CHAPTER III

MHD OSCILLATORY FREE CONVECTIVE FLOW PAST A VERTICAL PLATE IN SLIP-FLOW REGIME WITH VARIABLE SUCTION AND PERIODIC PLATE TEMPERATURE

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3.1:INTRODUCTION

The oscillatory free convective flow and heat transfer problems in the presence of magnetic field have attracted the attention of a number of scholars due to its important in technological applications. Unsteady oscillatory free convective flows play an important role in chemical engineering, turbo machinery and aerospace technology. Such flows arise due to either unsteady motion of a boundary or boundary temperature. The unsteadiness may also be due to oscillatory free stream velocity or temperature.

Soundalgekar (1973, 1975) studied the effects of the oscillatory free stream and free convection currents on the flow field past a vertical flat plate under a transverse magnetic field or without it when the temperature of the plate differs from the temperature of the stream, causing the flow of the free convection currents in the boundary layer. The analysis of an unsteady MHD free convection flow of a viscous incompressible electrically conducting fluid past on infinite vertical porous plate subjected to constant suction and heat sink was made by Sahoo *et al* (2003). Anwar (1998) studied the MHD unsteady free convection flow past a vertical porous plate. The transient free convection flow past an infinite vertical plate with periodic temperature variation was investigated analytically by Das *et al.* (1998). The effect of variable suction on transient free convective viscous incompressible flow past a variable plate with periodic temperature variation in a sleep flow regime was investigated by Sharma and Choudhary (2003). Sharma (2005) has further studied the influence of periodic temperature and concentration on unsteady free convective viscous incompressible flow and heat transfer past a vertical plate in slip flow regime. Recently Jain and Sharma

(2006) have investigated the effect of viscous heating on flow past a vertical plate in slip-flow regime with periodic temperature variations.

The object of the present work is to investigate the effect of the transverse magnetic field and suction parameter on the oscillatory free convective flow past a vertical plate in slip-flow regime with variable suction and periodic plate temperature. This work is an extension of the work done by Jain and Sharma (2006) to MHD case.

3.2: MATHEMATICAL FORMULATION

Let us consider a flow of an oscillatory free convective flow of an electrically conducting, viscous and incompressible fluid past a vertical plate in slip-flow regime with variable suction and periodic plate temperature under the action transverse magnetic field by making the following assumption:

(1) All fluid properties except the density in the buoyancy force term are constant.

(2) The magnetic dissipation term in the energy equation is negligible.

(3) The Eckert number E is small.

(4) The magnetic Reynolds Number is so small that the induced magnetic field can be neglected.

The X-axis is taken along the upward vertical plate and the Y-axis is taken perpendicular to it into the fluid region. Since the plate is of infinite length therefore all the physical quantities except the pressure p are independent of x. Under these assumptions, the physical quantities are function of y and t only.

The equations governing the flow are

Equation of continuity:

$$\frac{\partial \bar{v}}{\partial \bar{y}}=0$$

$\Rightarrow \bar{v}=-v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})$, a constant

Momentum equation:

$$\frac{\partial \bar{u}}{\partial t}-v_0(1+\varepsilon A e^{i\bar{\omega}\bar{t}})\frac{\partial \bar{u}}{\partial \bar{y}}=g\beta(\bar{T}-\bar{T}_\infty)+\upsilon\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}-\frac{\sigma B_0^{\,2}}{\rho}\bar{u} \qquad (3.2.1)$$

Energy equation:

$$c_p\left[\frac{\partial \bar{T}}{\partial \bar{t}}-v_0\left(1+\varepsilon A e^{i\bar{\omega}\bar{t}}\right)\frac{\partial \bar{T}}{\partial \bar{y}}\right]=\frac{k}{\rho}\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}+\upsilon\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \qquad (3.2\ 2)$$

The boundary conditions are

$$\left.\begin{array}{l} \text{at } \bar{y}=0;\bar{u}=\bar{h}\left(\dfrac{\partial \bar{u}}{\partial \bar{y}}\right),\bar{T}=\bar{T}_w+\varepsilon\left(\bar{T}_w-\bar{T}_\infty\right)e^{i\bar{\omega}\bar{t}} \\ \text{at } \bar{y}\to\infty;\ \bar{u}\to 0,\quad \bar{T}\to\bar{T}_\infty \end{array}\right\} \qquad (3\ 2.3)$$

We introduce the following non- dimensional quantities

$y=\dfrac{\bar{y}v_0}{\upsilon}$ (distance), $t=\dfrac{\bar{t}v_0^{\,2}}{4\upsilon}$ (time), $\omega=\dfrac{4\upsilon\bar{\omega}}{v_0^{\,2}}$ (frequency), $u=\dfrac{\bar{u}}{v_0}$ (velocity),

$\theta=\dfrac{\bar{T}-\bar{T}_\infty}{\bar{T}_w-\bar{T}_\infty}$ (fluid temperature), $G=g\beta\upsilon\dfrac{\bar{T}_w-\bar{T}_\infty}{v_0^{\,3}}$ (Grashof number),

$M=\dfrac{\sigma B_0^{\,2}\upsilon}{\rho v_0^{\,2}}$ (Hartmann number), $E=\dfrac{v_0^{\,2}}{c_p\left(\bar{T}_w-\bar{T}_\infty\right)}$ (Eckert number), $P=\dfrac{\mu c_p}{k}$

(Prandtl number), $h=\dfrac{v_0\bar{h}}{\upsilon}$ (Rarefaction parameter)

where, A is the Suction parameter, υ is the kinematics viscosity, ω is the frequency parameter, ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion for heat transfer, c_p is the specific heat at constant pressure, B_0 is the applied magnetic field, k is the coefficient of thermal conductivity , μ is the coefficient of viscosity, v_0 is the mean suction velocity, $\overline{T}_\infty$ is the free stream temperature, $\overline{T}_W$ is the mean plate temperature and the other symbols have their usual meanings.

The non-dimensional equations and boundary conditions are

$$\frac{1}{4}\frac{\partial u}{\partial t}-\left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y}=G\theta+\frac{\partial^2 u}{\partial y^2}-Mu \quad (3.2.4)$$

$$\frac{1}{4}\frac{\partial \theta}{\partial t}-\left(1+\varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y}=\frac{1}{P}\frac{\partial^2 \theta}{\partial y^2}+E\left(\frac{\partial u}{\partial y}\right)^2 \quad (3.2.5)$$

subject to

$$\left.\begin{array}{l} y=0; u=h\dfrac{\partial u}{\partial y}, \theta=1+\varepsilon e^{i\omega t} \\ y\to\infty; u=0, \theta=0 \end{array}\right\} \quad (3.2.6)$$

3.3: SOLUTION OF THE PROBLEM

Assuming the small amplitude oscillation ($\varepsilon <<1$), we represent the velocity u and temperature θ, near the plate as

$$\left.\begin{array}{l} u=u_0(y)+\varepsilon e^{i\omega t}u_1(y)+O(\varepsilon^2) \\ \theta=\theta_0(y)+\varepsilon e^{i\omega t}\theta_1(y)+O(\varepsilon^2) \end{array}\right\} \quad (3.3.1)$$

Substituting from the equations (3.3.1) in (3.2.4) and (3.2.5), equating the harmonic terms and neglecting ε^2, the following equations for u_0, u_1 and θ_0, θ_1 are obtained

$$u_0''+u_0'-Mu_0=-G\theta_0 \quad (3.3.2)$$

$$u_1'' + u_1' - \left(\frac{i\omega}{4} + M\right)u_1 = -G\theta_1 - Au_0' \tag{3.3.3}$$

$$\theta'' + P\theta' = -EPu_0'^2 \tag{3.3.4}$$

$$\theta_1'' + P\theta_1' - i\frac{P\omega\theta_1}{4} = -AP\theta_0' - 2EPu_0'u_1' \tag{3.3.5}$$

Subject to boundary conditions

$$\left.\begin{array}{l} y=0; u_0 = h\frac{\partial u_0}{\partial y}, \theta_0 = 1, u_1 = h\frac{\partial u_1}{\partial y}, \theta_1 = 1 \\ y \to \infty; u_0 = 0, \theta_0 = 0, u_1 = 0, \theta_1 = 0 \end{array}\right\} \tag{3.3.6}$$

Where dashes denote differentiation with respect to y.

The equations (3.3.2) to (3.3.5) are still coupled for the variables u_0, u_1, θ_0 and θ_1. To solve them we note that E <1 for all incompressible fluids and assume that

$$\left.\begin{array}{l} u_0 = u_{00} + Eu_{01} + O(E^2);\ u_1 = u_{10} + Eu_{11} + O(E^2) \\ \theta_0 = \theta_{00} + E\theta_{01} + O(E^2);\ \theta_1 = \theta_{10} + E\theta_{11} + O(E^2) \end{array}\right\} \tag{3.3.7}$$

Substituting from (3.3.7) in the equations (3.3.2) to (3.3.5), equating the terms independent of E and coefficient of E in each equation and neglecting E^2 the following equations are obtained

$$u_{00}'' + u_{00}' - Mu_{00} = -G\theta_{00} \tag{3.3.8}$$

$$u_{01}'' + u_{01}' - Mu_{01} = -G\theta_{01} \tag{3.3.9}$$

$$u_{10}'' + u_{10}' - \left(M + \frac{i\omega}{4}\right)u_{10} = -G\theta_{10} - Au_{00}' \tag{3.3.10}$$

$$u_{11}'' + u_{11}' - \left(M + \frac{i\omega}{4}\right)u_{11} = -G\theta_{11} - Au_{01}' \tag{3.3.11}$$

$$\theta_{00}'' + P\theta_{00}' = 0 \tag{3.3.12}$$

$$\theta_{01}'' + P\theta_{01}' = -P(u_{00}')^2 \tag{3.3.13}$$

$$\theta_{10}'' + P\theta_{10}' - \frac{iP\omega}{4}\theta_{10} = -AP\theta_{00}' \tag{3.3.14}$$

$$\theta_{11}'' + P\theta_{11}' - \frac{iP\omega}{4}\theta_{11} = -AP\theta_{01}' - 2Pu_{00}'u_{10}' \tag{3.3.15}$$

subject to boundary conditions

$$\left.\begin{array}{l} u_{00} = h\dfrac{\partial u_{00}}{\partial y}, \theta_{00} = 1, u_{10} = h\dfrac{\partial u_{10}}{\partial y}, \theta_{10} = 1 \\ u_{01} = h\dfrac{\partial u_{01}}{\partial y}, \theta_{01} = 0, u_{11} = h\dfrac{\partial u_{11}}{\partial y}, \theta_{11} = 0 \end{array}\right\} \text{at} \quad y = 0 \tag{3.3.16}$$

and

$$\left.\begin{array}{l} u_{00} = 0, \theta_{00} = 0, u_{10} = 0, \theta_{10} = 0 \\ u_{01} = 0, \theta_{01} = 0, u_{11} = 0, \theta_{11} = 0 \end{array}\right\} \text{as} \quad y \to \infty \tag{3.3.17}$$

Solving the equations from (3.3.8) to (3.3.15) with the help of boundary conditions (3.3.16) and (3.3.17) we get

$$\theta_{00}(y) = e^{-Py} \tag{3.3.18}$$

$$u_{00}(y) = A_1 e^{-Py} + A_2 e^{\lambda_1 y} \tag{3.3.19}$$

$$\theta_{01}(y) = A_3 e^{\lambda_2 y} + A_4 e^{\lambda_3 y} + A_5 e^{\lambda_4 y} + A_6 e^{-Py} \tag{3.3.20}$$

$$u_{01}(y) = A_7 e^{-Py} + A_8 e^{\lambda_2 y} + A_9 e^{\lambda_3 y} + A_{10} e^{\lambda_4 y} + A_{11} e^{\lambda_1 y} \tag{3.3.21}$$

$$\theta_{10}(y) = A_{13} e^{-Py} + A_{14} e^{\lambda_5 y} \tag{3.3.22}$$

$$u_{10}(y) = A_{16} e^{\lambda_5 y} + A_{17} e^{-Py} + A_{18} e^{\lambda_1 y} + A_{19} e^{\lambda_6 y} \tag{3.3 23}$$

$$\theta_{11}(y)=A_{31}e^{-Py}+\left(A_{32}+A_{38}\right)e^{\lambda_2 y}+\left(A_{33}+A_{37}\right)e^{\lambda_3 y}+\left(A_{34}+A_{41}\right)e^{\lambda_4 y}+A_{35}e^{\lambda_7 y}$$

$$+A_{36}e^{\lambda_8 y}+A_{39}e^{\lambda_9 y}+A_{40}e^{\lambda_{10} y}+A_{43}e^{\lambda_5 y} \qquad (3\ 3.24)$$

$$u_{11}(y)=A_{57}e^{\lambda_5 y}+A_{58}e^{-Py}+\left(A_{59}+A_{65}\right)e^{\lambda_2 y}+\left(A_{60}+A_{64}\right)e^{\lambda_3 y}+\left(A_{61}+A_{68}\right)e^{\lambda_4 y}+$$
$$A_{62}e^{\lambda_7 y}+A_{63}e^{\lambda_8 y}+A_{66}e^{\lambda_9 y}+A_{67}e^{\lambda_{10} y}+A_{69}e^{\lambda_1 y}+A_{72}e^{\lambda_6 y} \qquad (3.3.25)$$

where, $\lambda_1=\dfrac{-1-\sqrt{1+4M}}{2}$, $\lambda_2=2\lambda_1$, $\lambda_3=\lambda_1-P$, $\lambda_4=-2P$,

$\lambda_5=\dfrac{-P-\sqrt{P^2+iP\omega}}{2}$, $\lambda_6=\dfrac{-1-\sqrt{1+4M+i\omega}}{2}$, $\lambda_7=\lambda_1+\lambda_6$, $\lambda_8=\lambda_1+\lambda_5$,

$\lambda_9=\lambda_6-P$, $\lambda_{10}=\lambda_5-P$.

The other constants A_1, A_2, A_3,.. ....A_{71}, A_{72} are obtained and these are shown in Appendix.

The solutions of u_{00}, θ_{00}, u_{01}, θ_{01} are real and the solutions of u_{10}, u_{11}, θ_{10}, θ_{11} are complex.

Substituting equations (3.3.18) to (3.3.25) in equation (3.3.1) and splitting real and imaginary parts we get the expression for velocity and temperature profiles in the following form:

$$u(y,t)=u_{00}(y)+Eu_{01}(y)+\varepsilon\left\{\left(u_{10}{}^{R}+Eu_{11}{}^{R}\right)\cos\omega t-\left(u_{10}{}^{I}+Eu_{11}{}^{I}\right)\sin\omega t\right\} \qquad (3.3\ 26)$$

$$\theta(y,t)=\theta_{00}(y)+E\theta_{01}(y)+\varepsilon\left\{\left(\theta_{10}{}^{R}+E\theta_{11}{}^{R}\right)\cos\omega t-\left(\theta_{10}{}^{I}+E\theta_{11}{}^{I}\right)\sin\omega t\right\} \qquad (3.3.27)$$

Where, $\theta_{10}{}^{R}$ = Real part of $\theta_{10}(y)$

$$=e^{X_2 y}\left(X_4\cos Y_2 y-Y_4\sin Y_2 y\right) \qquad (3.3.28)$$

$\theta_{10}{}^{I}$ = Imaginary part of $\theta_{10}(y)$

$$= e^{X_2 y}\left(X_4 \sin Y_2 y + Y_4 \cos Y_2 y\right) + Y_3 e^{-Py} \qquad (3.3.29)$$

θ_{11}^{R} = Real part of $\theta_{11}(y)$

$$= e^{X_2 y}\left(X_{52} \cos Y_2 y - Y_{52} \sin Y_2 y\right) + e^{-Py} X_{30} + e^{\lambda_2 y}\left(X_{32} + X_{44}\right)$$

$$e^{\lambda_3 y}\left(X_{34} + X_{42}\right) + e^{\lambda_4 y}\left(X_{36} + X_{50}\right) + e^{X_{26} y}\left(X_{38} \cos Y_{26} y - Y_{38} \sin Y_{26} y\right)$$

$$e^{X_{27} y}\left(X_{40} \cos Y_{27} y - Y_{40} \sin Y_{27} y\right) + e^{X_{28} y}\left(X_{46} \cos Y_{28} y - Y_{46} \sin Y_{28} y\right)$$

$$e^{X_{29} y}\left(X_{48} \cos Y_{29} y - Y_{48} \sin Y_{29} y\right) \qquad (3.3.30)$$

θ_{11}^{I} = Imaginary part of $\theta_{11}(y)$

$$= e^{X_2 y}\left(X_{52} \sin Y_2 y + Y_{52} \cos Y_2 y\right) + e^{-Py} Y_{30} + e^{\lambda_2 y}\left(Y_{32} + Y_{44}\right)$$

$$e^{\lambda_3 y}\left(Y_{34} + Y_{42}\right) + e^{\lambda_4 y}\left(Y_{36} + Y_{50}\right) + e^{X_{26} y}\left(X_{38} \sin Y_{26} y + Y_{38} \cos Y_{26} y\right)$$

$$e^{X_{27} y}\left(X_{40} \sin Y_{27} y + Y_{40} \cos Y_{27} y\right) + e^{X_{28} y}\left(X_{46} \sin Y_{28} y + Y_{46} \cos Y_{28} y\right)$$

$$+ e^{X_{29} y}\left(X_{48} \sin Y_{29} y + Y_{48} \cos Y_{29} y\right) \qquad (3.3.31)$$

u_{10}^{R} = Real part of $u_{10}(y)$

$$= e^{\lambda_1 y} X_{15} + e^{-Py} X_{13} + e^{X_2 y}\left(X_{10} \cos Y_2 y - Y_{10} \sin Y_2 y\right)$$

$$+ e^{X_6 y}\left(X_{28} \cos Y_6 y - Y_{28} \sin Y_6 y\right) \qquad (3.3.32)$$

u_{10}^{I} = Imaginary part of $u_{10}(y)$

$$= e^{\lambda_1 y} Y_{15} + e^{-Py} Y_{13} + e^{X_2 y}\left(X_{10} \sin Y_2 y + Y_{10} \cos Y_2 y\right)$$

$$+ e^{X_6 y}\left(X_{28} \sin Y_6 y + Y_{28} \cos Y_6 y\right) \qquad (3.3\ 33)$$

u_{11}^{R} = Real part of $u_{11}(y)$

$$e^{X_6 y}\left(X_{91} \cos Y_6 y - Y_{91} \sin Y_6 y\right) + e^{X_2 y}\left(X_{66} \cos Y_2 y - Y_{66} \sin Y_2 y\right) +$$

$$e^{-Py}X_{68}+e^{\lambda_2 y}\left(X_{70}+X_{80}\right)+e^{\lambda_3 y}\left(X_{72}+X_{79}\right)+e^{\lambda_4 y}\left(X_{74}+X_{85}\right)+$$

$$e^{X_{26}y}\left(X_{76}\cos Y_{26}y-Y_{76}\sin Y_{26}y\right)+e^{X_{27}y}\left(X_{78}\cos Y_{27}y-Y_{78}\sin Y_{27}y\right)$$

$$e^{X_{28}y}\left(X_{82}\cos Y_{28}y-Y_{82}\sin Y_{28}y\right)+e^{X_{29}y}\left(X_{84}\cos Y_{29}y-Y_{84}\sin Y_{29}y\right)$$

$$+e^{\lambda_1 y}X_{86} \qquad (3.3\ 34)$$

u_{11}^{I} = Imaginary part of $u_{11}(y)$

$$=e^{X_6 y}\left(X_{91}\sin Y_6 y+Y_{91}\cos Y_6 y\right)+e^{X_2 y}\left(X_{66}\sin Y_2 y+Y_{66}\cos Y_2 y\right)+$$

$$e^{-Py}Y_{68}+e^{\lambda_2 y}\left(Y_{70}+Y_{80}\right)+e^{\lambda_3 y}\left(Y_{72}+Y_{79}\right)+e^{\lambda_4 y}\left(Y_{74}+Y_{85}\right)+$$

$$e^{X_{26}y}\left(X_{76}\sin Y_{26}y+Y_{76}\cos Y_{26}y\right)+e^{X_{27}y}\left(X_{78}\sin Y_{27}y+Y_{78}\cos Y_{27}y\right)$$

$$e^{X_{28}y}\left(X_{82}\sin Y_{28}y+Y_{82}\cos Y_{28}y\right)+e^{X_{29}y}\left(X_{84}\sin Y_{29}y+Y_{84}\cos Y_{29}y\right)$$

$$+e^{\lambda_1 y}Y_{86} \qquad (3.3.35)$$

where, X_1, X_2 ,.......... ......,X_{91} and Y_1, Y_2,.... .... , Y_{91} are constants and these are shown in Appendix.

3.4: COEFFICIENT OF SKIN-FRICTION

The coefficient of skin-friction at the plate is given by

$$\bar{\tau}=\mu\left.\frac{\partial\bar{u}}{\partial\bar{y}}\right]_{\bar{y}=0}$$, where μ is the viscosity.

The skin-friction in the non-dimensional form on the plate y = 0 is given by

$$\tau=\left.\frac{\mu\dfrac{\partial\bar{u}}{\partial\bar{y}}}{\rho v_0^{2}}\right]_{\bar{y}=0}=\left.\frac{\partial u}{\partial y}\right]_{y=0}=\left\{\frac{\partial u_0}{\partial y}+\varepsilon e^{i\omega t}\frac{\partial u_1}{\partial y}\right\}_{y=0}$$

$$= u_0'(0) + \varepsilon |B| \cos(\omega t + \alpha)$$

[Breaking into real and imaginary parts]

where $|B| = \sqrt{X_R^{\,2} + Y_R^{\,2}}$

$$X_R = u_{10}^{R'}(0) + E u_{11}^{R'}(0)$$

$$Y_R = u_{10}^{I'}(0) + E u_{11}^{I'}(0)$$

$$\tan\alpha = \frac{Y_R}{X_R}$$

3.5: COEFFICIENT OF HEAT-TRANSFER

The (Nusselt number) rate of heat transfer between the fluid and the plate is given by

$$\overline{N}u = -k\left(\frac{\partial \overline{T}}{\partial \overline{y}}\right)_{\overline{y}=0}$$

In the non-dimensional form it is given as below:

$$N_u = -\frac{\overline{N}u\,\upsilon}{k v_0\left(\overline{T}_\omega - \overline{T}_\infty\right)}$$

$$= -\frac{\upsilon}{k v_0\left(\overline{T}_\omega - \overline{T}_\infty\right)}(-k)\left(\frac{\partial \overline{T}}{\partial \overline{y}}\right)_{\overline{y}=0}$$

$$= \frac{\upsilon}{v_0\left(\overline{T}_\omega - \overline{T}_\infty\right)}\left(\overline{T}_\omega - \overline{T}_\infty\right)\left(\frac{\partial \theta}{\partial y}\right)_{y=0}\frac{v_0}{\upsilon}$$

$$= \left(\frac{\partial \theta}{\partial y}\right)_{y=0}$$

$$=\left\{\frac{\partial\theta_0}{\partial y}+\varepsilon e^{i\omega t}\frac{\partial\theta_1}{\partial y}\right\}_{y=0}$$

$$=\theta_0'(0)+\varepsilon|H|\cos(\omega t+\beta)$$ [Splitting into real and imaginary parts]

where, $|H|=\sqrt{P_R^{\,2}+Q_R^{\,2}}$

$$P_R=\theta_{10}^{R'}(0)+E\theta_{11}^{R'}(0)$$

$$Q_R=\theta_{10}^{I'}(0)+E\theta_{11}^{I'}(0)$$

$$\tan\beta=\frac{Q_R}{P_R}$$

3.6: RESULTS AND DISCUSSION

The graphs for amplitude **|B|** (verses A, h and ω) the phase $\tan\alpha$ (verses A and h), of the fluctuating part of the non-dimensional skin friction τ and the amplitude **|H|** (verses A and h), the phase $\tan\beta$ (verses A, h and ω) of the fluctuating part of the non-dimensional heat transfer N_u have been displayed.

Throughout our investigation the Prandtl number P is taken to be equal to 0.7, which corresponds to the air and the Eckert number E is assumed to be 0.01 or 0.05. The other values of the Hartmann number M, the free convection parameter G, the suction parameter A, the frequency parameter ω and the rarefaction parameter h are chosen arbitrary.

Figures (3.1), (3.2) and (3 3) demonstrate the behaviour of |B| against A, M, G and h. These three figures indicates that an increase in each of the values of M, A, h and ω causes |B| to decrease, whereas |B| increases as G is increased. It is also observed from figure (3.1) that the effect of M on |B| is insignificant for large values of A. Further figure (3.2) and figure (3.3) show that $|B| \to 0$ as $\omega \to \infty$ or $h \to \infty$ irrespective of values of |G| and M.

The variation of $\tan\alpha$ against A, M and h are presented in figure (3.4) and (3.5). It is inferred from these two figures that $\tan\alpha$ sharply increases due application of magnetic field. Figure (3.4) shows that an increase in A results a significant increase in $\tan\alpha$ whereas $\tan\alpha$ slowly decreases as h is increased.

The behaviour of |H| against A, M and h are exhibited in figures (3.6) and (3.7). It is seen from these two figures that |H| increases for the increase values of M, A and h. The effect of M on |H| seems to be insignificant for large values of h as observed from figure (3.7). Further from figure (3 6) we see that |H| becomes unpronounced for large M.

Figure (3.8), (3.9) and (3 10) exhibit the variation of $\tan\beta$ against A, M, ω, |G| and h. Figure (3.8) shows that an increase in M or A causes $\tan\beta$ to decrease steadily and $\tan\beta \to 0$ as $A \to \infty$. It is inferred from figure (3.9) that $\tan\beta$ sharply increases for small values of ω and it is increased slowly and steadily for large and moderate values of ω. The same figure also indicates that $\tan\beta$ increases when the magnitude of Grashof number is increased. Again in figure (3.10) we observe that $\tan\beta$ slowly and steadily decrease as h or M is increased From these three figures we further observe that $\tan\beta$ is lightly effected by M, |G| for large values of A, ω and h.

Fig 3.1: Skin friction amplitude |B| against suction parameter A when G=5, P=. 7, E= .01, h =.4, ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

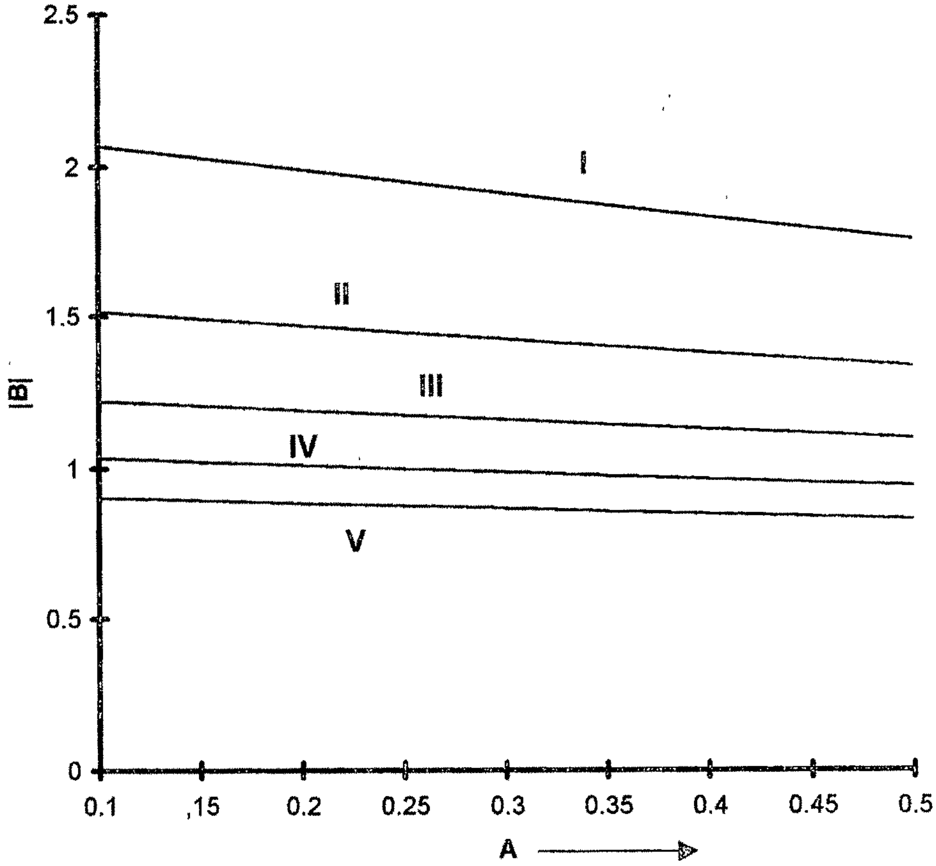


Fig 3.2: Skin friction amplitude |B| verses frequency ω when P=.7, E=.01, h=.4, A=.2, M =2

Curve	I	II	III	IV
G	±3	±5	±7	±10

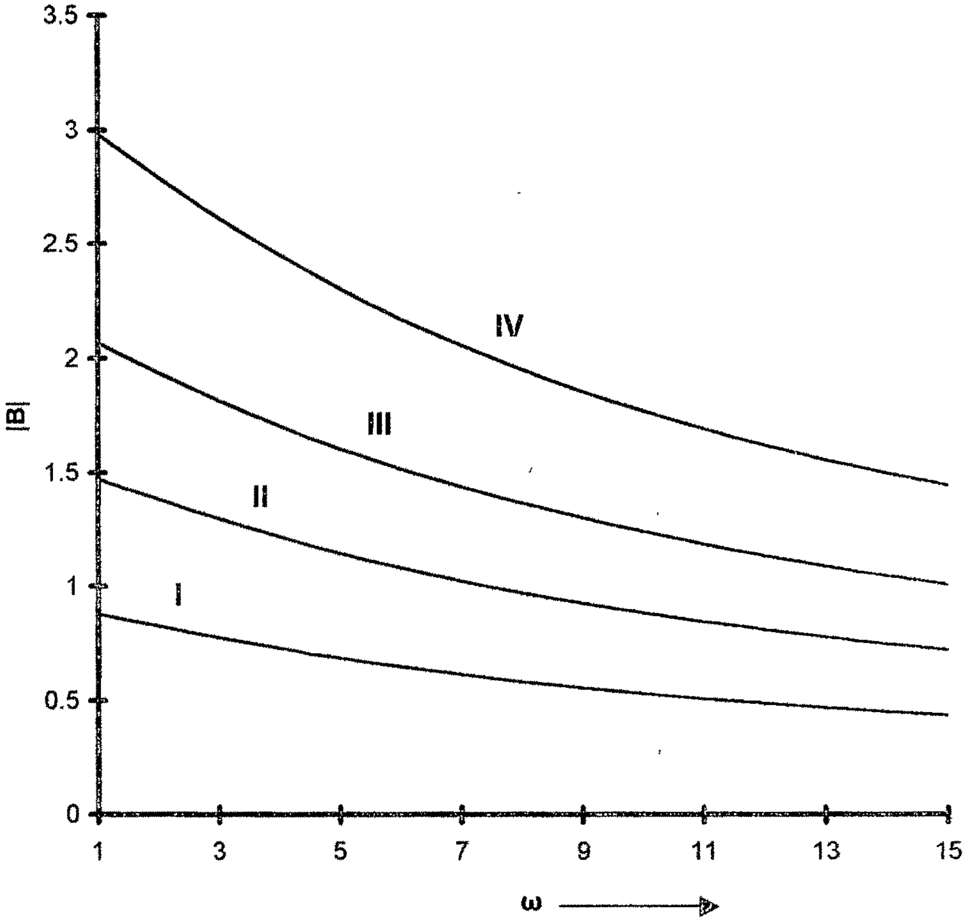


Fig 3.3: Skin friction amplitude |B| against rarefaction parameter h when G=5, P= .7, E= .01, A= .2, ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

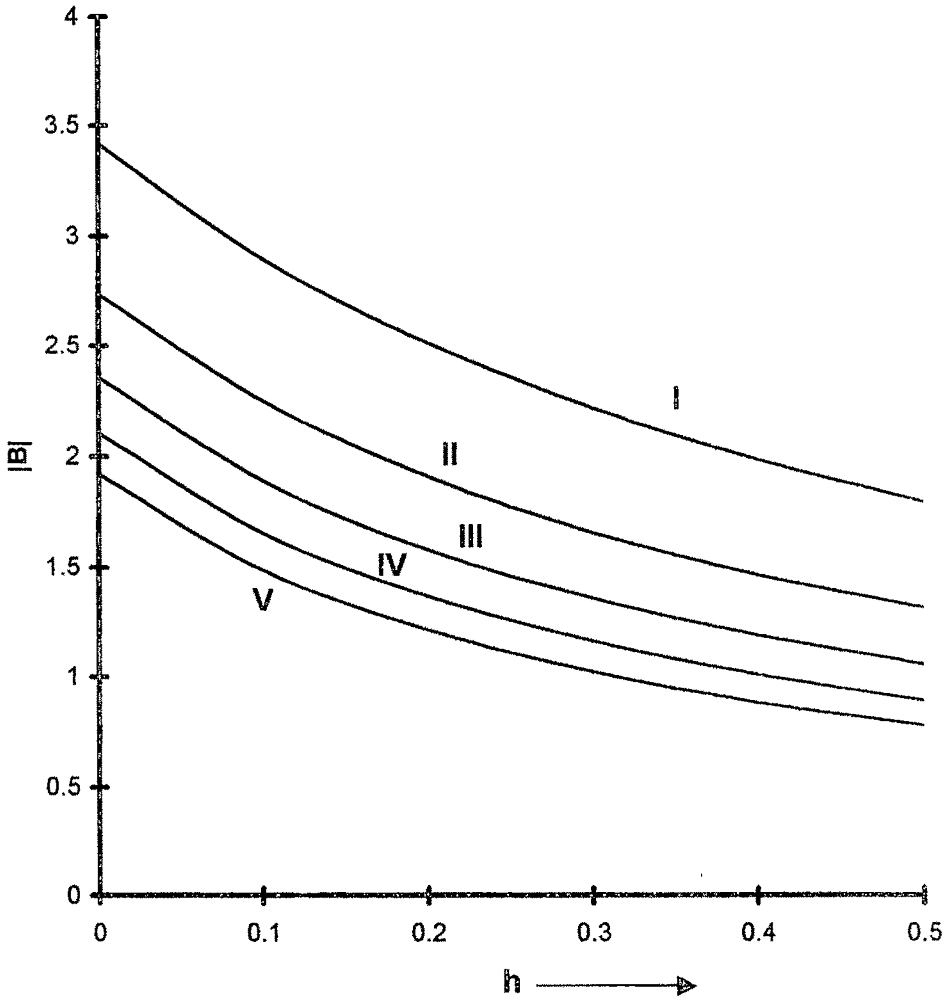


Fig 3.4: Skin friction phase tanα against suction parameter A when G=5, P=.7, E=.01, h=.4, ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

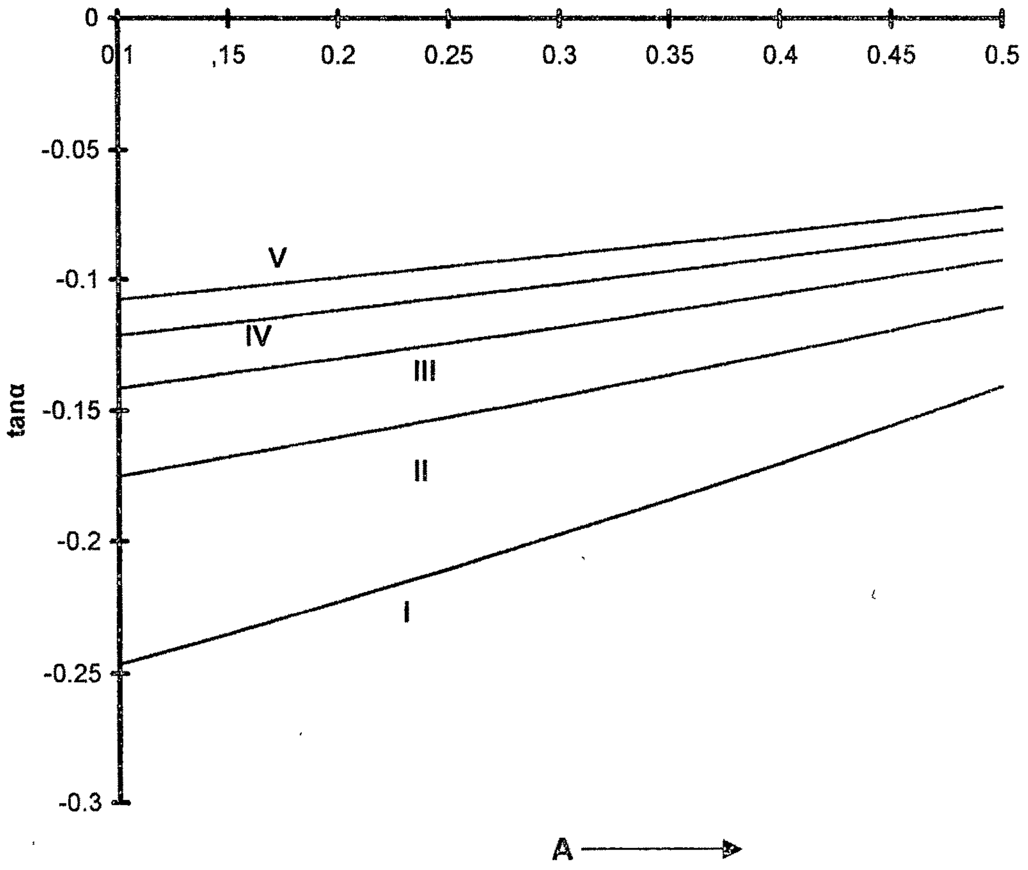


Fig 3.5: Skin friction phase $\tan\alpha$ against rarefaction parameter h when G=5, P=.7, E=.01, A=.2, ω=1

Curve	I	II	III	IV	V
M	1	2	3	4	5

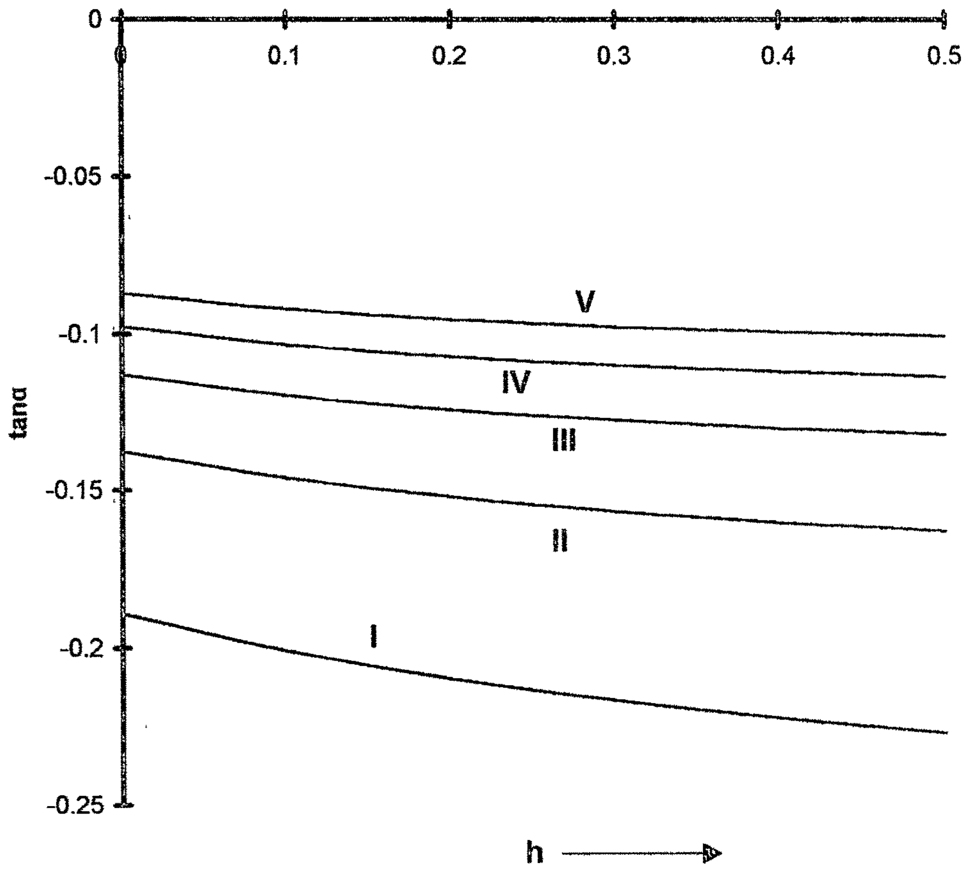


Fig 3.6: Heat transfer amplitude |H| against suction parameter A when G=10, P= .7, E=.01, h=.2 , ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

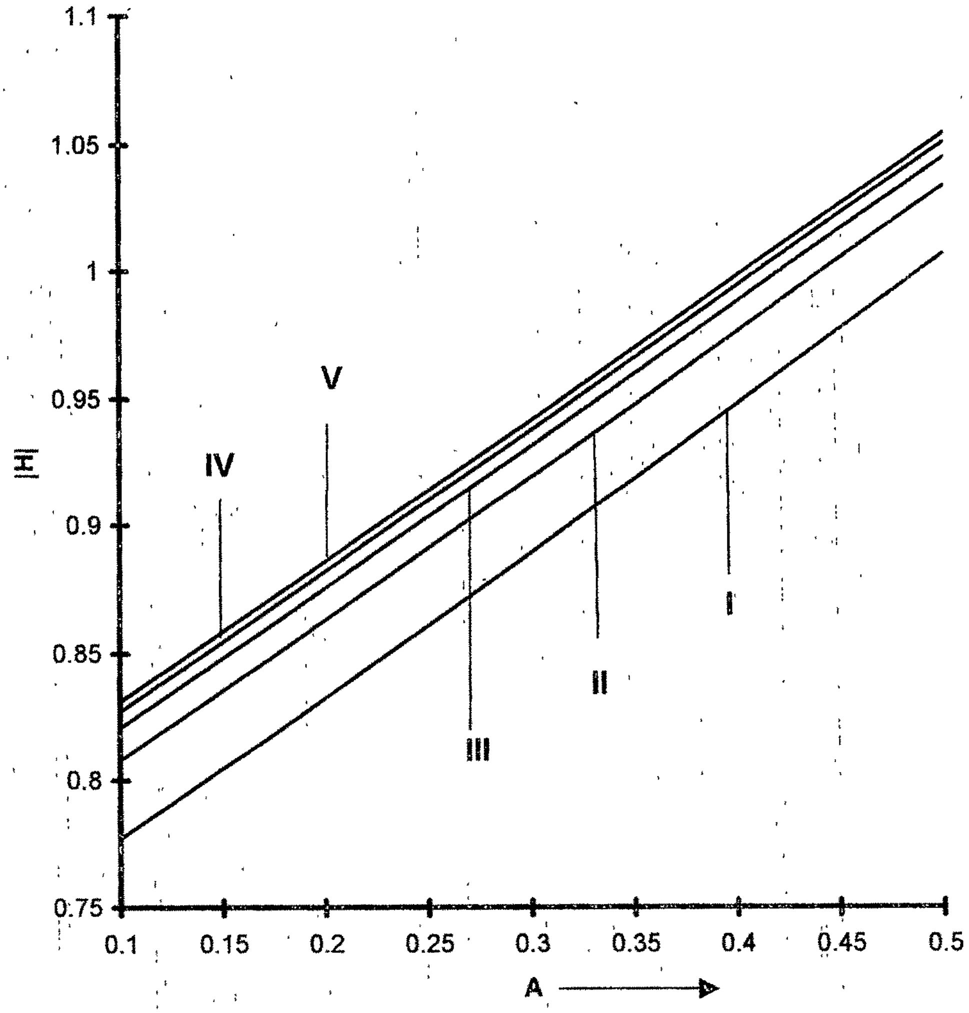


Fig 3.7: Heat transfer amplitude |H| against rarefaction parameter h when G=5, P= .7, E=.01, A=.2 , ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

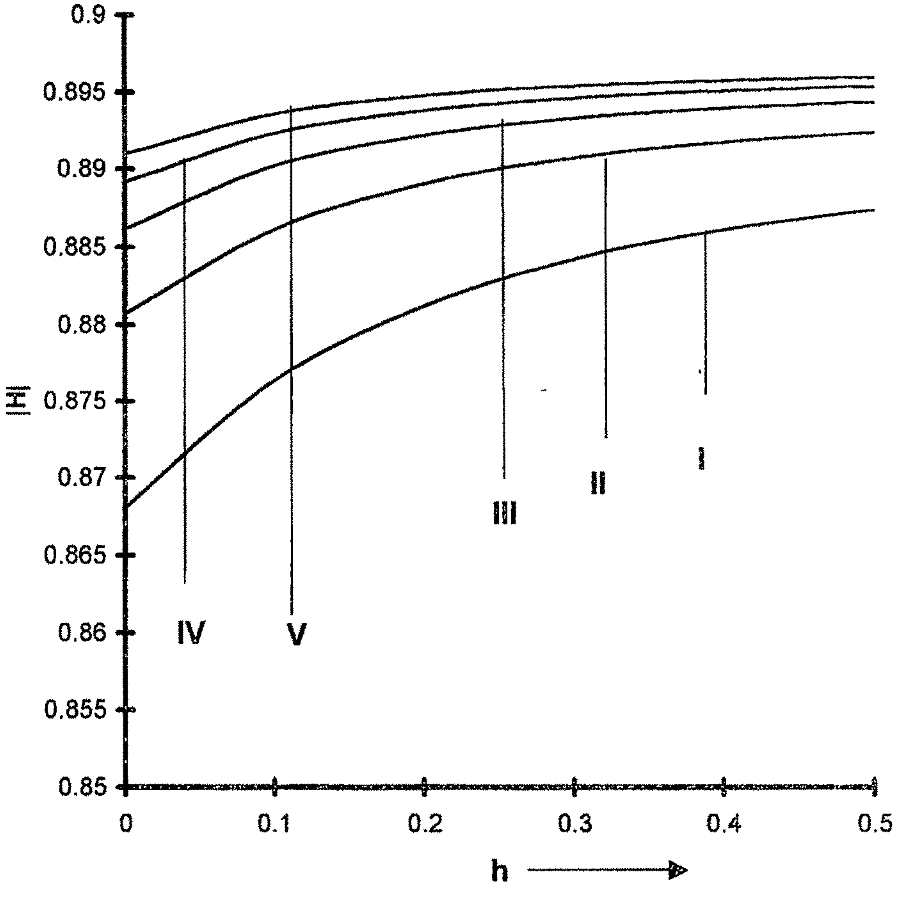


Fig 3.8: Heat transfer phase tanβ verses suction parameter A when
G=10, P= .7, E= .01, h = . 2 ω =1

Curve	I	II	III	IV	V
M	1	2	3	4	5

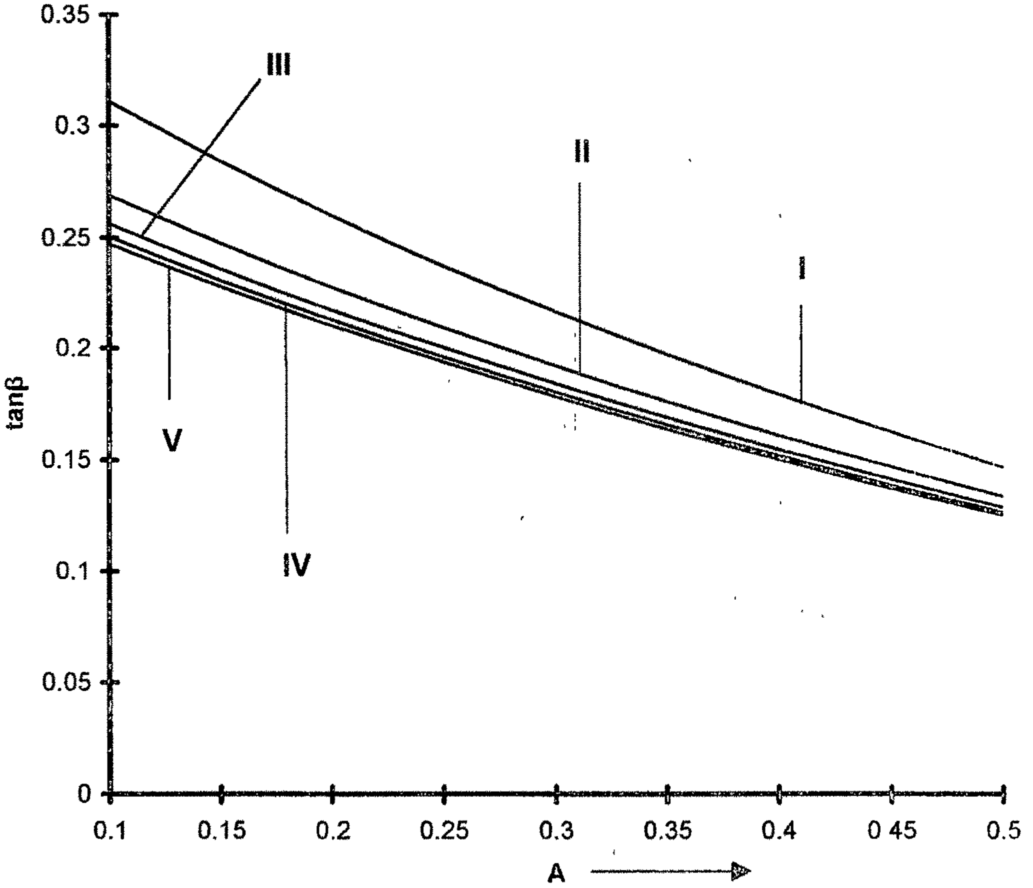


Fig 3.9: Heat transfer phase tan β verses frequency ω when P= .7, E=.05, h=.2, A=.2, M =2

Curve	I	II	III	IV
G	±3	±5	±7	±10

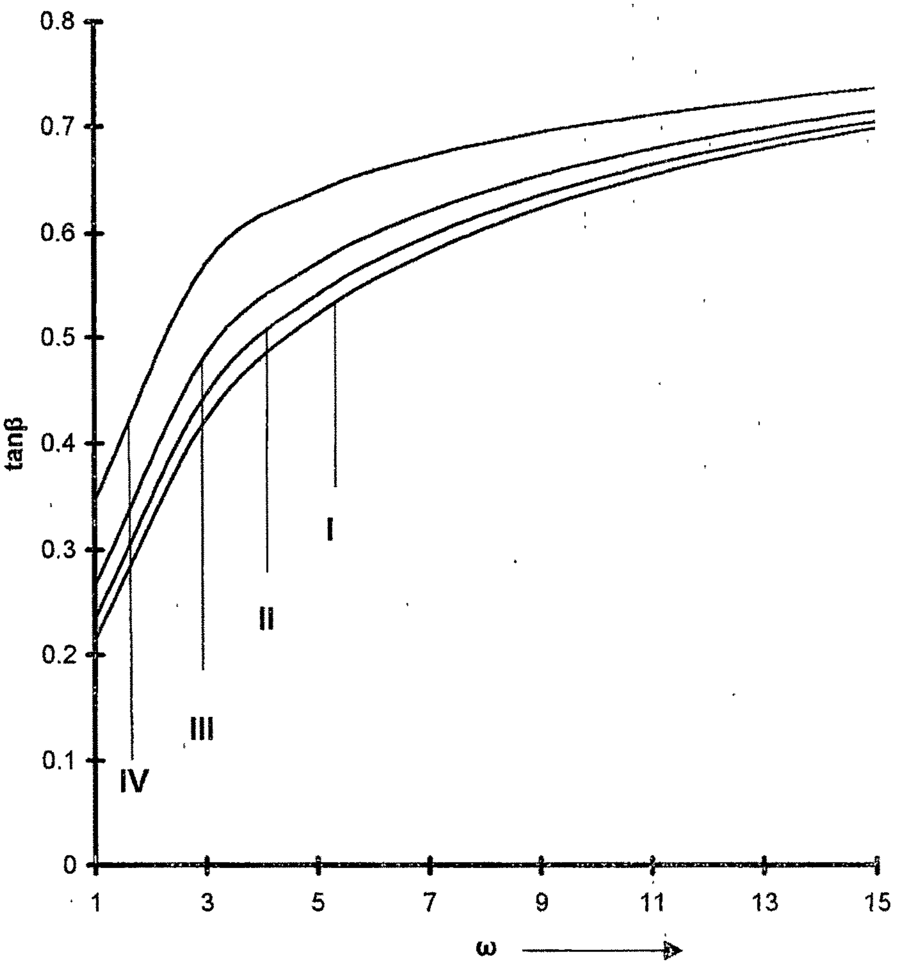
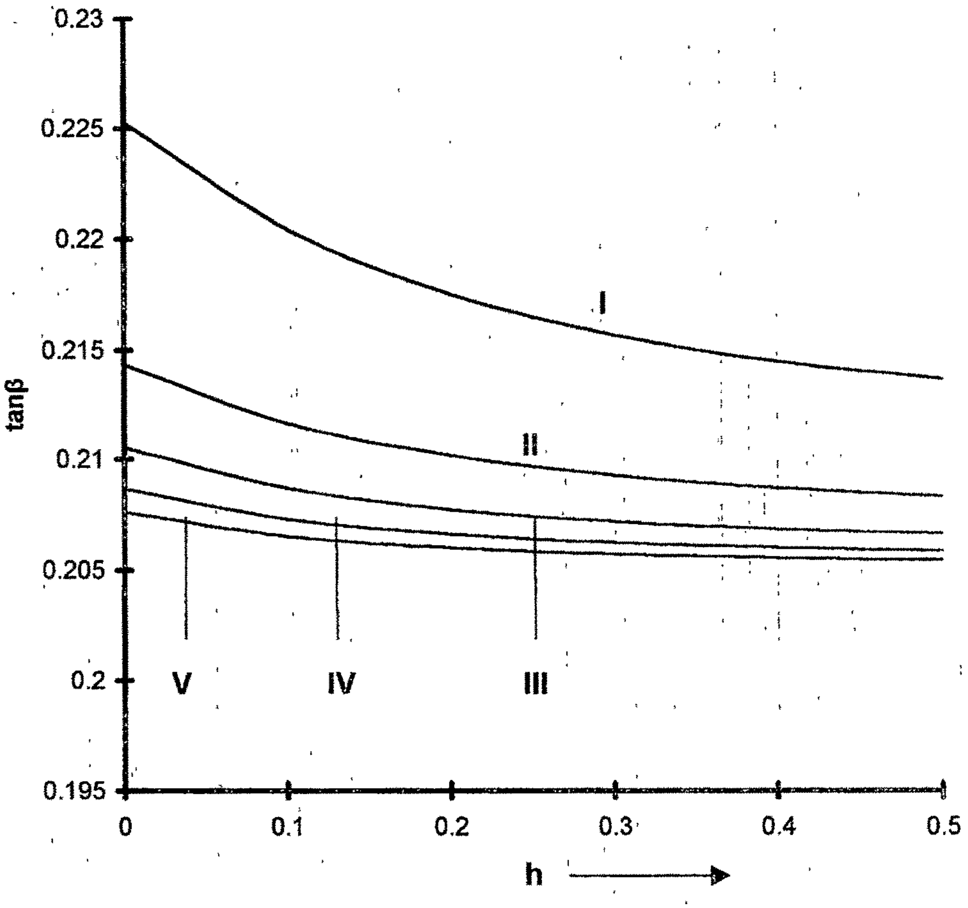


Fig 3.10: Heat transfer phase $\tan\beta$ against rarefaction parameter h when $G=5$, $P=.7$, $E=.01$, $A=.2$, $\omega=1$

Curve	I	II	III	IV	V
M	1	2	3	4	5



APPENDIX

Real constants

$$A_1 = \frac{-G}{P^2 - P - M},\ A_2 = \frac{A_1(Ph+1)}{h\lambda_1 - 1},\ A_3 = \frac{-PA_2^{\,2}\lambda_1^{\,2}}{4\lambda_1^{\,2} + 2P\lambda_1},\ A_4 = \frac{2P^2A_1A_2\lambda_1}{\lambda_3^{\,2} + P\lambda_3},$$

$$A_5 = -\frac{PA_1^{\,2}}{2},\ A_6 = -(A_3 + A_4 + A_5),\ A_6 = -(A_3 + A_4 + A_5),$$

$$A_8 = \frac{-GA_3}{\lambda_2^{\,2} + \lambda_2 - M},\ A_9 = \frac{-GA_4}{\lambda_4^{\,2} + \lambda_4 - M},\ A_{10} = \frac{-GA_5}{\lambda_4^{\,2} + \lambda_4 - M},$$

$$A_{11} = \frac{-A_7(Ph+1) + (h\lambda_2 - 1)A_8 + (h\lambda_3 - 1)A_9 + (h\lambda_4 - 1)A_{10}}{1 - h\lambda_1},$$

$$A_{20} = P^2A_6A,\ A_{21} = -P\lambda_2AA_3,\ A_{22} = -P\lambda_3AA_4,\ A_{23} = -P\lambda_4AA_5,$$

$$A_{56} = -\lambda_1AA_{11}$$

Complex constants with their real and imaginary parts

$$A_{12} = \frac{-iP\omega}{4} = X_1 + iY_1,\ \lambda_5 = \frac{-P - \sqrt{P^2 + iP\omega}}{2} = X_2 + iY_2,$$

$$A_{13} = \frac{AP^2}{A_{12}} = X_3 + iY_3,\ A_{14} = 1 - A_{13} = X_4 + iY_4,$$

$$\lambda_6 = \frac{-1 - \sqrt{1 + 4M + i\omega}}{2} = X_6 + iY_6,\ A_{15} = M + i\frac{\omega}{4} = X_7 + iY_7,$$

$$A_{16} = \frac{-GA_{14}}{\lambda_5^{\,2} + \lambda_5 - A_{15}} = X_{10} + iY_{10},\ A_{17} = \frac{PAA_1 - GA_{13}}{P^2 - P - A_{15}} = X_{13} + iY_{13},$$

$$A_{18} = \frac{-AA_2\lambda_1}{\lambda_1^{\,2} + \lambda_1 - A_{15}} = X_{15} + iY_{15},$$

$$A_{19}=\frac{(h\lambda_5-1)A_{16}-A_{17}(Ph+1)+(h\lambda_1-1)A_{18}}{1-h\lambda_6}=X_{18}+iY_{18},$$

$$A_{24}=-2P\lambda_1\lambda_6A_2A_{19}=X_{19}+iY_{19},\ A_{25}=-2P\lambda_1\lambda_5A_2A_{16}=X_{20}+iY_{20},$$

$$A_{26}=2P^2\lambda_1(A_2A_{17}+A_1A_{18})=X_{21}+iY_{21},\ A_{27}=-2P\lambda_1^2A_2A_{18}=X_{22}+iY_{22},$$

$$A_{28}=2P^2\lambda_6A_1A_{19}=X_{23}+iY_{23},\ A_{29}=2P^2\lambda_5A_1A_{16}=X_{24}+iY_{24},$$

$$A_{30}=-2P^3A_1A_{17}=X_{25}+iY_{25},\ \lambda_7=\lambda_1+\lambda_6=X_{26}+iY_{26},$$

$$\lambda_8=\lambda_1+\lambda_5=X_{27}+iY_{27},\ \lambda_9=\lambda_6-P=X_{28}+iY_{28},\ \lambda_{10}=\lambda_5-P=X_{29}+iY_{29},$$

$$A_{31}=\frac{A_{20}}{A_{12}}=X_{30}+iY_{30},\ A_{32}=\frac{A_{21}}{\lambda_2^2+P\lambda_2+A_{12}}=X_{32}+iY_{32},$$

$$A_{33}=\frac{A_{22}}{\lambda_3^2+P\lambda_3+A_{12}}=X_{34}+iY_{34},\ A_{34}=\frac{A_{23}}{\lambda_4^2+P\lambda_4+A_{12}}=X_{36}+iY_{36},$$

$$A_{35}=\frac{A_{24}}{\lambda_7^2+P\lambda_7+A_{12}}=X_{38}+iY_{38},\ A_{36}=\frac{A_{25}}{\lambda_8^2+P\lambda_8+A_{12}}=X_{40}+iY_{40},$$

$$A_{37}=\frac{A_{26}}{\lambda_3^2+P\lambda_3+A_{12}}=X_{42}+iY_{42},\ A_{38}=\frac{A_{27}}{\lambda_2^2+P\lambda_2+A_{12}}=X_{44}+iY_{44},$$

$$A_{39}=\frac{A_{28}}{\lambda_9^2+P\lambda_9+A_{12}}=X_{46}+iY_{46},\ A_{40}=\frac{A_{29}}{\lambda_{10}^2+P\lambda_{10}+A_{12}}=X_{48}+iY_{48},$$

$$A_{41}=\frac{A_{30}}{2P^2+A_{12}}=X_{50}+iY_{50},$$

$$A_{42}=A_{31}+A_{32}+A_{33}+A_{34}+A_{35}+A_{36}+A_{37}+A_{38}+A_{39}+A_{40}+A_{41}$$

$$=X_{51}+iY_{51},\ A_{43}=-A_{42}=X_{52}+iY_{52},\ A_{44}=-GA_{43}=X_{53}+iY_{53},$$

$$A_{45}=-GA_{31}+PAA_7=X_{54}+iY_{54},\ A_{46}=-GA_{32}-\lambda_2AA_8=X_{55}+iY_{55},$$

$$A_{47}=-GA_{33}-\lambda_3AA_9=X_{56}+iY_{56},\ A_{48}=-GA_{34}-\lambda_4AA_{10}=X_{57}+iY_{57},$$

$$A_{49}=-GA_{35}=X_{58}+iY_{58},\ A_{50}=-GA_{36}=X_{59}+iY_{59},$$

$$A_{51}=-GA_{37}=X_{60}+iY_{60},\ A_{52}=-GA_{38}=X_{61}+iY_{61},$$

$$A_{53}=-GA_{39}=X_{62}+iY_{62},\ A_{54}=-GA_{40}=X_{63}+iY_{63},$$

$$A_{55}=-GA_{41}=X_{64}+iY_{64},\ A_{57}=\frac{A_{44}}{\lambda_5^2+\lambda_5-A_{15}}=X_{66}+iY_{66},$$

$$A_{58}=\frac{A_{45}}{P^2-P-A_{15}}=X_{68}+iY_{68},\ A_{59}=\frac{A_{46}}{\lambda_2^2+\lambda_2-A_{15}}=X_{70}+iY_{70},$$

$$A_{60}=\frac{A_{47}}{\lambda_3^2+\lambda_3-A_{15}}=X_{72}+iY_{72},\ A_{61}=\frac{A_{48}}{\lambda_4^2+\lambda_4-A_{15}}=X_{74}+iY_{74},$$

$$A_{62}=\frac{A_{49}}{\lambda_7^2+\lambda_7-A_{15}}=X_{76}+iY_{76},\ A_{63}=\frac{A_{50}}{\lambda_8^2+\lambda_8-A_{15}}=X_{78}+iY_{78},$$

$$A_{64}=\frac{A_{51}}{\lambda_3^2+\lambda_3-A_{15}}=X_{79}+iY_{79},\ A_{65}=\frac{A_{52}}{\lambda_2^2+\lambda_2-A_{15}}=X_{80}+iY_{80},$$

$$A_{66}=\frac{A_{53}}{\lambda_9^2+\lambda_9-A_{15}}=X_{82}+iY_{82},\ A_{67}=\frac{A_{54}}{\lambda_{10}^2+\lambda_{10}-A_{15}}=X_{84}+iY_{84},$$

$$A_{68}=\frac{A_{55}}{4P^2-2P-A_{15}}=X_{85}+iY_{85},\ A_{69}=\frac{A_{56}}{\lambda_1^2+\lambda_1-A_{15}}=X_{86}+iY_{86},$$

$$A_{70}=\lambda_5A_{57}-PA_{58}+\lambda_2A_{59}+\lambda_3A_{60}+\lambda_4A_{61}+\lambda_7A_{62}+\lambda_8A_{63}+$$

$$\lambda_3A_{64}+\lambda_2A_{65}+\lambda_9A_{66}+\lambda_{10}A_{67}+\lambda_4A_{68}+\lambda_1A_{69}=X_{87}+iY_{87},$$

$$A_{71}=A_{57}+A_{58}+A_{59}+A_{60}+A_{61}+A_{62}+A_{63}+A_{64}+A_{65}+A_{66}+A_{67}$$

$$A_{68}+A_{69}=X_{88}+iY_{88},\ A_{72}=\frac{A_{71}-hA_{70}}{h\lambda_6-1}=X_{91}+iY_{91},\ X_1=0,$$

$$Y_1=-\frac{\omega P}{4},\ X_2=-\frac{P}{2}\left\{1+\sqrt{\frac{1+\sqrt{1+\frac{\omega^2}{P^2}}}{2}}\right\},\ Y_2=-\frac{P}{2}\left\{1+\sqrt{\frac{\sqrt{1+\frac{\omega^2}{P^2}}-1}{2}}\right\},$$

$$X_3=0,\ Y_3=\frac{4AP}{\omega},\ X_4=1,\ Y_4=-Y_3,$$

$$X_5=\frac{1}{\sqrt{2}}\sqrt{\left\{(1+4M)+\sqrt{(1+4M)^2+\omega^2}\right\}},$$

$$Y_5=\frac{1}{\sqrt{2}}\sqrt{\left\{\sqrt{(1+4M)^2+\omega^2}-(1+4M)\right\}},\ X_6=\frac{-1-X_5}{2},\ Y_6=\frac{-Y_5}{2},\ X_7=M,$$

$$Y_7=\frac{\omega}{4},\ X_8=-GX_5,\ Y_8=-GY_5,\ X_9=X_2{}^2-Y_2{}^2+X_2-X_7,$$

$$Y_9=2X_2Y_2+Y_2-Y_7,\ X_{10}=\frac{X_8X_9+Y_8Y_9}{X_9{}^2+Y_9{}^2},\ Y_{10}=\frac{X_9Y_8-X_8Y_9}{X_9{}^2+Y_9{}^2},$$

$$X_{11}=PAA_1,\ Y_{11}=-GY_3,\ X_{12}=P^2-P-X_7,\ X_{13}=\frac{X_{11}X_{12}+Y_{11}Y_{12}}{X_{12}{}^2+Y_{12}{}^2},$$

$$Y_{13}=\frac{Y_{11}X_{12}-X_{11}Y_{12}}{X_{12}{}^2+Y_{12}{}^2},\ X_{14}=\lambda_1{}^2+\lambda_1-X_7,\ Y_{14}=-Y_7,\ X_{15}=-\frac{AA_2\lambda_1X_{14}}{X_{14}{}^2+Y_{14}{}^2},$$

$$Y_{15}=\frac{AA_2\lambda_1Y_{14}}{X_{14}{}^2+Y_{14}{}^2},X_{16}=h\left(X_2X_{10}-Y_2Y_{10}\right)-X_{10}-(1+ph)X_{13}+(h\lambda_1-1)X_{15},$$

$$Y_{16}=h\left(Y_2X_{10}+X_2Y_{10}\right)-Y_{10}-(1+ph)Y_{13}+(h\lambda_1-1)Y_{15},\ X_{17}=1-hX_6,$$

$$Y_{17}=-hY_6,\ X_{18}=\frac{X_{16}X_{17}+Y_{16}Y_{17}}{X_{17}{}^2+Y_{17}{}^2},\ Y_{18}=\frac{Y_{16}X_{17}-X_{16}Y_{17}}{X_{17}{}^2+Y_{17}{}^2},$$

$$X_{19}=-2P\lambda_1A_2\left(X_6X_{18}-Y_6Y_{18}\right),\quad Y_{19}=-2P\lambda_1A_2\left(X_6Y_{18}+Y_6X_{18}\right),$$

$$X_{20}=-2P\lambda_1A_2\left(X_2X_{10}-Y_2Y_{10}\right),\quad Y_{20}=-2P\lambda_1A_2\left(X_{10}Y_2+Y_{10}X_2\right),$$

$$X_{21}=2P^2\lambda_1\left(A_2X_{13}+A_1X_{15}\right),\quad Y_{21}=2P^2\lambda_1\left(A_2Y_{13}+A_1Y_{15}\right),$$

$$X_{22}=-2P\lambda_1^2A_2X_{15},\quad Y_{22}=-2P\lambda_1^2A_2Y_{15},\quad X_{23}=2P^2A_1\left(X_6X_{18}-Y_6Y_{18}\right),$$

$$Y_{23}=2P^2A_1\left(Y_6X_{18}+X_6Y_{18}\right),\quad X_{24}=2P^2A_1\left(X_2X_{10}-Y_2Y_{10}\right),$$

$$Y_{24}=2P^2A_1\left(X_2Y_{10}+Y_2X_{10}\right),\quad X_{25}=-2P^3A_1X_{13},\quad Y_{25}=-2P^3A_1Y_{13},$$

$$X_{26}=\lambda_1+X_6,\quad Y_{26}=Y_6,\quad X_{27}=\lambda_1+X_2,\quad Y_{27}=Y_2,\quad X_{28}=X_6-P,$$

$$Y_{28}=Y_6,\quad X_{29}=X_2-P,\quad Y_{29}=Y_2,\quad X_{30}=0,\quad Y_{30}=\frac{-A_{20}}{Y_1},\quad X_{31}=\lambda_2{}^2+P\lambda_2,$$

$$Y_{31}=Y_1,\quad X_{32}=-\frac{A_{21}X_{31}}{X_{31}{}^2+Y_{31}{}^2},\quad Y_{32}=-\frac{A_{21}Y_{31}}{X_{31}{}^2+Y_{31}{}^2},\quad X_{33}=\lambda_3{}^2+P\lambda_3,\quad Y_{33}=Y_1,$$

$$X_{34}=\frac{A_{22}X_{33}}{X_{33}{}^2+Y_{33}{}^2},\quad Y_{34}=-\frac{A_{22}Y_{33}}{X_{33}{}^2+Y_{33}{}^2},\quad X_{35}=\lambda_4{}^2+P\lambda_4,\quad Y_{35}=Y_1,$$

$$X_{36}=\frac{A_{23}X_{35}}{X_{35}{}^2+Y_{35}{}^2},\quad Y_{36}=-\frac{A_{23}Y_{35}}{X_{35}{}^2+Y_{35}{}^2},\quad X_{37}=X_{26}{}^2-Y_{26}{}^2+PX_{26},$$

$$Y_{37}=2X_{26}Y_{26}+PY_{26}+Y_1,\quad X_{38}=\frac{X_{19}X_{37}+Y_{19}Y_{37}}{X_{37}{}^2+Y_{37}{}^2},\quad Y_{38}=\frac{Y_{19}X_{37}-X_{19}Y_{37}}{X_{37}{}^2+Y_{37}{}^2},$$

$$X_{39}=X_{27}{}^2-Y_{27}{}^2+PX_{27},\quad Y_{39}=2X_{27}Y_{27}+PY_{27}+Y_1,$$

$$X_{40}=\frac{X_{20}X_{39}+Y_{20}Y_{39}}{X_{39}{}^2+Y_{39}{}^2},\quad Y_{40}=\frac{Y_{20}X_{39}-X_{20}Y_{39}}{X_{39}{}^2+Y_{39}{}^2},\quad X_{41}=\lambda_3{}^2+P\lambda_3,$$

$$Y_{41}=Y_1,\quad X_{42}=\frac{X_{21}X_{41}+Y_{21}Y_{41}}{X_{41}{}^2+Y_{41}{}^2},\quad Y_{42}=\frac{X_{41}Y_{21}-X_{21}Y_{41}}{X_{41}{}^2+Y_{41}{}^2},\quad X_{43}=\lambda_2{}^2+P\lambda_2,$$

$$Y_{43}=Y_1,\ X_{44}=\frac{X_{22}X_{43}+Y_{22}Y_{43}}{X_{43}^2+Y_{43}^2},\ Y_{44}=\frac{X_{43}Y_{22}-X_{22}Y_{43}}{X_{43}^2+Y_{43}^2},$$

$$X_{45}=X_{28}^2-Y_{28}^2+PX_{28},\ Y_{45}=2X_{28}Y_{28}+PY_{28}+Y_1,$$

$$X_{46}=\frac{X_{23}X_{45}+Y_{23}Y_{45}}{X_{45}^2+Y_{45}^2},\ Y_{46}=\frac{X_{45}Y_{23}-X_{23}Y_{45}}{X_{45}^2+Y_{45}^2},\ X_{47}=X_{29}^2-Y_{29}^2+PX_{29},$$

$$Y_{47}=2X_{29}Y_{29}+PY_{29}+Y_1,\ X_{48}=\frac{X_{24}X_{47}+Y_{24}Y_{47}}{X_{47}^2+Y_{47}^2},\ Y_{48}=\frac{X_{47}Y_{24}-X_{24}Y_{47}}{X_{47}^2+Y_{47}^2},$$

$$X_{49}=2P^2,\ Y_{49}=Y_1,\ X_{50}=\frac{X_{25}X_{49}+Y_{25}Y_{49}}{X_{49}^2+Y_{49}^2},\ Y_{50}=\frac{X_{49}Y_{25}-X_{25}Y_{49}}{X_{49}^2+Y_{49}^2},$$

$$X_{51}=X_{32}+X_{34}+X_{36}+X_{38}+X_{40}+X_{42}+X_{44}+X_{46}+X_{48}+X_{50},$$

$$Y_{51}=Y_{32}+Y_{34}+Y_{36}+Y_{38}+Y_{40}+Y_{42}+Y_{44}+Y_{46}+Y_{48}+Y_{50},\ X_{52}=-X_{51},$$

$$Y_{52}=-Y_{51},\ X_{53}=-GX_{52},\ Y_{53}=-GY_{52},\ X_{54}=APA_7,\ Y_{54}=-GY_{30},$$

$$X_{55}=-GX_{32}-\lambda_2AA_8,\ Y_{55}=-GY_{32},\ X_{56}=-GX_{34}-A\lambda_3A_9,$$

$$Y_{56}=-GY_{34},\ X_{57}=-GX_{36}-A\lambda_4A_{10},\ Y_{57}=-GY_{36},\ X_{58}=-GX_{38},$$

$$Y_{58}=-GY_{38},\ X_{59}=-GX_{40},\ Y_{59}=-GY_{40},\ X_{60}=-GX_{42},\ Y_{60}=-GY_{42},$$

$$X_{61}=-GX_{44},\ Y_{61}=-GY_{44},\ X_{62}=-GX_{46},\ Y_{62}=-GY_{46},\ X_{63}=-GX_{48},$$

$$Y_{63}=-GY_{48},\ X_{64}=-GX_{50},\ Y_{64}=-GY_{50},\ X_{65}=X_2^2-Y_2^2+X_2-X_7,$$

$$Y_{65}=2X_2Y_2+Y_2-Y_7,\ X_{66}=\frac{X_{53}X_{65}+Y_{53}Y_{65}}{X_{65}^2+Y_{65}^2},\ Y_{66}=\frac{X_{65}Y_{53}-X_{53}Y_{65}}{X_{65}^2+Y_{65}^2},$$

$$X_{67}=P^2-P-X_7,\ Y_{67}=-Y_7,\ X_{68}=\frac{X_{54}X_{67}+Y_{54}Y_{67}}{X_{67}^2+Y_{67}^2},$$

$$Y_{68}=\frac{X_{67}Y_{54}-X_{54}Y_{67}}{X_{67}^2+Y_{67}^2},\ X_{69}=\lambda_2^2+\lambda_2-X_7,\ Y_{69}=-Y_7,$$

$$X_{70}=\frac{X_{55}X_{69}+Y_{55}Y_{69}}{X_{69}^2+Y_{69}^2},\ Y_{70}=\frac{X_{69}Y_{55}-X_{55}Y_{69}}{X_{69}^2+Y_{69}^2},\ X_{71}=\lambda_3^2+\lambda_3-X_7,$$

$$Y_{71}=-Y_7,\ X_{72}=\frac{X_{56}X_{71}+Y_{56}Y_{71}}{X_{71}^2+Y_{71}^2},\ Y_{72}=\frac{X_{71}Y_{56}-X_{56}Y_{71}}{X_{71}^2+Y_{71}^2},$$

$$X_{73}=\lambda_4^2+\lambda_4-X_7,\ Y_{73}=-Y_7,\ X_{74}=\frac{X_{57}X_{73}+Y_{57}Y_{73}}{X_{73}^2+Y_{73}^2},$$

$$Y_{74}=\frac{X_{73}Y_{57}-X_{57}Y_{73}}{X_{73}^2+Y_{73}^2},\ X_{75}=X_{26}^2-Y_{26}^2+X_{26}-X_7,$$

$$Y_{75}=2X_{26}Y_{26}+Y_{26}-Y_7,\ X_{76}=\frac{X_{58}X_{75}+Y_{58}Y_{75}}{X_{75}^2+Y_{75}^2},\ Y_{76}=\frac{X_{75}Y_{58}-X_{58}Y_{75}}{X_{75}^2+Y_{75}^2},$$

$$X_{77}=X_{27}^2-Y_{27}^2+X_{27}-X_7,\ Y_{77}=2X_{27}Y_{27}+Y_{27}-Y_7,$$

$$X_{78}=\frac{X_{59}X_{77}+Y_{59}Y_{77}}{X_{77}^2+Y_{77}^2},\ Y_{78}=\frac{X_{77}Y_{59}-X_{59}Y_{77}}{X_{77}^2+Y_{77}^2},\ X_{79}=\frac{X_{60}X_{71}+Y_{60}Y_{71}}{X_{71}^2+Y_{71}^2},$$

$$Y_{79}=\frac{X_{71}Y_{60}-X_{60}Y_{71}}{X_{71}^2+Y_{71}^2},\ X_{80}=\frac{X_{61}X_{69}+Y_{61}Y_{69}}{X_{69}^2+Y_{69}^2},\ Y_{80}=\frac{X_{69}Y_{61}-X_{61}Y_{69}}{X_{69}^2+Y_{69}^2},$$

$$X_{81}=X_{28}^2-Y_{28}^2+X_{28}-X_7,\ Y_{81}=2X_{28}Y_{28}+Y_{28}-Y_7,$$

$$X_{82}=\frac{X_{62}X_{81}+Y_{62}Y_{81}}{X_{81}^2+Y_{81}^2},\ Y_{82}=\frac{X_{81}Y_{62}-X_{62}Y_{81}}{X_{81}^2+Y_{81}^2},\ X_{83}=X_{29}^2-Y_{29}^2+X_{29}-X_7,$$

$$Y_{83}=2X_{29}Y_{29}+Y_{29}-Y_7,\ X_{84}=\frac{X_{63}X_{83}+Y_{63}Y_{83}}{X_{83}^2+Y_{83}^2},$$

$$Y_{84}=\frac{X_{83}Y_{63}-X_{63}Y_{83}}{X_{83}^2+Y_{83}^2},X_{85}=\frac{X_{64}X_{73}+Y_{64}Y_{73}}{X_{73}^2+Y_{73}^2},\ Y_{85}=\frac{X_{73}Y_{64}-X_{64}Y_{73}}{X_{73}^2+Y_{73}^2},$$

$$X_{86}=\frac{X_{69}A_{56}}{X_{69}^2+Y_{69}^2},\ Y_{86}=-\frac{Y_{69}A_{56}}{X_{69}^2+Y_{69}^2},$$

$$X_{87}=\left(X_2X_{66}-Y_2Y_{66}\right)-PX_{68}+\lambda_2X_{70}+\lambda_3X_{72}+$$

$$\lambda_4X_{74}+\left(X_{26}X_{76}-Y_{26}Y_{76}\right)+\left(X_{27}X_{78}-Y_{27}Y_{78}\right)+\lambda_3X_{79}+\lambda_2X_{80}+$$

$$\left(X_{28}X_{82}-Y_{28}Y_{82}\right)+\left(X_{29}X_{84}-Y_{29}Y_{84}\right)-2PX_{85}+\lambda_1X_{86},$$

$$Y_{87}=\left(X_2Y_{66}+Y_2X_{66}\right)-PY_{68}+\lambda_2Y_{70}+\lambda_3Y_{72}+\lambda_4Y_{74}+\left(X_{26}Y_{76}+X_{76}Y_{26}\right)$$

$$+\left(X_{27}Y_{78}+Y_{27}X_{78}\right)+\lambda_3Y_{79}+\lambda_2Y_{80}+\left(X_{28}Y_{82}+Y_{28}X_{82}\right)+\left(X_{29}Y_{84}+Y_{29}X_{84}\right)$$

$$-2PY_{85}+\lambda_1Y_{86},\ X_{88}=X_{66}+X_{68}+X_{70}+X_{72}+X_{74}+X_{76}+X_{78}+X_{79}+$$

$$X_{80}+X_{81}+X_{84}+X_{85}+X_{86},\ Y_{88}=Y_{66}+Y_{68}+Y_{70}+Y_{72}+Y_{74}+Y_{76}+Y_{78}$$

$$+Y_{79}+Y_{80}+Y_{81}+Y_{84}+Y_{85}+Y_{86},\ X_{89}=X_{88}-hX_{87};\ Y_{89}=Y_{88}-hY_{87},$$

$$X_{90}=hX_6-1,\ Y_{90}=hY_6,\ X_{91}=\frac{X_{90}X_{89}+Y_{89}Y_{90}}{X_{90}^2+Y_{90}^2},\ Y_{91}=\frac{X_{90}Y_{89}-X_{89}Y_{90}}{X_{90}^2+Y_{90}^2}.$$