to study any stability problem since nonlinear stability problems are very difficult. Even with the rapid advances of computer technology, numerical studies are still limited to cases of relatively mild instabilities, i.e., for cases whose instability parameters are not much beyond some critical values. It is found that as the instability parameters increase, the system often first develops regular organised patterns, then the patterns become more and more complex, and finally irregular and chaotic situation arises.

We know that electromagnetic waves and magnetoacoustic waves across a magnetic field are modulationally stable for all wave numbers (Kakutani and Sugimoto,1974). On the other hand, surface waves in structured magnetized plasmas (low - frequency) and the rf - wave damping at the Alfvén resonance (for laboratory plasmas), have important roles in the heating of solar corona and the supplementary heating of plasmas to fusion temperatures respectively (Cramer,1994). Kinetic effects on Alfvén waves have been studied to get regions of stability and monotonic instability (Bora and Talwar, 1993) and to show the influence of direction of propagation on the dispersion equation. In most cases of hydromagnetic instabilities (in particular, when the spatial field variations are large compared to the characteristic space), the Fourier representation of the fluctuating quantities which can permit the development of a dispersion relation, is appreciated for the eigenvalues to be obtained in order to examine the nature of stable or unstable wave mode.

Mathematically strongest and easiest tool for the stability analysis of a dynamical system is the consideration of the (x, t) dependence as proportional to $e^{i(kx-\omega t)}$ namely, the normal mode analysis. In the analysis, the growing mode leading to certain instability mechanism needs to be identified so as to detect the type of instability and the influence of the source of free enegy, available for the growth of unstable mode. Following the classification criteria of Mikhailovskii (1974), Hasegawa (1975), Cap (1976), we can follow the analysis scheme in case of kinetic Alfvén mode, under the effect of electron inertia through a fluid model related to the dynamics of motion. Low- frequency Kinetic Alfvén waves (KAW) can occur in ionospheric F- region satisfying the condition $1 >> \beta >> m_e / m_i$, where Buneman type of instability driven by electron inertia effects can generate several permanent type of energy spectrums. These KAWs are also responsible for large- scale plasma heating. Laboratory plasmas also encounter KAW. Guha and Asthana (1990) have studied parametric instabilities involving KAW. In this chapter, we consider the linear stability analysis of kinetic Alfvén waves under the drifting effect of electrons through the linear dispersion relations derived in chapters 4 and 5. Numerical calculations are carried out to see the effect of electrons' drift motion in the growth of the instability. An attempt is also made to differentiate the instability mechanism related to the warm situation of chapter 5 from that of the cold situation of chapter 4. Critical values of wave numbers have also been observed for the specific type of the instability produced in the simple process.

6.2 Mathematical formulation

The linear dispersion relation is of the form

$$D (k, \omega, v'_e) = 0 \tag{6.1}$$

which is algebraic in the case of fluid approach (easily applicable for the much denser ionospheric region near the earth) for plasma waves. The regions, for which $Im\,(\omega) > 0$, are called the regions of growth of the instability. If the corresponding $Re\,(\omega)$'s are non-zero, then the process corresponding to the perturbed motion of the plasma is called periodic. For the excitement of plasma oscillations during a certain interval of time, the behaviour of the perturbation is studied through the properties of $\omega = \omega (k, v'_e)$. The original form of the perturbation say $p (\mathbf{r})$ is given by $p (\mathbf{r}, \mathbf{v}) = \int \; p_k (\mathbf{v})\, e^{i\mathbf{k}\mathbf{r}}\, dk$, where the integration is to be performed over the variation in space, and comes under the kinetic treatment in plasma.

We first consider the following linear dispersion relation for the KAW in a cold plasma described by the set of equations (4.1) to (4. 8) in chapter 4 [Appendix I]

$$\left[(c^2/\omega^2_{pe})(k^2_x + k^2 v^2_A/c^2) + (1 + v^2_A/c^2)\right]\omega^2 - 2v'_e k_z (c^2/\omega^2_{pe})(k^2_x + k^2 v^2_A/c^2)\omega$$

$$+ (c^2/\omega^2_{pe})(v'^2_e - v^2_A\beta/2Q)(k^2_x + k^2 v^2_A/c^2)k^2_z - k^2_z v^2_A = 0. \qquad (6.2)$$

The equation (6.2) has real roots for $v'^2_e \le v^2_A \beta / 2Q$.

Taking v'_e as the parameter for the cause of an excitation, the small perturbation can be expressed as

$v'^2_e (1 + \varepsilon) = v^2_A \beta / 2Q$, (Infeld & Rowlands, 1992, p. 50)

where $1 >> \beta >> m_e / m_i$, $\quad 0 < \varepsilon << 1$.

Using this relation in (6.2), we get

$$\left\{(c^2/\omega^2_{pe})(k^2_x + k^2 v^2_A/c^2) + (1 + v^2_A/c^2)\right\}\omega^2 - 2v'_e k_z (c^2/\omega^2_{pe})(k^2_x + k^2 v^2_A/c^2)\omega$$

$$+ (c^2/\omega^2_{pe})(-\varepsilon v'^2_e)(k^2_x + k^2 v^2_A/c^2)k^2_z - k^2_z v^2_A = 0. \qquad (6.3)$$

Assuming that ω^2 is of the order of ε, we have from equation (6.3) out of the ε – order ,

$$\left\{(c^2/\omega^2_{pe})(k^2_x + k^2 v^2_A/c^2) + (1 + v^2_A/c^2)\right\}\omega^2 + (c^2/\omega^2_{pe})(v'^2_e - v^2_A\beta/2Q)$$

$$(k^2_x + k^2 v^2_A/c^2)k^2_z = 0.$$

For a stable mode, we must get real wave frequency ω. The condition for stability is obtained from the above relation, the second term of which must be negative, as

$$v'^2_e < v^2_A \beta / 2Q.$$

Let v_c be the critical value of the electrons' drift velocity; beyond which we expect instability and let ω_c be the wave frequency (real) corresponding to v_c. To study the characteristic change in the model of plasma due to the affecting parameter v'_e, we put

$v'_e = v_c + \varepsilon$; $\omega = \omega_c + \varepsilon\,\omega_1$. We have taken $O(\omega) < \varepsilon$.

Now, the dispersion relation (6.1) is actually of the form

$D(k, \omega_c + \varepsilon\omega_1, v_c + \varepsilon) = 0$, $\quad k$ being real.

Expanding it in a Taylor's series, we have

$$\frac{\partial D(k,\omega,v'_e)}{\partial v_c} + \frac{\partial D(k,\omega,v'_e)}{\partial \omega_c}\,\omega_1 = 0.$$

Putting $\frac{\partial D}{\partial \omega_c} = 0$, we see that

$$\left\{(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2) + (1 + v^2_A/c^2)\right\}\omega_c + (c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2)\,v'_e k_z = 0$$

so that

$$\omega_c = \frac{(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2)\,v'_e k_z}{\left\{(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2) + (1 + v^2_A/c^2)\right\}}$$

which is real since ω_1 is not present here.

To the second order expansion of the dispersion relation in Taylor's series with a new ordering of ω such that $\omega = \omega_c + (\varepsilon\,\omega_1)^{1/2}$ on the wave frequency ω, (Infeld and Rowlands, 1992, p.51), we have to the order of ε

$$\frac{\partial D}{\partial v_c} + \omega^2_1 \frac{\partial^2 D}{\partial \omega^2_c} = 0\,.$$

To generate instability through an affecting parameter (here v'_e), we expect complex root of ω for which it is essential to know the sign of ω^2_1 from the expansion upto the desired terms.

Here $\frac{\partial D}{\partial v_c} = 2(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2)\,(v'_e k^2_z - k_z\,\omega)$

and

$$\frac{\partial^2 D}{\partial \omega^2_c} = (c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2) + (1 + v^2_A/c^2).$$

Thus we solve for ω^2_1 as

$$\omega^2_1 = -2\,\frac{(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2)\,v'_e k^2_z}{(c^2/\omega^2_{pe})(k^2_x + k^2_z v^2_A/c^2) + (1 + v^2_A/c^2)}\,. \qquad (6.4)$$

The minus sign indicates the existence of instability sought for the marginal mode given by

$\omega = 0$ satisfying $\qquad \frac{\partial D}{\partial \omega} = 0.$

Here ω_1 is the growth rate of the instability.

For the warm kinetic Alfvén mode, the linear dispersion relation is of the degree four in ω obtainable from the set of equations (5.1) to (5.8) [Appendix II]. Taking ω^4 to the order of $\varepsilon^2 (\varepsilon << 1)$ we write

$$v'^2_e (1+\varepsilon)^{1/2} = v^2_A \beta / 2Q .$$

Retaining terms upto ε^2, we get from the dispersion relation

$$(1+ v^2_A/c^2)\omega^4 + \left[(1+ v^2_A/c^2)(c^2k^2_x/\omega^2_{pe}) + \left\{k^2_z v^2_A - k^2_z \alpha Q v'^2_e (1+ \varepsilon/2 + \varepsilon^2/2\right\}(k^2/\omega^2_{pe})\right]$$
$$\left[(\omega - v'_e k_z)^2 - k^2_z v^2_e (1+\varepsilon/2 + \varepsilon^2/2)\right]\omega^2 - k^2_z v^2_A + k^2_z v'^2_e \alpha Q(1 + \varepsilon/2 + \varepsilon^2/2) - (k^2_z v^2_A \alpha Q/c^2)$$
$$(1 + \varepsilon/2 + \varepsilon^2/2) + (\alpha\beta k^4_z v^4_A/2)\left[1 - (\omega - v'_e k_z)^2/\omega^2_{pe} - (k^2_z v^2_e/\omega^2_{pe})(1 + \varepsilon/2 + \varepsilon^2/2)\right] = 0.$$

Collection of the terms of order ε^2 from the left hand side gives

$$\left[(1 + v^2_A/c^2)(1 + c^2k^2_x/\omega^2_{pe}) + (k^2_z v^2_A/\omega^2_{pe})k^2\right]\omega^4$$
$$- \left[(k^2_z v'^2_e \alpha Q k^2 + k^4_z v^2_A v'^2_e k^2)/\omega^2_{pe}) + (1 + v^2_A/c^2) c^2k^2_x k^2_z v'^2_e/2)\right]\omega^2$$
$$+ (k^2_z Q/2)(v'^2_e \alpha - v^2_A \alpha/c^2 + k^4_z v^4_A \alpha\beta k^2_z v'^2_e/\omega^2_{pi}) = 0.$$

Here the second term is always negative. Therefore ω^2 is negative if

$$v'^2_e \alpha - v^2_A \alpha/c^2 + k^4_z v^4_A \alpha\beta k^2_z v'^2_e/\omega^2_{pi} < 0$$

as expected for complex root. This result gives the region of instability, subjected to suitable parametric domains..

6.3 Numerical Analysis of Buneman type of instability

In the stability analysis, we have concentrated mainly on the effects of the drift velocity of electrons parallel to the direction of the ambient magnetic field which in turn speaks for the shearing effects on the background plasma. Mitarai *et al.* (1980) have shown that even for electrostatic waves under the effect of a strong magnetic field, Buneman instability can exist in the range between 0^0 and 90^0 (approx.) in the plane of motion of the wave. Recently, Iranpour *et al.* (1997) have numerically obtained a dispersion relation by considering linearized continuity, momentum equations for electrons and ions, with density gradient in the dynamics, coupled through Poisson equation. The real part of this dispersion relation agrees with the results of Farley- Buneman type linear instabilities studied in 1963, while the growth rates have been found to be somewhat modified. In the later part of their studies, they have compared results from ROSE F4 rocket data analysis to see some contradiction with such a linear dispersion relation. In our case of Alfvén wave instability, consideration of Buneman type of instability is realistic (Hasegawa,1975, p. 50). The involvement of the entire electrons through its momentum equation in the instability mechanism can lead to violence and show quite high values of ω, the wave frequency (Hasegawa,1975, p. 34).

We write

$VE = v'_e / v_A$, where $v_A = 4 \times 10^7$ meters / sec., $v_0 = v_A / c$, $c_0 = c / \omega_{pe}$, $\omega_o = v_A / \omega_{pe}$

$k^2_x = KX$, $k^2_z = KZ$. Here c_0 is taken to be equal to 0.42226.

Case I : For cold kinetic Alfvén wave mode

The numerical results agree with the relation $VE \leq (\beta / 2Q)^{1/2}$ obtained from the theoretical analysis above and show that the wave mode becomes unstable after VE crosses the value 8.6. The expression of ω^2_1 in equation (6.4) can be rewritten as

$$\omega^2_1 = -2 \frac{c^2_o \left(KX + k^2 v^2_o \right) KZ\, v'_e}{c^2_o \left(KX + k^2 v^2_o \right) + \left(1 + v^2_o \right)} \qquad (6.5)$$

In table **1**, we have shown the growth rates obtained from numerical calculation and those from equation (6.5).The growth rate after the marginal state, subjected to the designated parameter v'_e with well-suited smallness parameter ε, has reflected the pattern of the instability in the tabulated values of Table-1.

TABLE - 1

Comparison of growth rates from equation (6.4) with actual values of $Im(\omega)$ out of the solution of the dispersion relation.

KX	*VE*	ω_1	**Growth rate**
0.25	10	2999.396	9185499.0
0.36	10	3545.302	1.51742×10^7
0.49	10	4077.692	1.91046×10^7
0.64	10	4593.676	2.18029×10^7
0.81	10	5091.456	2.344095×10^7

We have used $\beta = 0.08$, $Q = 0.00054$.

The finite larmour- radius effect is increased through the increase in the parameter k_x ($KX > 0.05$).

An increase in the value of β results in corresponding increase in KX for a complex root to occur.

Growth rate, i. e. $Im(\omega)$ of the instability increases with the increase in VE as well as in KX.

Case II : For warm kinetic Alfvén wave

For a stable mode,

$$VE < (\beta/2Q)^{1/2} \quad \text{(for the cold approximation)}$$

gives $VE < 8.60663$ when $\beta = 0.08$. From numerical computations we see with the help of the discriminant condition for the fourth degree corresponding dispersion relation

$$(F1)\omega^4 + (F2)\omega^3 + (F3)\omega^2 + (F4)\omega + (F5) = 0, \quad (\omega \text{ is normalized by } k_z v_A)$$

where

$$F1 = (1 + v_o^2)(1 + c_o^2 KX) + (1 - \alpha\beta/2)\, KZ\, \omega_o^2,$$

$$F2 = -2(VE)((1 + v_o^2)\, c_o^2 KX + (1 - \alpha\beta/2)\, KZ\, \omega_o^2),$$

$$F3 = \left[(VE)^2 - \beta/2Q\right]\left[(1 + v_o^2)\, c_o^2 KX + (1 - \alpha\beta/2)\, KZ\, \omega_o^2\right] - (1 - \alpha\beta/2)\, k^2\, \omega_o^2 \alpha\beta/2(1/c_o^2 + 1),$$

$$F4 = \omega_o^2 \alpha\beta (VE) k^2$$

$$F5 = (\omega_o^4 \alpha\beta/2)\left[(\omega_{pe}/v_A)^4 - (\omega_{pe}/v_A)^2\{(VE)^2 - \beta/2Q\}\right] k^2$$

that instability starts when $VE > 8.45$ for $\beta = 0.08$, $k_x > 0.05$. But for higher values of k_x, this lower limit of VE gets lowered. We have solved the equation for complex ω even at $VE = 7.0$ when $k_x > 0.5$ and also have seen that complex ω occurs for all k_z for a particular value of k_x.

At $VE = 8$, $\beta = 0.08.$, $\alpha = 0.05$, we have the region of instability for all k_z as shown below :

$0.5 < KX < 1.01$	complex ω
$1.01 < KX < 1.35$	No complex root
$1.35 < KX$ -----	complex ω

$Im(\omega)$ increases as KX increases ($KX > 0.5$) for $VE = 8.0$.

6.4 Discussion

Case I. The kinetic Alfvén mode, with the consideration of the total nonlinearity of the ion motion in the zx-plane under the drifting effect of the electrons in the direction of *z*-axis which is also parallel to the direction of the ambient magnetic field ***B***, becomes unstable when the drifting electrons' initial velocity v'_e has an order greater than that of the shear Alfvén velocity v_A $(=B^2/\mu_0 n_i m_i$, μ_0 is the permeability of free space). In this persuit, our dispersion relation (6.2) retains the finite Larmour radius effects through the term $(c^2/\omega^2_{pe})KX$ where (c/ω_{pe}) is the plasma wave length i.e., the electron inertial length for consideration of electron inertia (Q) such that $c^2/\omega_{pe}^{\ 2} = Qc^2/\ \omega_{pi}^{\ 2}$. This shows that the Larmour radius effect (i.e., the parameter KX) on the ion motion must increase to generate instability of the wave mode. Numerically, we have seen that for $KX > 0.2$ with $VE > 9.5$, we get the desired complex root with $Im\ (\ \omega\ ) > 0$, when $KZ = 0.25$ and $\beta = 0.08$. The instability that develops here is macroscopic and absolute (Cap, 1976). Due to this macroscopic character, the fluid model adopted in our consideration portrays a clear picture (through numerical calculation) of the growth rates for different sets of parametric values. The continuous increase in the values of $Im\ (\ \omega\ )$ also specifies the absolute character of the instability. The wave frequency of such unstable wave mode is seen to increase with the increase in VE. As is evident from the geometry of the shear effect, the least admissible value of the wave number k becomes smaller for higher drifting velocity VE and remains almost constant after some high value of VE [Fig. 1]. For a certain propagation direction of the unstable mode of kinetic Alfvén wave given by KZ, we see that the wave number k as well as the wave frequency increases with the increase in KX.

Case II. We know that the kinetic Alfvén wave is the outcome of kinetic effects in the plasma. We have considered the bunching effect of the drifting electrons that leads to a coherent interaction between the highly magnetized electrons and the stationary warm ions with inherent total nonlinearity (resulting from the neglect of polarization drift of ions) in the zx-plane. The interaction is basically due to the exchange of energy from the unidirectional electrons to the warm ions. Although the Alfvén wave is defined for $T_i \rightarrow 0$, the effect of ion temperature T_i $(<<T_e)$ is to increase the frequency of oscillation. We have noticed the instability, occurring at a lower value of VE as compared to the case of cold ions with $T_i \cong 0$. For $KX = 0.36$ when $KZ = 0.5$ and $\beta = 0.08$, the consideration of ion temperature ($\alpha = \mathbf{0.05}$) gives the

resultant instability for $VE = 8.45$ while for cold ions ($\alpha = 0$) in case I, this value of VE is 8.95. Moreover as KX increases, this lower limit of VE becomes smaller [Fig.2]. This lowering of values of VE is mainly due to the introduction of the finite ion temperature which is very small (Hasegawa, 1975, p.34). The ion temperature is also seen to affect the direction of propagation of the unstable wave mode (if KZ increases then α decreases to result a complex root ω). Figure (3) shows the variation of the growth rate for different values of α over the threshold of electron drift. This establishes the compatibility of the kinetic effects produced by the drifting electrons and those produced due to the increase in temperature effect. Figure (4) shows only a small region of stability of the Alfvén waves and that too for $KX = 0.5$.

Finally from figures (5) and (6), we get the difference in the growth pattern and the rate of instability for different initial velocities (VE) of the drifting electrons. At $VE = 7.5$, $k > 1$ for the unstable mode to occur [Fig.5] while in case of $VE = 8.5$, both $k > 1$ and $k < 1$ are allowed for the instability with some region of stable wave mode [Fig.6]. For $k < 1$ in case of $VE = 8.5$, the complete instability pattern resembles to that for $k > 1$ in case of $VE = 7.5$. This pattern for $VE = 8.5$ changes for $k > 1$ after crossing a stable region, as shown by the shaded region in figure (6). Thus we conclude that the linear instability breaks after attaining a maximum growth near the transition region of the ion oscillation for $k < 1$ and also for $k > 1$ when the value of VE is increased.

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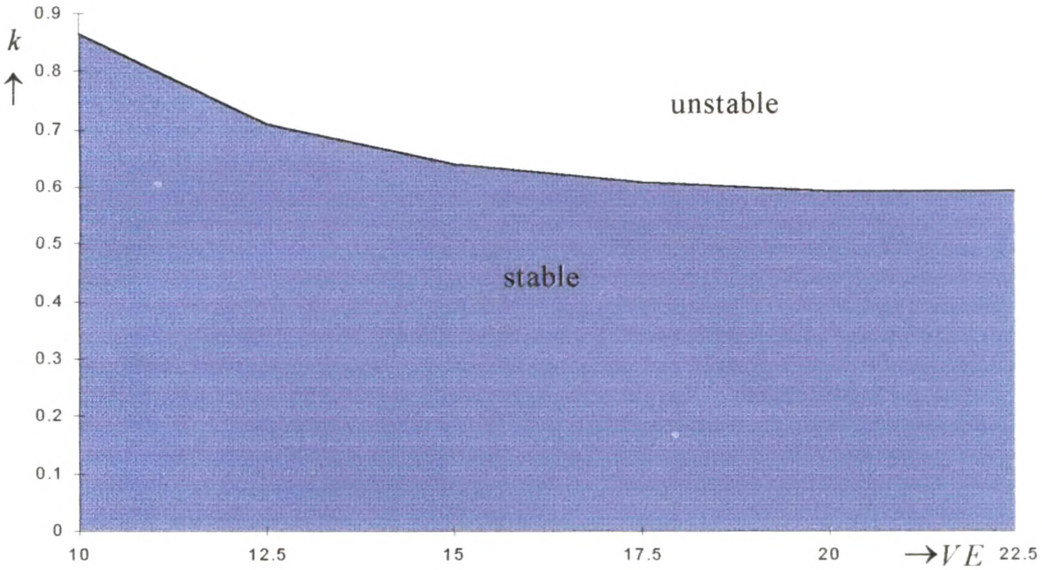

Fig.1. Region of instability versus the drift velocity VE of the electrons for cold plasma.

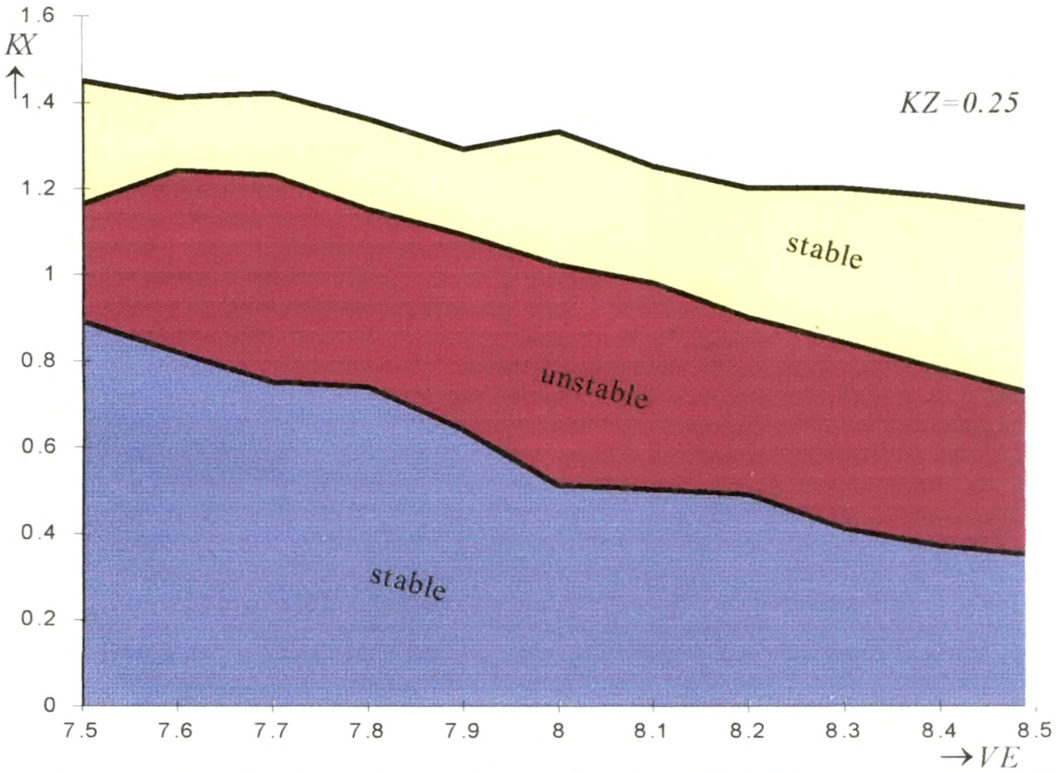

Fig.2. It shows the region of instability for fixed value of $\alpha = 0.05$. The lowest curve gives the lowest possible values of KX for the lowest possible value of the drifting velocity (VE), for an instability to occur. The descending nature of the curve shows that the lowest possible value of KX for an instabilty decreases with the increase in VE.

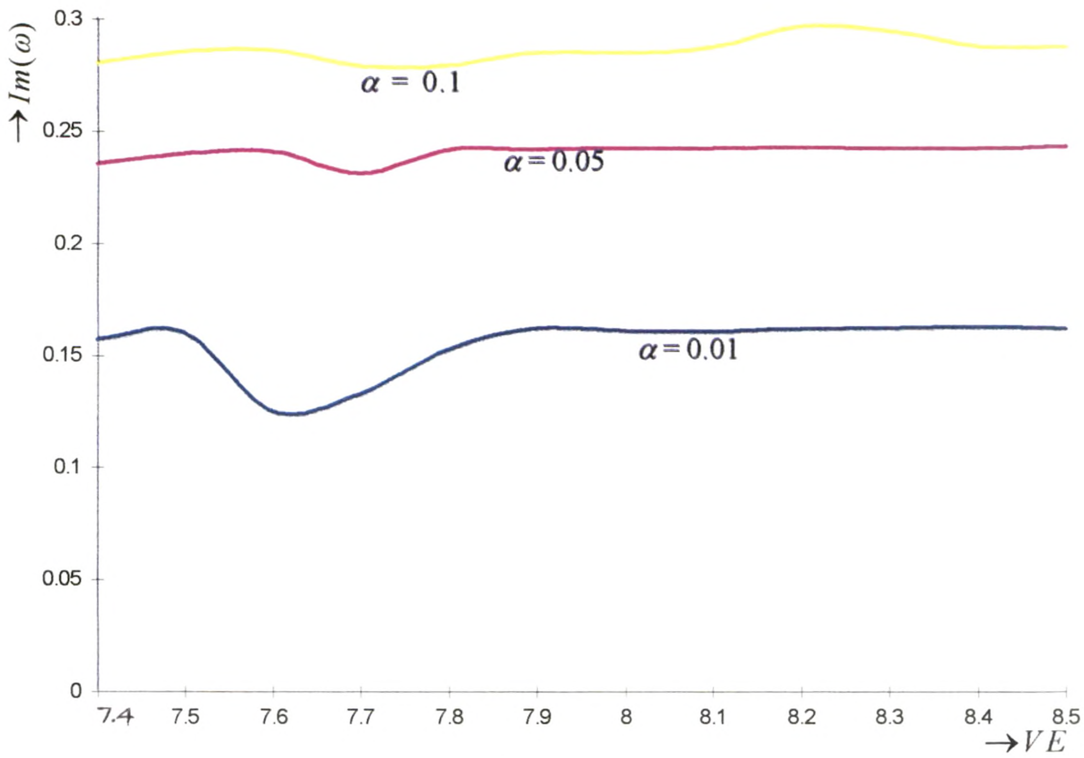

Fig.3. Variation of growth rates Im (ω) versus drifting velocity (VE) at different temperature ratios (α) for β =0.08.

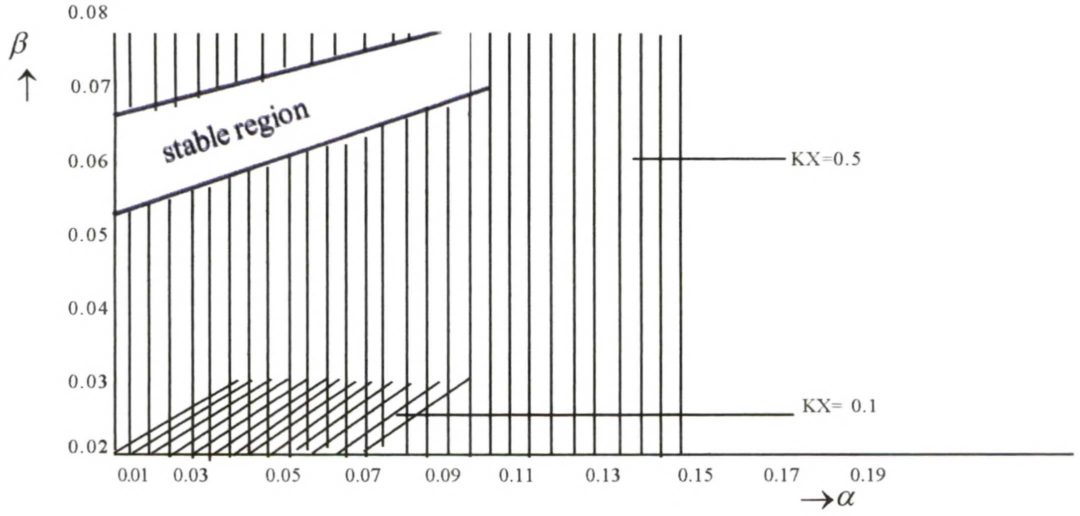

Fig.4. (α -β) phase space for the Alfvén wave instability (linear) at VE = 8.0 and KZ = 0.1. Shaded regions denote instabily at KX = 0.1 and 0.5.

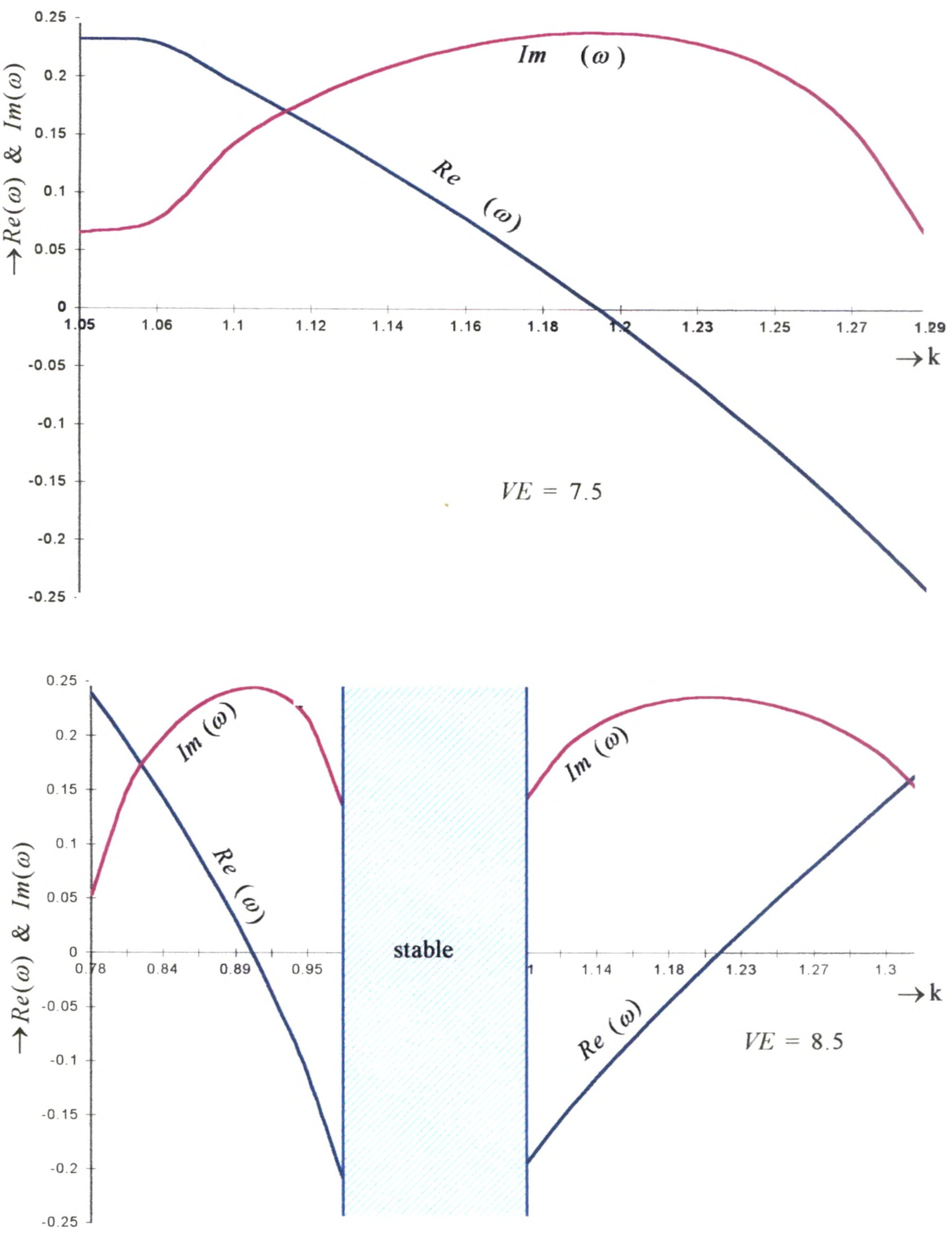

Fig.5 (above) and Fig.6 (below) : Plots of *Re* (ω) and *Im* (ω) *versus wave number k* at $\beta = 0.08$ and $\alpha = 0.05$.