CHAPTER 4

KINETIC ALFVÉN SOLITONS IN A LOW-β PLASMA UNDER THE INFLUENCE OF ELECTRON DRIFT MOTION

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4.1 Introduction

Kinetic Alfvén waves (KAWs) in plasma result from a mode found between the fast and slow Alfvén modes arising in the context of the magnetohydrodynamic approximation. Such waves appear when kinetic effects are taken into account. The extra mode occurring below the ion cyclotron frequency and above the magnetic drift frequency, is known as the kinetic (shear) Alfvén mode. The restoring forces, due to both particle and magnetic pressures, are responsible for excluding the shear mode of propagation from the MHD model. Such a wave carries energy across the magnetic field to some extent and affects the details of the energy absorption profile. The effect of these restoring forces is determined by the plasma parameter $\beta = 8\pi n_0 T / B^2_G$. This is an important parameter in testing the efficiency of magnetic fields in confining plasma in tokamak research. Electron thermal motion is the main factor in considering a low- β plasma model in the study of the propogation of Kinetic Alfvén Waves (KAWs) because the phase velocity of the long time attenuated wave remains much less than the electron thermal velocity when $\beta >> (m_e / m_i)^{1/2}$ (the square root of the electron-to-ion mass ratio). According to Stix (1980), if the condition $\beta > (m_e / m_i)^{1/2}$ is not satisfied then a cold plasma model gives better results in predicting KAWs. Shear Alfvén waves (SAWs) having a solitary wave character have been investigated under different situations in low-β plasmas. In most works, parallel ion inertia and the displacement current are ignored in a plasma model where the hot electrons obey the Boltzmann distribution. Some cases are dealt with using Ohm's law including electron enertia (Yu and Shukla, 1978; Kalita and Kalita, 1986). It should be borne in mind that in a nonuniform magnetic field, particles with opposite charges have drift velocities in opposite directions but with the same magnitude. We can ignore these polarization drifts when the frequency of the resulting motion is much less than the cyclotron frequency ($\omega \ll \Omega_{ci}$) in the plasma, which is generally true for Alfvénic cases. The current density gradient which is created by the polarization drift of ions can then be ignored. Some workers have considered such situations with the inclusion of the electron fluid equation for the propagation of different types of the Alfvén waves (Shukla *et al.*,1982) in plasma.

The possibility of experimentally creating a significant flow of electrons through a plasma immersed in a strong magnetic field by applying a positive voltage step by means of grids, allows the study of situations similar to those of dc discharges in both laboratory and cosmic plasmas. In a double plasma device, a plasma is produced in the source region by electron impact ionization with neutrals by energetic electrons emitted thermoionically from tungsten filaments. Energetic electrons flowing from the source region to the target region, produce a plasma in the target chamber. This motion of hot electrons through the plasma with the initial velocity has already been shown to lead to fascinating effects in both unmagnetized (Leven and Steinmann,1979) and magnetized (Kalita *et al.*, 1986) plasmas. Khabibrakhmanov *et al.*, (1992) using a nonlinear kinetic treatment for a general geometry and parallel waves, have obtained results suggesting the presence in a magnetized plasma of heat fluxes, resulting in a current density gradient, in contrast to the high conductivity in a fully ionized low-β plasma.

In this chapter, we account for the small electron mass m_e with an initial drift velocity v'_e and its subsequent effect on the propagation of kinetic Alfvén waves in a plasma. The smaller MHD time scale is obvious for the drifting electrons, whose equation of motion contains a convective term. In the case of the polarization drift of the ions, the Lorentz force $e\boldsymbol{v} \times \boldsymbol{B}$ pushes the electrons in the direction of $\boldsymbol{k}$. However, when polarization drift is neglected there is zero displacement current, and hence the drifting electrons have motions along the direction of the magnetic field with an initial drift velocity v'_e; the magnetic field direction and the initial drift velocity are taken to be along the z- axis. The force associated with the motion of the drifting electrons has a bunching effect and thus the low-frequency ions subjected to the dc effect (both are considered in our model) give rise to amplification of the wave amplitude.

4.2 Basic equations

We consider a cold plasma in the presence of an external magnetic field $B\hat{z}$ along the z axis. The electrons are assumed to drift with a uniform initial velocity v'_e in the $\hat{z}$ direction and the electron temperature T_e is assumed to be constant. Under the low -β (<<1) assumption, the use of two potentials ϕ and ψ with $E_x = -\partial\phi/\partial x$ and $E_z = -\partial\psi/\partial z$, is applicable (Kadomtsev,1965, p.82).

All variations are assumed to be in the (z, x) plane. The basic equations governing the motion in the plasma are

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial z}(n_e v_{ez}) = 0, \tag{4.1}$$

$$\frac{\partial v_{ez}}{\partial t} + v_{ez}\frac{\partial v_{ez}}{\partial z} = \frac{\beta}{2Q}\left(\frac{\partial \psi}{\partial z} - \frac{1}{n_e}\frac{\partial n_e}{\partial z}\right), \tag{4.2}$$

for the electrons

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{ix}) + \frac{\partial}{\partial z}(n_i v_{iz}) = 0 , \tag{4.3}$$

$$\frac{\partial v_{ix}}{\partial t} + v_{ix}\frac{\partial v_{ix}}{\partial x} + v_{iz}\frac{\partial v_{ix}}{\partial z} = -\frac{\beta}{2}\frac{\partial \phi}{\partial x} + v_{iy} , \tag{4.4}$$

$$\frac{\partial v_{iy}}{\partial t} + v_{ix}\frac{\partial v_{iy}}{\partial x} + v_{iz}\frac{\partial v_{iy}}{\partial z} = -v_{ix} , \tag{4.5}$$

$$\frac{\partial v_{iz}}{\partial t} + v_{ix}\frac{\partial v_{iz}}{\partial x} + v_{iz}\frac{\partial v_{iz}}{\partial z} = -\frac{\beta}{2}\frac{\partial \psi}{\partial z}, \tag{4.6}$$

for the ions,

$$\frac{\partial B_y}{\partial t} = \frac{\beta}{2}\frac{\partial^2}{\partial x\,\partial z}(\phi - \psi) , \tag{4.7}$$

$$\frac{\partial B_y}{\partial x} = J_z, \tag{4.8}$$

$$J = en_i v_i - en_e v_e , \tag{4.9}$$

from the Maxwell's equation and

$$\varepsilon\left(\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\psi}{\partial z^2}\right)=n_e - n_i, \tag{4.10}$$

is the Poisson equation, where $\varepsilon = c_s^2 / c^2$ and $Q = m_e / m_i$.

We combine the equations (4.7)–(4.9) with (4.1) as follows:

Differentiating both sides of (4.7) with respect to x and z, we have

$$\frac{\partial^3 B_y}{\partial t\,\partial x\,\partial z} = -\frac{\beta}{2}\frac{\partial^4}{\partial x^2\partial z^2}(\phi-\psi).$$

Using (4.8), after differentiating both sides with respect to t and z in this equation, we get

$$\frac{\partial^2 J_z}{\partial t\partial z} = \frac{\beta}{2}\frac{\partial^4}{\partial x^2\,\partial z^2}(\phi-\psi)$$

or

$$\frac{\partial^4(\phi-\psi)}{\partial x^2\,\partial z^2} = \frac{2}{\beta}\left[\frac{\partial^2 n_e}{\partial t^2} + \frac{\partial^2(n_i v_{iz})}{\partial t\,\partial z}\right]. \tag{4.11}$$

We have normalized densities by the equilibrium plasma density n_o, time by the inverse of the ion cyclotron frequency Ω_{ci}^{-1} , velocities by the Alfvén velocity $v_A = cB_0/(4\pi n_0 m_i)^{1/2}$, space by $\rho_s = c/\omega_{pi}$, and the potentials ϕ and ψ by T_e/ e.

From the original set of equations (4.1)–(4.6) and (4.10)–(4.11), the linear dispersion relation is obtained in the form [derivation is given in Appendix I]

$$k_z^2 v_A^2 - \omega^2\left(1+\frac{v_A^2}{c^2}\right) = \frac{\lambda^2 c^2}{\omega_{pe}^2}\left[\frac{k_z^2 v_A^2}{c^2} + k_x^2\left(1+\frac{v_A^2}{c^2}\right)\right],$$

with

$$\lambda^2 = (\omega - v_e' k_z)^2 - c_s^2 k_z^2 / Q.$$

This dispersion relation for kinetic Alfvén waves reduces to

$$\omega^2 = k_z^2 v_A^2 \left(1 + v_A^2 / c^2\right)^{-1},$$

in the limit as $m_e \to 0$ (i.e. $Q \to 0$), $T_e \to 0$ for parallel propagating shear Alfvén waves. Further, when $v_A^2 / c^2 << 1$, it takes the form $\omega = k_z v_A$, as required for the description of simple Alfvén waves.

4.3 Derivation of the Sagdeev potential

For plane waves propagating obliquely to the external magnetic field, taken to be in the $\hat{z}$ direction, the stationary independent variable ξ is defined as

$$\xi = xk_x + zk_z - Mt, \text{ where } M = \frac{V}{v_A}, \; k_x^2 + k_z^2 = 1,$$

V being the pulse speed (in the laboratory frame). By means of this transformation, equations (4.1)(–(4.6) and (4.11) can be simplified to give

$$n_e = (k_z v_e' - M) / (k_z v_{ez} - M), \tag{4.12}$$

$$\log n_e = \psi - \frac{Q}{k_z^2 \beta} \left[(k_z v_{ez} - M)^2 - (k_z v_e' - M)^2 \right], \tag{4.13}$$

$$k_x v_{ix} + k_z v_{iz} = M\left(1 - \frac{1}{n_i}\right), \tag{4.14}$$

$$\left[\frac{M}{-(k_x v_{ix} + k_z v_{iz})}\right] \frac{\partial v_{ix}}{\partial \xi} = \frac{\beta}{2} k_x \frac{\partial \phi}{\partial \xi} - v_{iy}, \tag{4.15}$$

$$\left[\frac{M}{-(k_x v_{ix} + k_z v_{iz})}\right] \frac{\partial v_{iy}}{\partial \xi} = v_{ix}, \tag{4.16}$$

$$\left[\frac{M}{-(k_x v_{ix} + k_z v_{iz})}\right] \frac{\partial v_{iz}}{\partial \xi} = \frac{\beta}{2} k_z \frac{\partial \psi}{\partial \xi}, \tag{4.17}$$

$$k_x^2 k_z^2 \frac{\partial^4}{\partial \xi^4} (\phi - \psi) = \frac{2}{\beta} \left[M^2 \frac{\partial^2 n_e}{\partial \xi^2} - M k_z \frac{\partial^2}{\partial \xi^2} (n_i v_{iz}) \right], \tag{4.18}$$

under the boundary conditions $v_{ix} = v_{iz} = 0$, $v_{ez} = v_e'$ (initial drift velocity), $\phi = \psi = 0$ at $n_i = n_e = 1$ as $|\xi| \to \infty$.

Equations (4.12) and (4.13) can be reduced to

$$n_e = e^{\psi} \left\{ \exp\left[A\left(1 - \frac{1}{n_e^2}\right)\right] \right\}, \tag{4.19}$$

with

$$A = (Q/\beta)(v_e' - M/k_z)^2. \tag{4.20}$$

When the Boltzmann distribution for the hot isothermal electrons is used, A becomes equal to zero (Kalita *et al.*, 1986). Clearly, the electron mass m_e, though small, along with the initial drift velocity of the electrons along the $\hat{z}$ direction infer to the variation of n_e through the relation (4.19) which is different from the usual Boltzmann distrubution function.

Equations (4.15) – (4.17) become with the uses of (4.14) respectively

$$\frac{M}{n_i}\frac{\partial v_{ix}}{\partial \xi} = \frac{\beta}{2}k_x\frac{\partial \phi}{\partial \xi} - v_{iy}, \tag{4.21}$$

$$\frac{M}{n_i}\frac{\partial v_{iy}}{\partial \xi} = v_{ix}, \tag{4.22}$$

and

$$\frac{M}{n_i}\frac{\partial v_{iz}}{\partial \xi} = \frac{\beta}{2}k_z\frac{\partial \psi}{\partial \xi}, \tag{4.23}$$

The v_{ix} eliminant of (4.14) and (4.22) is

$$\frac{M}{n_i}\frac{\partial v_{iy}}{\partial \xi} = \frac{1}{k_x}(M - \frac{M}{n_i} - k_z v_{iz}). \tag{4.24}$$

Differentiating (4.19) with respect to ξ and simplifying

$$\frac{\partial \psi}{\partial \xi} = [\frac{1}{n_e} - \frac{2A}{n_e^3}]\frac{\partial n_e}{\partial \xi}, \tag{4.25}$$

(4.21) $\times k_x$ + (4.23) $\times k_z$ $\Rightarrow$

$$\frac{M^2}{n_i^3}\frac{\partial n_i}{\partial \xi} = \frac{\beta}{2}(k^2_x\frac{\partial \phi}{\partial \xi} + k^2_z\frac{\partial \psi}{\partial \xi}) - k_x v_{iy},$$

where we have used (4.14) after differentiation with respect to ξ. Differenting the above result with respect to ξ once and using the value of k_x ($\partial v_{iy} / \partial \xi$) from (4.24) the following is obtained

$$\frac{\partial}{\partial \xi}(\frac{M^2}{n_i^3}\frac{\partial n_i}{\partial \xi}) = \frac{\beta}{2}(k_x^2\frac{\partial^2 \phi}{\partial \xi^2} + k_z^2\frac{\partial^2 \psi}{\partial \xi^2}) - \frac{n_i}{M}(M - \frac{M}{n_i} - k_z v_{iz}). \tag{4.26}$$

From now onwards the charge neutrality condition namely $n_i = n_e = n$ will be used for the rigorous solution of the equations required for the derivation of the Sagdeev potential.

Integrating (4.18) with respect to ξ under the boundary conditions $\phi = \psi = 0$, $v_{iz} = 0$, $\partial n/\partial \xi = 0$, $n = 1$ as $|\xi| \rightarrow \infty$, successively for two times we get

$$k^2_x k^2_z\frac{\partial^2}{\partial \xi^2}(\phi - \psi) = \frac{2}{\beta}[M^2 n - Mk_z n v_{iz} - M^2]. \tag{4.27}$$

Using (4.23) in (4.25), the equation

$$\frac{M}{n}\frac{\partial v_{iz}}{\partial \xi} = \frac{\beta k_z}{2}(\frac{1}{n} - \frac{2A}{n^3})\frac{\partial n}{\partial \xi}$$ is obtained which gives, on integration

under the boundary conditions $v_{iz} = 0$ at $n = 1$ as $|\xi| \rightarrow \infty$,

$$v_{iz} = \frac{\beta k_z}{2M}\left[(n-1) + 2A\left(\frac{1}{n} - 1\right)\right] \tag{4.28}$$

Putting the value of v_{iz} from (4.28) in the right hand side of (4.27) we have

$$k_x^2 k_z^2 \frac{\partial^2}{\partial \xi^2}(\phi - \psi) = \frac{2}{\beta}\left[M^2 n - Mk_z n\,\frac{\beta k_z}{2M}\left\{(n-1) + 2A\left(\frac{1}{n} - 1\right)\right\} - M^2\right]. \tag{4.29}$$

Similarly from (4.26) and (4.28), the following relation is obtained.

$$\frac{\beta}{2}\left(k_x^2 \frac{\partial^2\phi}{\partial \xi^2} + k_z^2 \frac{\partial^2\psi}{\partial \xi^2}\right) = \frac{\partial}{\partial \xi}\left(\frac{M^2}{n}\frac{\partial n}{\partial \xi}\right) + \frac{n}{M}\left[M - \frac{M}{n} - \frac{\beta k_z^2}{2M}\left\{(n-1) + 2A\left(\frac{1}{n} - 1\right)\right\}\right]. \tag{4.30}$$

Eliminating ϕ between (4.29) and (4.30) we have, on simplification (by using the value of $\frac{\partial^2\psi}{\partial\xi^2}$ from the differentiated equation of (4.25))

$$k_z^2\left\{\frac{2M^2}{\beta n^3} - \left(\frac{1}{n} - \frac{2A}{n^3}\right)\right\}\frac{\partial^2 n}{\partial \xi^2} - k_z^2\left\{\frac{6M^2}{\beta n^4} + \left(\frac{6A}{n^4} - \frac{1}{n^2}\right)\right\}\left(\frac{\partial n}{\partial \xi}\right)^2 =$$

$$k_z^2(n-1)\left[\frac{2}{\beta}\left(\frac{M^2}{k_z^2} - 1\right) - \left(1 - \frac{2A}{n}\right)n\left(1 - \frac{k_z^2}{M^2}\right)\right]$$

or

$$\frac{\partial}{\partial \xi}\left[\left\{\frac{2k_z^2 M^2}{\beta n^3} - k_z^2\left(\frac{1}{n} - \frac{2A}{n^3}\right)\right\}\frac{\partial n}{\partial \xi}\right] = (n-1)\left(1 - \frac{k_z^2}{M^2}\right)\left\{\frac{2M^2}{\beta} - (n-2A)k_z^2\right\}. \tag{4.31}$$

Multiplying both sides of (4.31) by the term in the parantheses on the left hand side, it can be integrated once to give

$$\frac{1}{2}\left(\frac{\partial n}{\partial \xi}\right)^2 + K(n, \beta, M, k_z) = 0, \tag{4.32}$$

where

$$\begin{aligned} K(n, \beta, M, k_z) = &- \frac{n^4(1 - k_z^2/M^2)}{k_z^2\left[2M^2/\beta - (n^2 - 2A)\right]^2}\left\{\frac{k_z^2 n^4}{2} - \left[\frac{2M^2}{\beta} + k_z^2(1+2A)\right]n^3\right. \\ &+ \frac{2M^2}{\beta}(1 - k_z^2)\,n^2\log n + \left[\frac{2M^2}{\beta}\left\{(A+1)(k_z^2+1) + \frac{M^2}{\beta}\right\}\right. \\ &\left.+ \left[(1+2A)^2 + 4A\right]\frac{k_z^2}{2}\right]n^2 - \left\{\frac{2M^2}{\beta}\left[\left(\frac{2M^2}{\beta} + 2A\right)\right.\right. \\ &\left.\left.\left.+ (1+2A)k_z^2\right] + 2(1+2A)Ak_z^2\right\}n + \left[\frac{2M^2}{\beta}\left(A(k_z^2 + 1) + \frac{M^2}{\beta}\right) + 2k_z^2 A^2\right]\right\} \end{aligned} \tag{4.33}$$

is the Sagdeev potential.

In deriving (4.32), the boundary condition $\partial n / \partial \xi = 0$ at $n = 1$ has been used.

If $n = N$ is the maximum variation of density (i.e. amplitude), then the nonlinear dispersion relation is given by

$$K(N, \beta, M, k_z) = 0 ,$$

so that

$$\frac{k^2_z}{2} N^4 - \left[\frac{2M^2}{\beta} + k^2_z (1+2A)\right] N^3 + \frac{2M^2}{\beta}(1-k^2_z) N^2 \log N$$

$$+ \left\{\frac{2M^2}{\beta}\left[(A+1)(k^2_z+1) + \frac{M^2}{\beta}\right] + \left[(1+2A)^2 + 4A\right]\frac{k^2_z}{2}\right\} N^2$$

$$- \left\{\frac{2M^2}{\beta}\left[\left(\frac{2M^2}{\beta} + 2A\right) + (1+2A) k^2_z\right] + 2(1+2A) A k^2_z\right\} N$$

$$+ \left\{\frac{2M^2}{\beta}\left[A(k^2_z + 1) + \frac{M^2}{\beta}\right] + 2k^2_z A^2\right\} = 0 . \tag{4.34}$$

Solution of this equation gives the amplitude N of the waves for different values of the parameters β, M, k_z and A.

4.4 Criteria for the existence of solitary Alfvén waves

Analysis of the potential $K(n)$ given by (4.33) leads to the conclusion that $K(1) = K(N) = \partial K/\partial n|_{n=1} = 0$; the condition $K(N) = 0$ has already been set in the nonlinear dispersion relation (4.34). The above conclusions, together with the condition $K(n) < 0$ between $n = 1$ and $n = N$, are necessary for the existence of a localized solitary-wave solution of the energy integral (4.32) of a classical particle in a potential well in terms of the Sagdeev potential $K(n)$.

The conditions to make $K(n) < 0$ near $n = 1$ are essentially required to reflect the character of $K(n)$. For this, we expand $K(n)$ in Taylor series near $n = 1$ and $n = N$ to give

$$K(n) = \frac{(n-1)^2 (1-k^2_z/M^2)}{(\beta k^2_z/M^2)[2M^2/\beta - (1-2A)]^2} \left[1 - \frac{\beta k^2_z}{2M^2}(1-2A)\right]\left[(1-2A) - \frac{2M^2}{\beta}\right] , \tag{4.35}$$

and

$$K(n) = - \frac{(n-N)(N-1)N^3(1-k_z^2/M^2)}{k^2_z [2M^2/\beta - (N^2-2A)]^2} \left[\frac{2M^2}{\beta} - k_z^2 (N-2A)\right]\left[\frac{2M^2}{\beta} - (N^2-2A)\right] \tag{4.36}$$

respectively.

From (4.35) and (4.36), it is seen that $K(n)$ is negative near $n=1$ and $n=N$ if

$$1-2A > \frac{2M^2}{\beta} > k_z^2(1-2A) \qquad \text{for } M < k_z \qquad (4.37)$$

so that

$$N > \frac{2M^2}{\beta k_z^2} + 2A > \left(\frac{2M^2}{\beta}+2A\right)^{1/2} \qquad \text{for } N > 1 \qquad (4.38)$$

and

$$N < \left(\frac{2M^2}{\beta}+2A\right)^{1/2} < \frac{2M^2}{\beta k_z^2}+2A \qquad \text{for } N < 1 \qquad (4.39)$$

Thus, both compressive and rarefactive solitary Alfvén waves exist for $M < k_z$.

When $M > k_z$, for $K(n)$ to be negative, (4.35) and (4.36) give, near $n=1$ and $n=N$

$$\frac{2M^2}{\beta} > 1-2A > k_z^2(1-2A), \qquad (4.40)$$

leading to

$$\left(\frac{2M^2}{\beta}+2A\right)^{1/2} < N < \frac{2M^2}{\beta k_z^2}+2A \qquad \text{for } N > 1. \qquad (4.41)$$

This shows that only a compressive Alfvén soliton exists when $M > k_z$.

The width (Δ) of the Alfvén soliton of amplitude N is given by $N/\sqrt{d}$, d is the maximum depth of the Sagdeev potential $K(n, \beta, k_z, M, A)$.

4.5 Discussion

Under the drifting effect of electrons in the direction of the external magnetic field, both compressive and rarefactive Alfvén solitons of various amplitudes are found to exist in a magnetized plasma. The amplitude $N\,(>1)$ of the compressive Alfvén soliton gradually increases with increasing drift velocity v'_e in its acceptable domain for $M < K_z$ and fixed values of K_z and β (Fig.1). As the amplitude increases, the direction of propagation deviates from the direction of the magnetic field. It is observed that the drifting effect of the electrons favours the formation of a rarefactive Alfvén soliton.

The sharp increase in the amplitude of the compressive Alfvén soliton away from the magnetic field ($k_z << 1$) changes its character finally to a slow increase with M when the direction of propagation approaches to that of the magnetic field [Figs.2a, b]. For each value of k_z, a small amplitude soliton is produced at the respective lower limit of M in its domain of existence. When the amplitude increases, the parameter v'_e is found to vary in almost equally spaced intervals (that extend only marginally with A) for each case of assigned values of the parameters β, A and k_z [Fig.3] for small k_z. However, for each set of values of β and A, these intervals contract with increasing k_z. It can be seen from figure 4 that the width of the compressive Alfvén soliton for $M < k_z$ increases slightly with mach number M. Of course, the width is greater at smaller drift velocity v'_e, characterized by the small value of M for fixed β .It is noteworthy that the decrease in width of the compressive Alfvén soliton for small A (=0.05) [Fig.5] is found to be more rapid with v'_e.

On the other hand, the amplitude of the rarefactive (N<1) Alfvén soliton increases considerably and almost linearly with M (< k_z) [Fig.6 (a)] for a fixed value of k_z. The lower limit of M is found to increase with increasing k_z, which also increases the amplitude of the rarefactive Alfvén soliton. It is worth mentioning that the mach number M is relatively higher in case of a rarefactive Alfvén soliton, but in both the cases of Alfvén solitons, it does not exceed 0.2. Further, numerical calculations reveal that the widths of the compressive Alfvén solitons for $M > k_z$ and that of rarefactive solitons for $M < k_z$ increases with M, taking only very small values.

References

Kadomtsev, B.B. (1965) Plasma Turbulence. Academic Press, New York.

Kalita, M. K. & Kalita, B. C. (1986) J. Plasma Phys. **35**, 267

Kalita, B. C., Kalita, M. K. & Chutia, J. (1986) J. Phys. A : Math. Gen. **19**, 3559

Khabibrakhmanov, K. H., Galinsky, V. L. & Verheest, F. (1992) Phys. Fluids **4**, 2538

Leven, R. and Steinmann, W. (1979) Plasma Phys. **3/80**, 195

Shukla, P. K., Rahaman, H. U. & Sarma, R. P. (1982) J. Plasma Phys. **28**, 125

Stix, T.H.(1980) Heating in toroidal plasma. Proceedings of the 2nd Joint Grenoble- Varenna International Symposium, 1980 **Vol. 2**, p. 631. Commission of the European Communities,Brussels.

Yu, M. Y. & Shukla, P. K. (1978) Phys. Fluids **21**, 1457

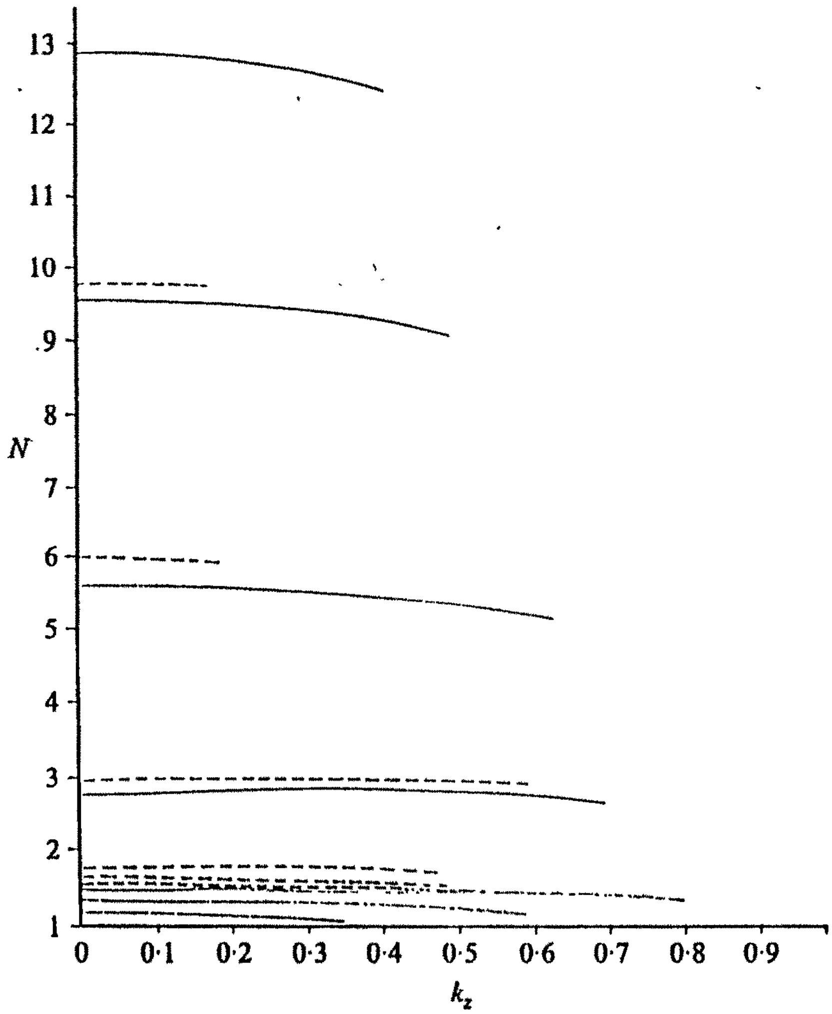

FIG.1. Variation of amplitude of an Alfvén soliton with k_z for a fixed value of $\beta = 0{\cdot}08$ ——, $A = 0{\cdot}05$ and $v_e' = 2{\cdot}92$, 2·93, 2·94, 3·0, 3·1, 3·2, 3·27; ––––, $A = 0{\cdot}45$ and $v_e' = 8{\cdot}3$, 8·31 8·32, 8·4, 8·52, 8·62.

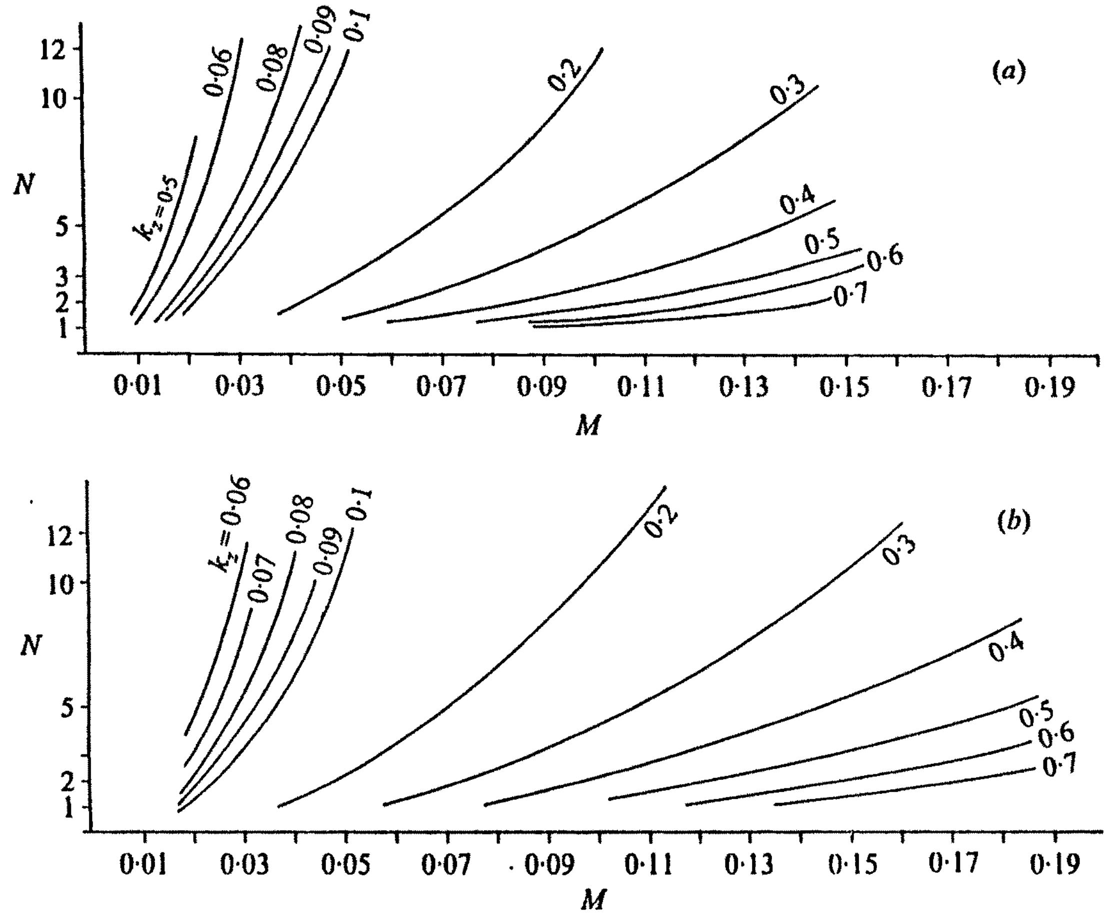

FIG.2. Amplitude N versus Mach number M ($< k_z$) of an Alfvén soliton for a fixed value of $\beta = 0{\cdot}08$: (a) $A = 0{\cdot}25$; (b) $A = 0{\cdot}05$.

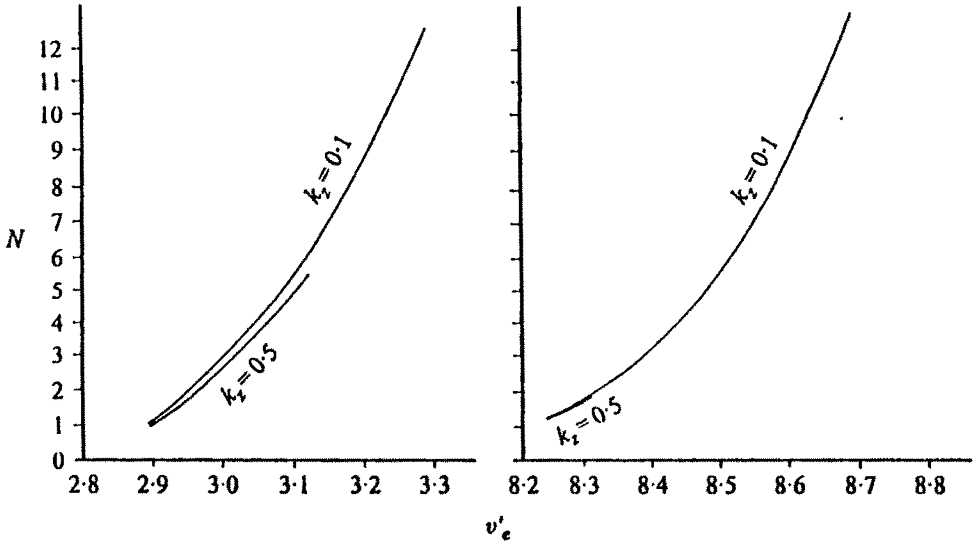

FIG.3. Variation of solition amplitude N with v'_e, showing regions of v'_e at fixed $\beta = 0{\cdot}08$ for different values of k_z.

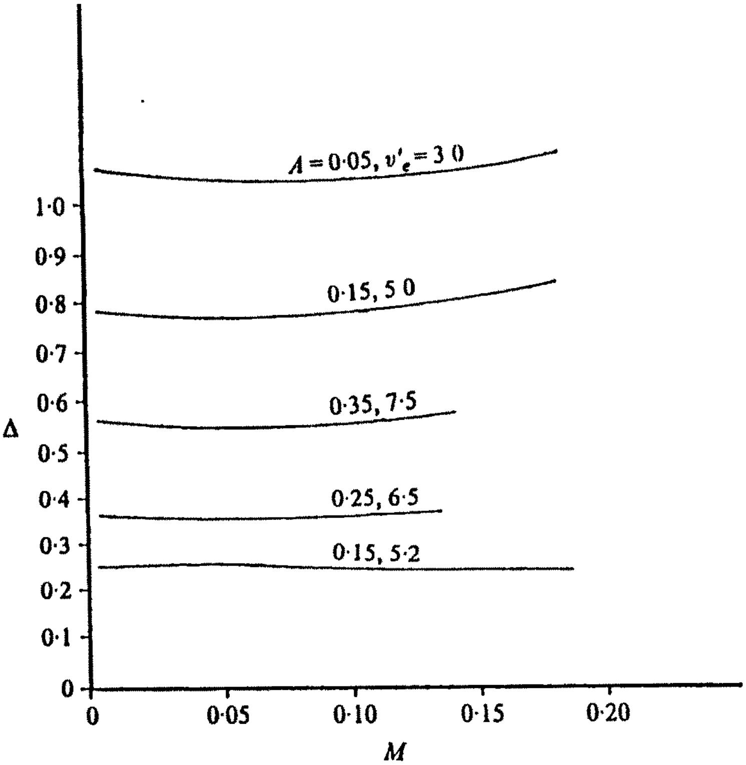

FIG.4. Width Δ versus Mach number M for a fixed $\beta = 0{\cdot}08$; values of A and v'_e are shown against the curves.

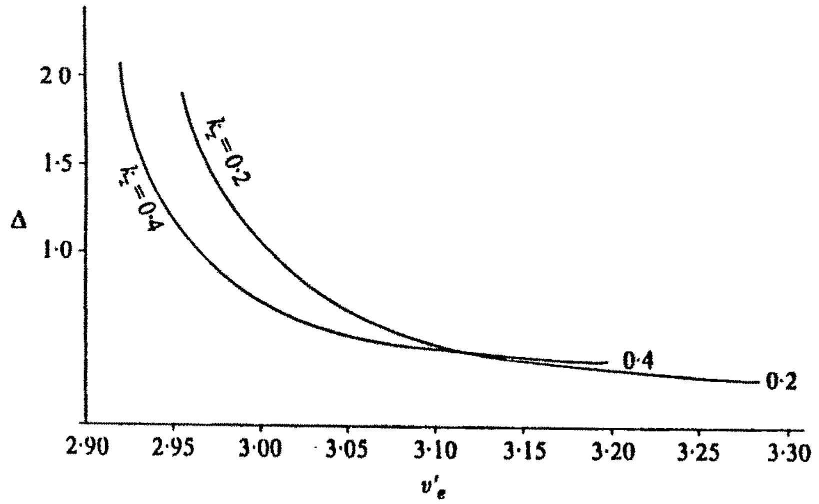

FIG 5. Soliton width Δ versus drift velocity v'_e of electrons for a fixed $\beta = 0·08$ and $A = 0·05$; values of k_z are shown against the curves.

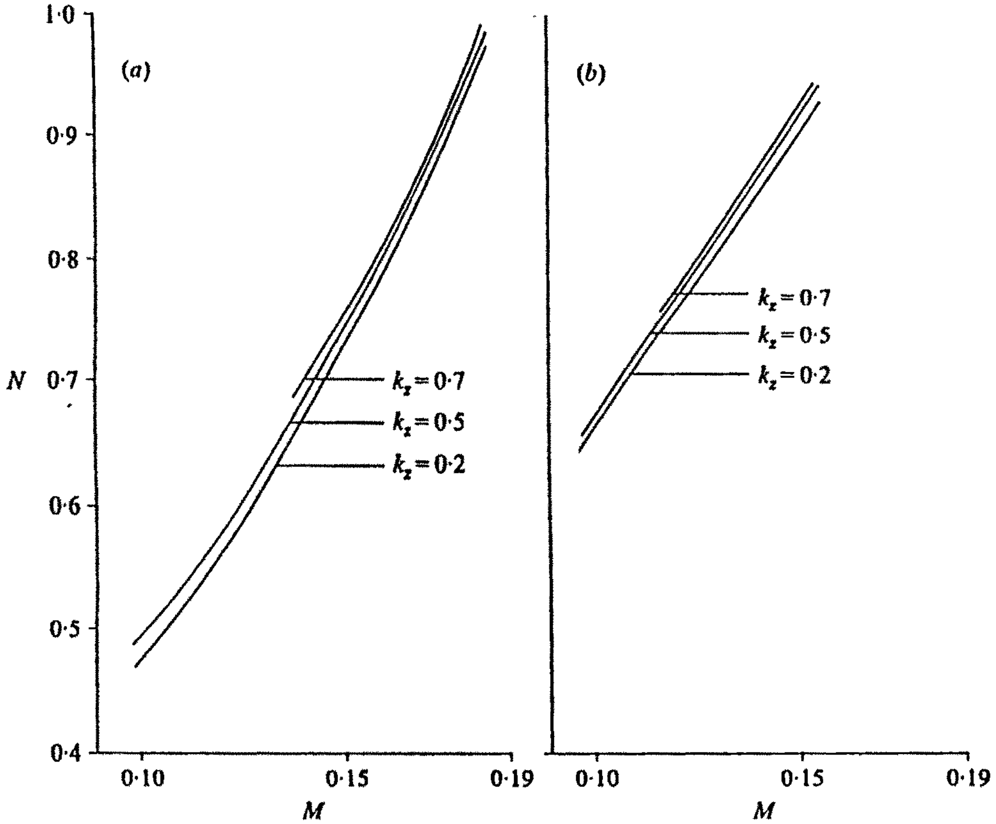

FIG.6. Amplitude $N(< 1)$ of an Alfvén soliton versus Mach number M for a fixed $\beta = 0·08$ and (a) $A = 0·05$ and (b) $A = 0·15$ for different k_z (shown against the curves).