Synopsis

In 1922, S. Banach propounded the first fixed point theorem in metric space for contraction mapping. Contraction mapping gives rise to several other mappings, namely contractive, non expansive, Lipschitz etc. and all of them are continuous. Nearly four decades after introduction of Banach's fixed point theorem, Edelstein (1961) made an extensive generalization of it and established a class of new fixed point theorems for a special class of mappings in metric spaces. Since then, a number of generalizations of contraction mapping principle have been established by different mathematicians leading to a volume of fixed point theorems in metric spaces and it is continued till now. Of course, there are other fixed point theorems such as Caristi's (1975, 1976) fixed point theorem related to arbitrary mapping.

Fixed point theorems are the most important tools for providing existence and uniqueness of the solution of various mathematical models (differential, integral, partial differential equation and variational inequalities) representing different phenomena suitable for different fields such as steady-state temperature distribution, chemical reactions, neutron transport theory, economic theories, epidemics and flow of fluids. At present this field has been recognized as one of the active fields of research.

With the introduction of the concept "Fuzzy Set" by Lofty A. Zadeh in 1965, a new horizon was opened in the sky of human knowledge. His way of defining 'fuzziness' is closely related to process of human thinking and reasoning and so is a tool for generating decisions in uncertainty. In one word, fuzzy set provides scope to express
uncertainty or vagueness suitably in a mathematical way. Gradually fuzzy set theory has entered into almost all the disciplines of science, technology and humanities and now-a-days it is an extremely versatile interdisciplinary research area. People have been experiencing the fruits of application of fuzzy logic and fuzzy set theory at large by adopting them everywhere from household appliances to traffic controlling system of high speed train. Fuzzy set theory is also widely used in information technology and resource management analysis. Every application of fuzzy logic can realize some benefits such as performance, simplicity, lower cost, productivity.

The fuzzy concept in metric space was introduced for the first time in 1975 by Kramosil and Michalek generalizing the concept of probabilistic metric space in fuzzy situation. In 1979, Ecreg introduced a definition of fuzzy metric space using the concept of lattices. In 1981, Heilpern developed the notion of fuzzy mapping and proved a fixed point theorem for fuzzy mapping. Z. Deng, in 1982 introduced fuzzy pseudo metric space. In 1984, Kaleva and Seikkala introduced the notion of fuzzy metric by setting the distance between two points to be a non negative fuzzy number. Ordering and triangular inequality of fuzzy numbers were also defined. The duo gave a new turn to the α-level set introduced by Zadeh . On the basis of α-level set they discussed some properties of fuzzy numbers and fuzzy metric space and accordingly explained the existence of a Hausdorff topology in fuzzy metric space. Under certain restrictions on the fuzzy metric space, this Hausdorff topology gives a Hausdorff uniformity for which a fuzzy metric space can be represented as a fuzzy uniform space. Finally Kaleva and Seikkala proved fixed point theorems in fuzzy metric spaces.

In 1995, P. Das and T. Basu geared up the idea of Hausdorff uniformity in fuzzy metric space discussed by Kaleva and Seikkala and added more advancement in fuzzy uniform space. The duo also proved fuzzy analogue of Banach and Ciric's fixed point theorem with the help of fuzzy uniformity.

Being inspired by the work of Kaleva and Seikkala, P. Das and T. Basu and C. Felbin, we have taken up the task of transformation of fixed point theorems in fuzzy metric spaces. In our work, we have attempted to establish some fixed point theorems in fuzzy metric spaces in a unified approach. Most of the theorems related to our task, belong to generalized contraction mapping. There is one fuzzy analogue of fixed point theorem for arbitrary mapping in fuzzy metric space. While establishing the fixed point theorems we have followed the path of Kaleva and Seikkala and also follow the direction of P. Das and T. Basu. In the process, we have developed the notions such as fuzzy $\varepsilon$-chainable metric space, fuzzy $(\varepsilon, k)$-uniformly locally contractive mapping and fuzzy $\varepsilon$-contractive mapping in the same setting. We have also attempted to establish fixed point theorems in complete fuzzy metric space representing it as uniform space. For that
purpose we have redeveloped the notion of fuzzy \((e, k)\) - uniformly locally contractive mapping to suit with fuzzy uniform space.

It is an outcome based on systematic study of fixed point theorems both in ordinary metric spaces and fuzzy metric spaces. The motivation behind this endeavour is

(i) to enrich the existing literature on fixed point theorem in fuzzy metric space.

(ii) to represent complete fuzzy metric space as fuzzy Hausdorff uniform space and accordingly establish fixed point theorem there.

(iii) to open a direction for application of fixed point theorems in socio-economic context as the wheel of time rolls on.

The thesis “Some Fixed point Theorems in Fuzzy Metric Spaces” is furnished with six chapters.

Chapter one of this thesis is a collection of some definitions and known results without proof from various texts, papers and dissertations on fuzzy set, fuzzy sets induced by mappings, fuzzy real number, fuzzy metric spaces, fuzzy normed linear space, fuzzy uniform space, fixed point theorems in fuzzy metric spaces and fixed point theorem in uniform space, which are used in the subsequent work of the thesis.

Chapter two briefly sketches the relevant parts of the historical development of contraction mapping principle and as well as fixed point theorems in metric spaces; uniform space and Hausdorff uniformity; fuzzification of metric space, normed linear space and uniform space along with some fixed point theorems in these structure. Common fixed point theorems are also discussed here.
Chapter three is furnished with the development of the notions of fuzzy $\varepsilon$-chainable metric space, fuzzy $(\varepsilon, k)$-uniformly locally contractive mapping, fuzzy $\varepsilon$-contractive mapping and establishment of two fixed point theorems related to them.

Chapter four deals with the establishment of three fixed point theorems in complete fuzzy metric spaces - one is for an arbitrary mapping, second one for generalized contractive mapping, and the third theorem for generalized contraction mapping.

Chapter five is devoted to the establishment of two fixed point theorems in complete fuzzy metric space representing it as uniform space. Fuzzy $(\varepsilon, k)$-uniformly locally contractive mapping has been redeveloped here to suit with fuzzy uniform space.

Chapter six incorporates again the fuzzy analogue of one of the latest fixed point theorems (2005) for generalized contraction mapping principle. The fuzzy analogue of the other three established fixed point theorems are corollaries of this theorem.