CHAPTER X

THERMAL DIFFUSION EFFECT ON A THREE-DIMENSIONAL MHD FREE CONVECTION WITH MASS TRANSFER FLOW FROM A POROUS VERTICAL PLATE

10.1: INTRODUCTION

Many natural phenomena and technological problems are susceptible to MHD analysis. Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields. Engineers employ MHD principle in the design of heat exchange pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating naval power generating systems etc. From technological point of view, MHD convection flow problems are also very significant in the fields of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. Many have made model studies of the above phenomena of MHD convection. Some of them are Sanyal and Bhattacharya (1992), Ferraro and Plumpton (1966) and Cramer and Pai (1973). On the other hand, along with free convection currents, caused by the temperature difference, the flow is also affected by the difference in concentrations on material constitutions. Many investigators have studied the phenomena of MHD free convection and mass transfer flow of whom the names of Singh and Singh (2000) and Singh *et al.* (2007) are worth mentioning.

The effect of the three dimensional flow caused by the periodic suction perpendicular to the main flow when the difference between the wall temperature and free stream temperature gives rise to buoyancy force in the direction of the free stream on heat transfer characteristics was investigated by Singh *et. al.* (1988), Ahmed and Sarma (1997), Chaudhury and Chand (2002). The effects of transverse sinusoidal injection velocity distribution on the three dimensional free convective couette flow of a viscous incompressible fluid in slip flow regime under the influence of heat source has been recently studied by Jain and Gupta (2006). Ahmed *et. al.* (2006) obtained an analytical solution to the problem of the three-dimensional free convective flow of an incompressible viscous fluid past a porous vertical plate with transverse sinusoidal suction velocity taking into account the presence of species concentration. However in the above studies the effect of thermal diffusion was ignored. This assumption is true when the concentration level is very low. There are, however exceptions. The thermal diffusion effect (known as Soret effect) is applied for isotope separation and in mixtures between gases with very light molecular weight (H_2, H_e) and medium molecular weight (N_2, air) where the diffusion – thermo effect is found to be of a magnitude such that it can not be neglected. Sattar and Allam (1994) studied the joint effect of thermal diffusion and transpiration on MHD free convection and mass transfer flow past an accelerated vertical plate having a variable suction. The problem of two dimensional MHD free convection and mass transfer flow past an infinite vertical porous plate taking into account the combined effect of heat sources and thermal diffusion has been studied recently by Singh *et al.* (2007).

In view of the importance of the joint effect of the magnetic field and thermal diffusion, it is proposed to study a problem of three-dimensional MHD free convection with mass transfer from a porous vertical plate taking into account the effect of thermal diffusion. In the present work our main objectives are to study the Soret effect as well as the magnetic field effect on the flow, heat and mass transfer.

10.2: BASIC EQUATIONS

The equations governing the steady motion of an incompressible viscous electrically conducting fluid in presence of a magnetic field are

The equation of continuity:

$$\operatorname{div}\vec{q}=0 \tag{10.2.1}$$

The Gauss's law of magnetism:

$$\operatorname{div}\overline{B}=0 \tag{10.2.2}$$

The momentum equation:

$$\left(\vec{q}.\overline{\nabla}\right)\vec{q}=-\frac{1}{\rho}\overline{\nabla}p+\frac{\vec{J}\times\overline{B}}{\rho}+\upsilon\nabla^{2}\vec{q}+\vec{g} \tag{10.2.3}$$

The energy equation:

$$\rho C_{p}\left[\left(\vec{q}.\overline{\nabla}\right)\overline{T}\right]=k\nabla^{2}\overline{T}+\Phi+\frac{\vec{J}^{2}}{\sigma} \tag{10.2.4}$$

The species continuity equation:

$$\left(\vec{q}.\overline{\nabla}\right)\overline{C}=D_{m}\nabla^{2}\overline{C}+D_{T}\nabla^{2}\overline{T} \tag{10.2.5}$$

The Ohm's law:

$$\vec{J}=\sigma\left[\overline{E}+\vec{q}\times\overline{B}\right] \tag{10.2.6}$$

where $\vec{q}$ is the velocity vector, p is the pressure, $\vec{g}$ is the acceleration due to gravity, υ is the kinematic viscosity, $\overline{T}$ is the temperature, $\overline{B}$ is the magnetic induction vector, $\vec{J}$ is the electric current density, $\overline{E}$ is the electric field, (here assumed to be zero) σ is the electrical conductivity, k is the thermal conductivity , Φ is the viscous dissipation of energy per unit volume, C_p is the specific heat at constant pressure, ρ is the density of the fluid, $\vec{J}\times\overline{B}$ is the Lorentz force per unit volume, $\overline{C}$ is the species concentration, D_m is the co-efficient of chemical molecular diffusivity, D_T is the co-efficient of chemical thermal diffusivity and the other symbols have their usual meanings.

We now consider the steady free convection flow of an incompressible viscous electrically conducting fluid taking into account species concentration and thermal diffusion past a vertical porous plate with transverse sinusoidal suction velocity a by making the following assumptions:

(i) All the fluid properties except the density in the buoyancy force term are constant.

(ii) A magnetic field of uniform strength B_0 is applied transversely to the direction of the main stream.

(iii) The magnetic Reynolds number is so small that the induced magnetic fluid can be neglected.

(iv) The viscous dissipation and magnetic dissipations of energy are negligible

(v) $\overline{T}_W > \overline{T}_\infty$ and $\overline{C}_W > \overline{C}_\infty$

We introduce a co-ordinate system $(\overline{x},\overline{y},\overline{z})$ with $X-\text{axis}$ vertically upwards along the plate, $Y-\text{axis}$ perpendicular to it and directed into the fluid region and $Z-\text{axis}$ along the width of the plate.

Let $\vec{q}=\hat{i}\overline{u}+\hat{j}\overline{v}+\hat{k}\overline{w}$ be the fluid velocity at the point $(\overline{x},\overline{y},\overline{z})$ and $\vec{B}=B_o\hat{j}$ be the applied magnetic field, $\hat{i}$, $\hat{j}$, $\hat{k}$ being the unit vectors along positive $X-\text{axis}$, $Y-\text{axis}$ and $Z-\text{axis}$ respectively.

The suction velocity is taken as follows:

$$\overline{v}_W(\overline{z})=-V_o\left[1+\varepsilon\,\text{Cos}\frac{\pi\overline{z}}{L}\right] \qquad (10.2.7)$$

where ε is small reference parameter such that $\varepsilon << 1$ and L is the wave length of the periodic suction.

Since the plate is infinite in length in $X-$direction, therefore all the quantities except possibly the pressure are assumed to be independent of $\overline{x}$.

With the foregoing assumptions and under the usual boundary layer and Boussinesq approximations, the equations (10.2.1), (10.2.3), (10.2.4) and (10.2.5) reduce to

Equation of continuity

$$\frac{\partial \overline{v}}{\partial \overline{y}} + \frac{\partial \overline{w}}{\partial \overline{z}} = 0 \tag{10.2.8}$$

Momentum equations

$$\overline{v}\frac{\partial \overline{u}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{u}}{\partial \overline{z}} = g\beta\left(\overline{T} - \overline{T}_\infty\right) + g\overline{\beta}\left(\overline{C} - \overline{C}_\infty\right) + \upsilon\left(\frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{u}}{\partial \overline{z}^2}\right) - \frac{\sigma {B_0}^2 \overline{u}}{\rho} \tag{10.2.9}$$

$$\overline{v}\frac{\partial \overline{v}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{v}}{\partial \overline{z}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{y}} + \upsilon\left(\frac{\partial^2 \overline{v}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{v}}{\partial \overline{z}^2}\right) \tag{10.2.10}$$

$$\overline{v}\frac{\partial \overline{w}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{w}}{\partial \overline{z}} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial \overline{z}} + \upsilon\left(\frac{\partial^2 \overline{w}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{w}}{\partial \overline{z}^2}\right) - \frac{\sigma {B_0}^2 \overline{w}}{\rho} \tag{10.2.11}$$

Energy equation:

$$\overline{v}\frac{\partial \overline{T}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{T}}{\partial \overline{z}} = \alpha\left(\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{z}^2}\right) \tag{10.2.12}$$

Species continuity equation

$$\overline{v}\frac{\partial \overline{C}}{\partial \overline{y}} + \overline{w}\frac{\partial \overline{C}}{\partial \overline{z}} = D_m\left(\frac{\partial^2 \overline{C}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{C}}{\partial \overline{z}^2}\right) + D_T\left(\frac{\partial^2 \overline{T}}{\partial \overline{y}^2} + \frac{\partial^2 \overline{T}}{\partial \overline{z}^2}\right) \tag{10.2.13}$$

where, β is the coefficient of volume expansion for heat transfer

$\overline{\beta}$ is the coefficient of volume expansion for mass transfer

α is the thermal diffusivity

$\overline{T}_\infty$ is the fluid temperature in the free stream

$\overline{C}_\infty$ is the species concentration in the free stream and the other symbols have their usual meanings.

The equation (10.2.2) is satisfied by $\vec{B}=B_0\hat{j}$

The relevant boundary conditions are

$$\overline{y}=0:\ \overline{u}=0,\ \overline{v}=\overline{v}_w,\ \overline{w}=0,\ \overline{T}=\overline{T}_w,\ \overline{C}=\overline{C}_w \tag{10.2.14}$$

$$\overline{y}\to\infty:\ \overline{u}=0,\ \overline{v}=-V_0,\ \overline{w}=0,\ \overline{T}=\overline{T}_\infty,\ \overline{C}=\overline{C}_\infty,\ \overline{p}=\overline{p}_\infty \tag{10.2.15}$$

We introduce the following non-dimensional quantities

$y=\dfrac{\overline{y}}{L}$ (distance), $z=\dfrac{\overline{z}}{L}$ (distance), $u=\dfrac{\overline{u}}{V_0}$ (velocity), $v=\dfrac{\overline{v}}{V_0}$ (velocity), $w=\dfrac{\overline{w}}{V_0}$ (velocity), $\theta=\dfrac{\overline{T}-\overline{T}_\infty}{\overline{T}_w-\overline{T}_\infty}$ (fluid temperature), $\phi=\dfrac{\overline{C}-\overline{C}_\infty}{\overline{C}_w-\overline{C}_\infty}$ (species concentration), $P=\dfrac{\upsilon}{\alpha}$ (Prandtl number), $S_c=\dfrac{\upsilon}{D_m}$ (Schmidt number), $S_o=\dfrac{D_T\left(\overline{T}_w-\overline{T}_\infty\right)}{\upsilon\left(\overline{C}_w-\overline{C}_\infty\right)}$ (Soret number), $G_r=\dfrac{Lg\beta\left(\overline{T}_w-T_\infty\right)}{V_0^2}$ (Grashof number for heat transfer), $G_m=\dfrac{Lg\beta\left(\overline{C}_w-\overline{C}_\infty\right)}{V_0^2}$ (Grashof number for mass transfer), $M=\dfrac{\sigma B_0^2\upsilon}{\rho v_0^2}$ (Hartmann number), $R=\dfrac{V_0L}{\upsilon}$ (Reynolds number), $p=\dfrac{\overline{p}}{\rho\left(\frac{\upsilon}{L}\right)^2}$, $p_\infty=\dfrac{\overline{p}_\infty}{\rho\left(\frac{\upsilon}{L}\right)^2}$

where, p_∞ is the pressure at large distance from the plate.

The non-dimensional forms of the equations (10.2.8), (10.2.9), (10.2.10), (10.2.11), (10.2.12) and (10.2.13) are

$$\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{10.2 16}$$

$$v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=G_r\theta+G_m\phi+\frac{1}{R}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)-MRu \tag{10.2 17}$$

$$v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{R^2}\frac{\partial p}{\partial y}+\frac{1}{R}\left(\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \tag{10.2.18}$$

$$v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{R^2}\frac{\partial p}{\partial z}+\frac{1}{R}\left(\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-MRw \tag{10.2.19}$$

$$v\frac{\partial \theta}{\partial y}+w\frac{\partial \theta}{\partial z}=\frac{1}{PR}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \tag{10.2.20}$$

$$v\frac{\partial \phi}{\partial y}+w\frac{\partial \phi}{\partial z}=\frac{1}{S_cR}\left(\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}\right)+\frac{S_0}{R}\left(\frac{\partial^2 \theta}{\partial y^2}+\frac{\partial^2 \theta}{\partial z^2}\right) \tag{10.2.21}$$

with relevant boundary conditions

$$y=0:\ u=0,\ v=-(1+\varepsilon\,\mathrm{Cos}\pi z),\ w=0,\ \theta=1, \phi=1 \tag{10.2.22}$$

$$y\to\infty: u=0,\ v=-1,\ w=0, \theta=0, \phi=0, P=P_\infty \tag{10.2.23}$$

10.3: METHOD OF SOLUTION

We assume the solutions of the equations (10.2.16) to (10.2.21) to be of the form

$$u=u_0(y)+\varepsilon u_1(y,z)+0(\varepsilon^2) \tag{10.3.1}$$

$$v=v_0(y)+\varepsilon v_1(y,z)+0(\varepsilon^2) \tag{10.3.2}$$

$$w=w_0(y)+\varepsilon w_1(y,z)+0(\varepsilon^2) \tag{10.3.3}$$

$$p = p_0(y) + \varepsilon p_1(y,z) + 0(\varepsilon^2) \tag{10.3.4}$$

$$\theta = \theta_0(y) + \varepsilon \theta_1(y,z) + 0(\varepsilon^2) \tag{10.3.5}$$

$$\phi = \phi_0(y) + \varepsilon \phi_1(y,z) + 0(\varepsilon^2) \tag{10.3.6}$$

with $p_0 = p_\infty$, $w_0 = 0$

Substituting these in the equations (10.2.16) to (10.2.21) and by equating the co-efficient of the similar terms and neglecting ε^2 the following differential equations are obtained.

Zeroth Order equations

$$\frac{dv_0}{dy} = 0 \tag{10.3.7}$$

$$v_0 \frac{du_0}{dy} = G_r\theta_0 + G_m\phi_0 + \frac{1}{R}\frac{d^2u_0}{dy^2} - MRu \tag{10.3.8}$$

$$v_0 \frac{d\theta_0}{dy} = \frac{1}{PR}\frac{d^2\theta_0}{dy^2} \tag{10.3.9}$$

$$v_0 \frac{d\phi_0}{dy} = \frac{1}{S_cR}\frac{d^2\phi_0}{dy^2} + \frac{S_0}{R}\frac{d^2\theta_0}{dy^2} \tag{10.3.10}$$

First order equations

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \tag{10.3.11}$$

$$-\frac{\partial u_1}{\partial y} + v_1\frac{du_0}{dy} = G_r\theta_1 + G_m\phi_1 + \frac{1}{R}\left(\frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) - MRu_1 \tag{10.3.12}$$

$$-\frac{\partial v_1}{\partial y} = -\frac{1}{R^2}\frac{\partial p_1}{\partial y} + \frac{1}{R}\left(\frac{\partial^2 v_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2}\right) \tag{10.3.13}$$

$$-\frac{\partial w_1}{\partial y} = -\frac{1}{R^2}\frac{\partial p_1}{\partial z} + \frac{1}{R}\left(\frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2}\right) - MRw_1 \tag{10.3.14}$$

$$-\frac{\partial \theta_1}{\partial y} + v_1\frac{d\theta_0}{dy} = \frac{1}{PR}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) \tag{10.3.15}$$

$$-\frac{\partial \phi_1}{\partial y} + v_1\frac{d\phi_0}{dy} = \frac{1}{S_c R}\left(\frac{\partial^2 \phi_1}{\partial y^2} + \frac{\partial^2 \phi_1}{\partial z^2}\right) + \frac{S_o}{R}\left(\frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2}\right) \tag{10.3.16}$$

Subject to the boundary conditions

$$\begin{aligned} y = 0\,;\, u_0 = 0, v_0 = -1, \theta_0 = 1, \phi_0 = 1, u_1 = 0, \\ v_1 = -\cos\pi z, w_1 = 0, \theta_1 = 0,\ \phi_1 = 0 \end{aligned} \tag{10.3.17}$$

$$\begin{aligned} y \to \infty\,;\, u_0 = 0, v_0 = -1, \theta_0 = 0, \phi_0 = 0, u_1 = 0, \\ v_1 = 0, w_1 = 0, p_1 = 0, \theta_1 = 0,\ \phi_1 = 0 \end{aligned} \tag{10.3.18}$$

The solution of the equations (10.3.7) to (10.3.10) under the boundary conditions (10.3.17) and (10.3.18) are

$$v_0 = -1 \tag{10.3.19}$$

$$\theta_0 = e^{-PRy} \tag{10.3.20}$$

$$\phi_0 = (1-a)e^{-S_c Ry} + ae^{-PRy} \quad \text{for } S_c \neq P \tag{10.3.21}$$

$$= \left(1 + P^2 S_0 Ry\right)e^{-PRy} \quad \text{for } S_c = P \tag{10.3.22}$$

$$u_0 = A_1 e^{-PRy} + A_2 e^{-S_c Ry} + (-A_1 - A_2)e^{-\lambda Ry} \text{ for } S_c \neq P \tag{10.3.23}$$

$$= A_6\left(e^{-\lambda Ry} - e^{-PRy}\right) - A_5 y e^{-PRy} \quad \text{for } S_c = P \tag{10.3.24}$$

where, $A_1 = \dfrac{G_m P S_c S_o}{R\left(P^2 - P - M\right)\left(P - S_c\right)} - \dfrac{G_r}{R\left(P^2 - P - M\right)}$,

$A_2 = \dfrac{-G_m}{R\left(S_c^2 - S_c - M\right)}\left[1 + \dfrac{PS_c S_o}{P - S_c}\right]$, $a = \dfrac{PS_c S_o}{S_c - P}$, $\lambda = \dfrac{1 + \sqrt{1 + 4M}}{2}$, $A_6 = A_3 + A_4$,

$A_5 = G_m \eta P^2 S_o R$, $A_4 = G_m \eta \{1 + \eta R (2P-1) S_o P^2\}$, $A_3 = G_r \eta$,

$$\eta = \frac{1}{R(P^2 - P - M)}$$

10.4: CROSS FLOW SOLUTION

We shall first consider the equations (10.3.11), (10.3.13), (10.3.14) for $v_1(y,z)$, $w_1(y,z)$ and $p_1(y,z)$ which are independent of the main flow component u_1, temperature field θ_1 and the concentration field ϕ_1.

The suction velocity $V_w = -(1 + \varepsilon \cos \pi z)$ consists of a basic uniform distribution -1 with superimposed weak sinusoidal distribution $\varepsilon \cos \pi z$. Hence the velocity components v, w and p are also separated into mean and small sinusoidal components v_1, w_1 and p_1.

We assume v_1, w_1 and p_1 to be of the following forms

$$v_1 = -\pi v_{11}(y) \cos \pi z \quad (10.4.1)$$

$$w_1 = v'_{11}(y) \sin \pi z \quad (10.4.2)$$

$$p_1 = R^2 p_{11}(y) \cos \pi z \quad (10.4.3)$$

On substitution of (10.4.1), (10.4.2) and (10.4.3), the equation (10.3.11) is satisfied and the equations (10.3.13) and (10.3.14) reduce to the following ordinary differential equations

$$v''_{11} + R v'_{11} - \pi^2 v_{11} = -\frac{R p'_{11}}{\pi} \quad (10.4.4)$$

$$v'''_{11} + R v''_{11} - (\pi^2 + MR^2) v'_{11} = -R \pi p_{11} \quad (10.4.5)$$

with relevant boundary conditions

$$y=0;\quad v_{11}=\frac{1}{\pi},\, v'_{11}=0 \tag{10.4.6}$$

$$y\to\infty;\; v_{11}=0,\, v'_{11}=0 \tag{10.4.7}$$

The solutions of these equations are

$$v_{11}=\frac{1}{\pi\left(\bar{\xi}-\xi\right)}\left[\bar{\xi}e^{-\xi y}-\xi e^{-\bar{\xi}y}\right] \tag{10.4.8}$$

$$p_{11}=\frac{\xi\bar{\xi}}{R\pi^2\left(\bar{\xi}-\xi\right)}\left[\left(\pi^2+MR^2+R\bar{\xi}-\bar{\xi}^2\right)e^{-\bar{\xi}y}\right.$$

$$\left.-\left(\pi^2+MR^2+R\xi-\xi^2\right)e^{-\xi y}\right] \tag{10.4.9}$$

where, $\xi=\frac{R\lambda+\sqrt{R^2\lambda^2+4\pi^2}}{2}$, $\bar{\xi}=\frac{R\bar{\lambda}+\sqrt{R^2\bar{\lambda}^2+4\pi^2}}{2}$, $\lambda=\frac{1+\sqrt{1+4M}}{2}$, $\bar{\lambda}=\frac{1-\sqrt{1+4M}}{2}$.

Hence the solutions of the velocity components v_1 ; w_1 and pressure p_1 are as follows

$$v_1=\frac{1}{\xi-\bar{\xi}}\left[\bar{\xi}e^{-\xi y}-\xi e^{-\bar{\xi}y}\right]\cos\pi z \tag{10.4.9}$$

$$w_1=\frac{\xi\bar{\xi}}{\pi\left(\bar{\xi}-\xi\right)}\left[\bar{\xi}e^{-\bar{\xi}y}-e^{-\xi y}\right]\sin\pi z \tag{10.4.10}$$

$$p_1=\frac{R\xi\bar{\xi}}{\pi^2\left(\bar{\xi}-\xi\right)}\left[\bar{\xi}_1e^{-\bar{\xi}y}-\xi_1e^{-\xi y}\right]\cos\pi z \tag{10.4.11}$$

where, $\bar{\xi}_1=\pi^2+MR^2+R\bar{\xi}-\bar{\xi}^2$, $\xi_1=\pi^2+MR^2+R\xi-\xi^2$

10.5: SOLUTIONS FOR THE FIRST ORDER FLOW, CONCENTRATION AND TEMPERATURE FIELD

We now consider the equations (10.3.12), (10.3.15) and (10.3.16). The solutions for velocity components u, temperature field θ and concentration field ϕ are also separated into mean and sinusoidal components u_1, θ_1 and ϕ_1. To reduce the partial differential equations (10.3.12), (10.3.15), (10.3.16) into ordinary differential equations, we consider the following forms for u_1, θ_1 and ϕ_1

$$u_1 = u_{11}(y)\cos\pi z \tag{10.5.1}$$

$$\theta_1 = \theta_{11}(y)\cos\pi z \tag{10.5.2}$$

$$\phi_1 = \phi_{11}(y)\cos\pi z \tag{10.5.3}$$

Using the expressions for v_1, u_1, θ_1, ϕ_1 in (10.3.12), (10.3.15) and (10.3.16), we get the following ordinary differential equations

$$u_{11}'' + Ru_{11}' - \left(\pi^2 + MR^2\right)u_{11} = -\pi R v_{11} u_0' - G_r R\theta_{11} - G_m R\phi_{11} \tag{10.5.4}$$

$$\theta_{11}' + PR\theta_{11}' - \pi^2\theta_{11} = -\pi PRv_{11}\theta_0' \tag{10.5.5}$$

$$\phi_{11}'' + S_c R\phi_{11}' - \pi^2\phi_{11} = -S_c R\pi v_{11}\phi_0' - S_c S_0\left(\theta_{11}'' - \pi^2\theta_{11}\right) \tag{10.5 6}$$

with the boundary conditions

$$y = 0: \quad u_{11} = 0,\ \theta_{11} = 0,\ \phi_{11} = 0 \tag{10.5.7}$$

$$y \to \infty: u_{11} = 0,\ \theta_{11} = 0,\ \phi_{11} = 0 \tag{10.5.8}$$

The solutions of the equations (10.5.4), (10.5.5) and (10.5.6) subject to the boundary conditions (10.5.7) and (10.5.8) are

$$\theta_{11} = G_0 e^{-hy} + G_1 e^{-(\xi+PR)y} + G_2 e^{-(\xi+PR)y} \tag{10.5.9}$$

$$\phi_{11} = H_0 e^{-my} + H_1 e^{-hy} + H_2 e^{-(\xi+PR)y} + H_3 e^{-(\bar{\xi}+PR)y} + H_4 e^{-(\xi+S_cR)y} + H_5 e^{-(\bar{\xi}+S_cR)y} \quad \text{for } S_c \neq P \qquad (10.5.10)$$

$$\phi_{11} = \eta_0 e^{-hy} + \eta_1 e^{-(\xi+PR)y} + \eta_2 e^{-(\bar{\xi}+PR)y} + \eta_3 y e^{-hy} + \eta_4 y e^{-(\xi+PR)y} + \eta_5 y e^{-(\bar{\xi}+PR)y} \quad \text{for } S_c = P \qquad (10.5.11)$$

$$u_{11} = M_0 e^{-ny} + M_1 e^{-hy} + M_2 e^{-my} + M_3 e^{-(\xi+PR)y} + M_4 e^{-(\bar{\xi}+PR)y} + M_5 e^{-(\xi+S_cR)y} + M_6 e^{-(\bar{\xi}+S_cR)y} + M_7 e^{-(\xi+\lambda R)y} + M_8 e^{-(\bar{\xi}+\lambda R)y} \quad \text{for } S_c \neq P \; (10.5.11)$$

$$u_{11} = B_{10} e^{-ny} + B_{11} e^{-(\xi+\lambda R)y} + B_{22} e^{-(\xi+PR)y} + B_{13} e^{-(\bar{\xi}+\lambda R)y} + B_{23} e^{-(\bar{\xi}+PR)y} + B_{24} e^{-hy} + B_{16} y e^{-(\bar{\xi}+PR)y} + B_{18} y e^{-(\xi+PR)y} + B_{20} y e^{-hy} \quad \text{for } S_c = P \qquad (10.5.12)$$

where, $G_1 = \dfrac{P^2R^2\bar{\xi}}{(\bar{\xi}-\xi)(\xi^2+PR\xi-\pi^2)}$, $G_2 = -\dfrac{P^2R^2\xi}{(\bar{\xi}-\xi)(\bar{\xi}^2+PR\bar{\xi}-\pi^2)}$,

$$h = \frac{PR+\sqrt{P^2R^2+4\pi^2}}{2}, \; H_0 = -\sum_{k=1}^{5} H_k, \; H_1 = \frac{E_1}{h^2 - SRh - \pi^2},$$

$$H_2 = \frac{E_2+B_2}{(\xi+PR)^2 - S_cR(\xi+PR) - \pi^2}, \; H_3 = \frac{E_3+B_4}{(\bar{\xi}+PR)^2 - S_cR(\bar{\xi}+PR) - \pi^2},$$

$$G_0 = -(G_1+G_2), \; H_4 = \frac{B_1}{(\xi+S_cR)^2 - S_cR(\xi+S_cR) - \pi^2},$$

$$H_5 = \frac{B_3}{(\bar{\xi}+S_cR)^2 - S_cR(\bar{\xi}+S_cR) - \pi^2}, \; B_1 = \frac{S_c^2R^2(1-a)\pi\bar{\xi}}{\pi(\bar{\xi}-\xi)}, \; B_2 = \frac{S_cR^2aP\bar{\xi}}{\bar{\xi}-\xi},$$

$$B_3 = \frac{-S_c^2R^2\xi(1-a)}{\bar{\xi}-\xi}, \; B_4 = \frac{-S_caR^2\xi P}{\bar{\xi}-\xi}, \; E_1 = -S_cS_0\left[h^2-\pi^2\right]G_0,$$

$$E_2=-S_cS_0\left[G_1(\xi+PR)^2-\pi^2G_1\right],\ E_3=-S_cS_0\left[\left(\bar{\xi}+PR\right)^2-\pi^2\right]G_2,$$

$$m=\frac{S_cR+\sqrt{S^2{}_cR^2+4\pi^2}}{2},\ n=\frac{R+\sqrt{R^2+4\left(\pi^2+MR^2\right)}}{2},\ M_0=-\sum_{k=1}^{8}M_k,$$

$$M_1=\frac{L_1}{h^2-Rh-\pi^2-MR^2},\ M_2=\frac{L_2}{m^2-Rm-\pi^2-MR^2},$$

$$M_3=\frac{L_3}{(\xi+PR)^2-R(\xi+PR)-\pi^2-MR^2},\ M_4=\frac{L_4}{\left(\bar{\xi}+PR\right)^2-R\left(\bar{\xi}+PR\right)-\pi^2-MR^2},$$

$$M_5=\frac{L_5}{(\xi+S_cR)^2-R(\xi+S_cR)-\pi^2-MR^2},M_6=\frac{L_6}{\left(\bar{\xi}+S_cR\right)^2-R\left(\bar{\xi}+S_cR\right)-\pi^2-MR^2},$$

$$M_7=\frac{K_3}{(\xi+\lambda R)^2-R(\xi+\lambda R)-\pi^2-MR^2},\ M_8=\frac{K_6}{\left(\bar{\xi}+\lambda R\right)^2-R\left(\bar{\xi}+\lambda R\right)-\pi^2-MR^2},$$

$$L_1=-G_rRG_0-G_mRH_1,\ L_2=-G_mRH_0,\ L_3=K_1-G_mRH_2-G_rRG_1,$$

$$L_4=K_4-G_rRG_2-G_mRH_3,\ L_5=K_2-G_mRH_4,\ L_6=K_5-G_mRH_5,$$

$$K_1=\frac{R^2A_1P\bar{\xi}}{\bar{\xi}-\xi},\ K_2=\frac{R^2A_2S_c\bar{\xi}}{\bar{\xi}-\xi},\ K_3=\frac{\bar{\xi}(A_1+A_2)R}{\bar{\xi}-\xi},\ K_4=-\frac{\xi A_1PR^2}{\bar{\xi}-\xi},$$

$$K_5=\frac{-\xi A_2S_cR^2}{\bar{\xi}-\xi},\ K_6=\frac{-R\xi(A_1+A_2)}{\bar{\xi}-\xi},\ \eta_0=-(\eta_1+\eta_2),\ \eta_1=C_2+\eta_4J_1,$$

$$\eta_2=C_3+\eta_5J_2,\ \eta_3=\frac{a_1}{PR-2h},\ \eta_4=\frac{b_3}{\xi(\xi+PR)},\ \eta_5=\frac{b_4}{\bar{\xi}\left(\bar{\xi}+PR\right)},$$

$$C_2=\frac{b_1}{(\xi+PR)^2-PR(\xi+PR)-\pi^2},\ C_3=\frac{b_2}{\left(\bar{\xi}+PR\right)^2-PR\left(\bar{\xi}+PR\right)-\pi^2},$$

$$J_1=\frac{PR+2\xi}{\xi(\xi+PR)},\ J_2=\frac{PR+2\bar{\xi}}{\bar{\xi}\left(\bar{\xi}+PR\right)},\ b_1=a_2-\frac{P^2R^2A_7}{\bar{\xi}-\xi},\ b_2=a_3+\frac{P^2R^2}{\bar{\xi}-\xi}A_9,$$

$$b_3 = \frac{P^2R^2A_8}{\bar{\xi}-\xi},\ b_4 = -\frac{P^2R^2A_{10}}{\bar{\xi}-\xi},\ a_1 = -PS_0\left(h^2-\pi^2\right)G_0,$$

$$a_2 = -PS_0G_1\left[\left(\xi+PR\right)^2-\pi^2\right],\ a_3 = -PS_oG_2\left[\left(\xi+PR\right)^2-\pi^2\right],\ A_7 = \left(PS_0-1\right)\bar{\xi},$$

$$A_8 = p^2S_0R\bar{\xi},,\ A_9 = \left(PS_0-1\right)\xi,\ A_{10} = P^2S_0R\xi,\ A_{11} = \frac{\lambda A_6R^2\bar{\xi}}{\bar{\xi}-\xi},$$

$$A_{12} = \frac{-R\bar{\xi}\left(A_6PR-A_5\right)}{\bar{\xi}-\xi},\ A_{13} = \frac{-R^2\xi\lambda A_6}{\bar{\xi}-\xi},\ A_{14} = \frac{R}{\bar{\xi}-\xi}\left(A_6PR-A_5\right),$$

$$A_{15} = \frac{A_5PR^2\bar{\xi}}{\bar{\xi}-\xi},\ A_{16} = -\frac{A_5PR^2\bar{\xi}}{\bar{\xi}-\xi},\ A_{17} = A_{12}-G_rRG_1-G_mR\eta_1,$$

$$A_{18} = A_{14}-G_rRG_2-G_mR\eta_2,\ \ A_{19} = A_{15}-\eta_5G_mR,\ A_{20} = A_{16}-G_mR\eta_4,$$

$$A_{21} = G_rRG_0-G_mR\eta_0,A_{22} = -G_mR\eta_3,$$

$$B_{11} = \frac{A_{11}}{\left(\xi+\lambda R\right)^2-R\left(\xi+\lambda R\right)-\pi^2-MR^2},B_{12} = \frac{A_{17}}{\left(\xi+PR\right)^2-R\left(\xi+PR\right)-\pi^2-MR^2},$$

$$B_{13} = \frac{A_{13}}{\left(\bar{\xi}+\lambda R\right)^2-R\left(\bar{\xi}+\lambda R\right)-\pi^2-MR^2},\ B_{14} = \frac{A_{18}}{\left(\bar{\xi}+PR\right)^2-R\left(\bar{\xi}+PR\right)-\pi^2-MR^2},$$

$$B_{15} = \frac{A_{21}}{h^2-Rh-\pi^2-MR^2},\ B_{16} = \frac{A_{19}}{R_1},\ B_{17} = \frac{-A_{19}\left(R-2\bar{\xi}-2PR\right)}{R_1^{\ 2}},$$

$$R_1 = \left(\bar{\xi}+PR\right)^2-R\left(\bar{\xi}+PR\right)-\pi^2-MR^2,\ B_{18} = \frac{A_{20}}{R_2},\ B_{19} = \frac{-A_{20}\left(R-2\xi-2PR\right)}{R_2^{\ 2}},$$

$$R_2 = \left(\xi+PR\right)^2-R\left(\xi+PR\right)-\pi^2-MR^2,\ B_{20} = \frac{A_{22}}{R_3},\ B_{21} = \frac{-RA_{22}}{R_3^{\ 2}},$$

$$R_3 = h^2-Rh-\pi^2-MR^2,\ B_{22} = B_{12}+B_{19},\ B_{23} = B_{14}+B_{17},\ B_{24} = B_{25}+B_{21},$$

$$B_{10} = -\left(B_{11}+B_{22}+B_{13}+B_{23}+B_{24}+B_{20}\right).$$

10.6: SKIN FRICTION AT THE PLATE

The non-dimensional skin-friction at the plate in the direction of the free stream is given by

$$\tau = -\frac{\mu \frac{\partial \bar{u}}{\partial \bar{y}}\Big]_{\bar{y}=0}}{\rho V_0^2} = -\frac{1}{R}\frac{\partial u}{\partial y}\Big]_{y=0} = -\frac{1}{R}\left[u_0'(0) + \varepsilon u_{11}'(0)\text{Cos}\pi z\right]$$

$$= \tau_0 + \varepsilon Q_1 \cos \pi z \qquad (10.6.1)$$

where, $\tau_0 = -\frac{1}{R}u_0'(0) = PA_1 + SA_2 - \lambda(A_1 + A_2)$ for $P \neq S_c$ (10.6.2)

$$= \frac{1}{R}\{A_5 + A_6 R(\lambda - P)\} \quad \text{for } P = S_c \qquad (10.6.3)$$

and $Q_1 = -\frac{1}{R}u_{11}'(0) = \frac{1}{R}\left[nM_0 + hM_1 + mM_2 + (\xi + PR)M_3\right.$

$$+ (\bar{\xi} + PR)M_4 + (\xi + S_c R)M_5 + (\bar{\xi} + S_c R)M_6 + (\xi + \lambda R)M_7$$

$$\left. + (\bar{\xi} + \lambda R)M_8\right] \quad \text{for } P \neq S_c \qquad (10.6.4)$$

$$= \frac{1}{R}[\eta B_{10} + (\xi + \lambda R)B_{11} + (\xi + PR)B_{22} + (\bar{\xi} + \lambda R)B_{13}$$

$$+ (\bar{\xi} + PR)B_{23} + hB_{24} - B_{16} - B_{18} - B_{20}] \quad \text{for } P = S_c \qquad (10.6.5)$$

10.7: CO-EFFICIENT OF RATE OF HEAT TRANSFER

The heat flux from the plate to the fluid in terms of Nusselt number N_u is given by

$$N_u = -\frac{k}{\rho V_0 C_p(\bar{T}w - \bar{T}\infty)}\left(\frac{\partial \bar{T}}{\partial \bar{y}}\right)_{\bar{y}=0} = -\frac{1}{PR}\frac{\partial \theta}{\partial y}\Big]_{y=0} = Nu_0 + \varepsilon Q_2 \text{Cos}\pi z \qquad (10.7.1)$$

where $Nu_0 = \frac{-1}{PR}\theta_0'(0) = 1$

and $Q_2 = -\frac{\theta_{11}'(0)}{PR} = \frac{1}{PR}\left[hG_0 + (\xi + PR)G_1 + \left(\bar{\xi} + PR\right)G_2\right].$ (10.7 2)

10.8: CO-EFFICIENT OF MASS TRANSFER

The mass flux at the wall $y = 0$ in terms of Sherwood number Sh is given by

$$Sh = \frac{-D_m}{V_0\left(\bar{C}_w - \bar{C}_\infty\right)}\left(\frac{\partial \bar{C}}{\partial \bar{y}}\right)_{\bar{y}=0} = \frac{-1}{S_c R}\frac{\partial \phi}{\partial y}\bigg]_{y=0} = \frac{1}{S_c R}\left[\phi_0'(0) + \varepsilon\phi_{11}'(0)\mathrm{Cos}\pi z\right]$$

$$= Sh_0 + \varepsilon Q_3 \cos \pi z \qquad (10.8.1)$$

where, $Sh_0 = \frac{1}{S_c R}\phi_0'(0) = 1 + \frac{PS_0(1 - S_c)}{S_c - P}$ for $S_c \neq P$ (10.8.2)

$= 1 - PS_0$ for $S_c = P$ (10.8.3)

$$Q_3 = \frac{1}{S_c R}[mH_0 + hH_1 + H_2(\xi + PR) + H_3\left(\bar{\xi} + PR\right)$$

$$+ (\xi + S_c R)H_4 + \left(\bar{\xi} + S_c R\right)H_5\Big] \quad \text{for } S_c \neq P \qquad (10.8\ 4)$$

$$= \frac{1}{PR}[\eta_0 h + \eta_1(\xi + PR) + \eta_2\left(\bar{\xi} + PR\right)$$

$$-\eta_3 - \eta_4 - \eta_5\Big] \quad \text{for } S_c = P \qquad (10.8.5)$$

10.9: CURRENT DENSITY

The current density $\vec{J}$ is given by

$\vec{J} = \sigma\vec{q} \times \vec{B}$

$$= \sigma B_0\left(-\hat{i}\,\overline{w}+\hat{k}\,\overline{u}\right) \tag{10.9.1}$$

The magnitude of $\vec{J}$ is given by

$$|\vec{J}| = \sigma B_0\sqrt{\overline{w}^2+\overline{u}^2}$$

$$= \sigma B_0 V_0\sqrt{u^2+w^2} \tag{10.9.2}$$

The current density (in magnitude) in non-dimensional form is given by

$$J_c = \frac{|\vec{J}|}{\sigma B_0 V_0} = \sqrt{u^2+w^2} = u\sqrt{1+\left(\frac{w}{u}\right)^2} = u \quad \left(\text{since } \frac{w}{u} << 1\right)$$

That is the magnitude of the non-dimensional current density is proportional to the boundary layer velocity.

10.10: RESULTS AND DISCUSSION

In order to get physical insight into the problem, we have carried out the numerical calculations for Q_1, Q_2 and Q_3 which are respectively the amplitudes of the perturbed parts of the skin friction, Nusselt number and Sherwood number at the plate $y=0$ and their values are demonstrated in graphs. We have restricted our investigation to **P** (Prandtl number) equal to 0.7 and 7, which correspond to air and water vapour respectively. The value of the Grashof number G_m for heat transfer has been chosen as 10 (externally cooled plate) whereas the value of Grashof number G_m for mass transfer is considered to be 15 and the free stream velocity is selected to be 1. Since water vapour is used as a diffusing chemical interest in air, the value of S_c is taken to 0.60 (water vapour). The value of S_0, M and **R** are chosen arbitrary.

Figures 10.1 to 10.4 depict the variation of Q_1 versus the Reynolds number **R**. From these figures we observe that the magnetic field effect causes Q_1 to decrease irrespective of the fluid being air $(P=0.7)$ or water$(P=7)$. However the Soret effect results a steady increase in Q_1 for $(P=7)$, but the reverse effect is seen to occur for $(P=0.7)$. There is an indication from these figures that Q_1 asymptotically descends to its zero value as **R** tends to infinity. That is for low viscosity Q_1 is not significantly affected by S_0, **M** for both the fluids.

Figures 10.5 and 10.6 show how the amplitude Q_2 of the perturbed part of the Nusselt number N_u is affected by the magnetic field. It is inferred from figure 10.5 that for $(P=0.7)$ there is a slight growth in Q_2 due to the application of the magnetic field for large **R**, whereas the effect of **M** on Q_2 is almost negligible for all **R** in case of $(P=7)$. Both the figures clearly indicate that an increase in **R** leads to an increase in Q_2.

Figures 10.7 to 10.10 exhibit the change of behaviour of $|Q_3|$ (amplitude of the periodic perturbed part of the Sherwood number) under the influence of **M, R,** S_0. These figures show that $|Q_3|$ is increased due to thermal diffusion effect or magnitude field effect. The same figures also indicate that an increase in Reynolds number results a sharp increase in $|Q_3|$.

The variations of the zeroth order skin friction τ_0 at the plate y=0 against the Hartmann number **M** and Soret number S_0 are presented in figures 10.11, 10.12, 10.13 and 10.14. From the figures we see that magnitude of the drag force at the plate is diminished due to application of the magnetic field whereas the thermal diffusion effect causes it to increase for both the cases $(P=0.7)$ and $(P=7)$. The same figures also indicate that $|\tau_0|$ decreases asymptotically to its zero value as $\mathbf{R}\to\infty$.

10.11: CONCLUSIONS

(i) The thermal diffusion causes Q_1 to increase for water whereas it results a decrease in Q_1 for air.

(ii) The low viscosity and applied magnetic field lead to a steady decrease in Q_1.

(iii) $|Q_2|$ is large for small viscosity.

(iv) Q_2 is not significantly affected by the applied magnetic field.

(v) Soret effect and magnetic field effects increase $|Q_3|$.

(vi) An increase in R results a sharp increase in $|Q_3|$.

(vii) $|\tau_0|$ is reduced to its zero value asymptotically as $\mathbf{R}\to\infty$

(viii) Application of the magnetic field or Soret effect leads to an increase in $|\tau_0|$.

Fig 10.1: The amplitude Q_1 of the first order skin friction for P=0.7 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15, M=1

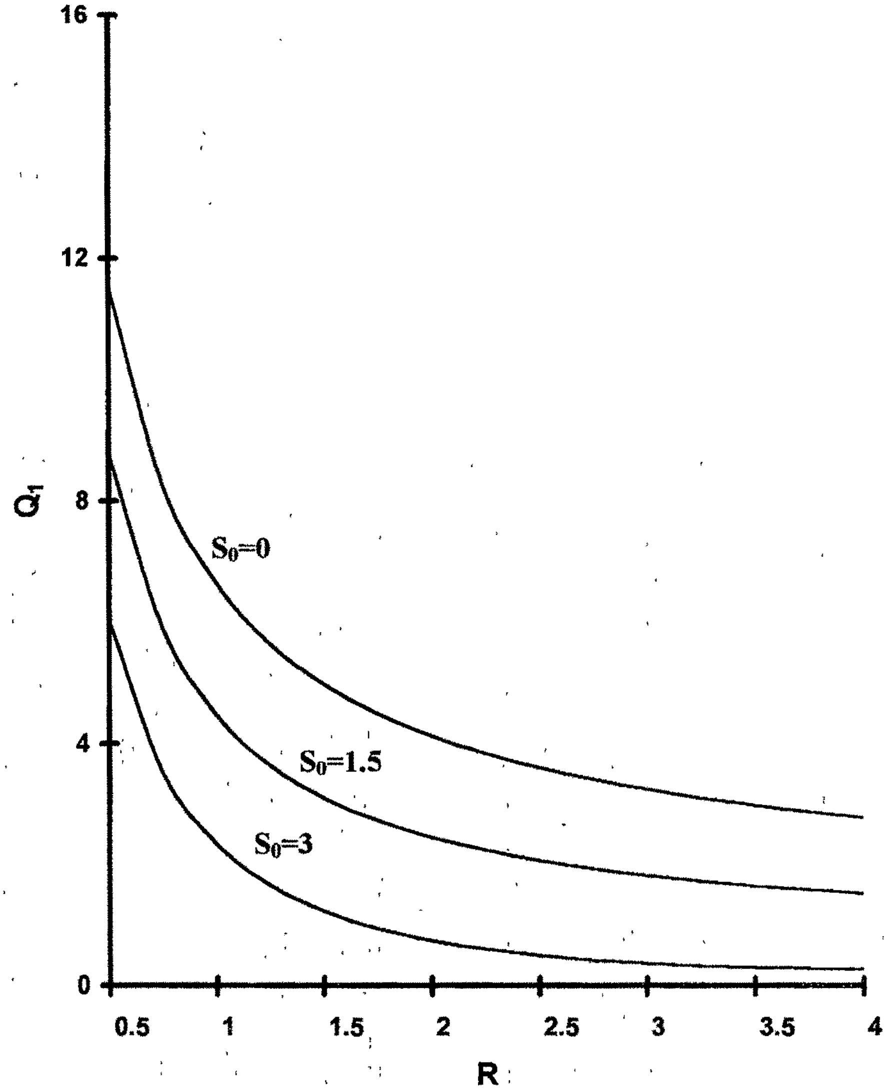

Fig 10.2: The amplitude Q_1 of the first order skin friction for P=7.0 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15, M=1

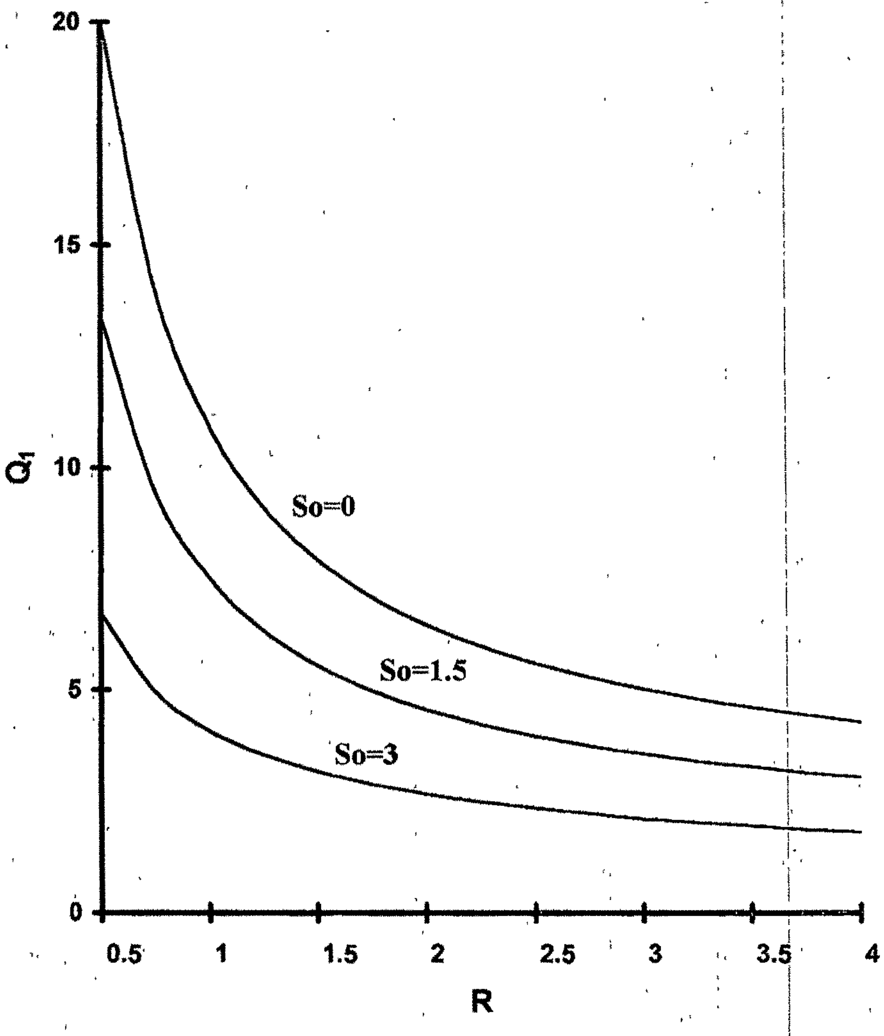

Fig 10.3: The amplitude Q_1 of the first order skin friction for P=0.7 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15, S_0=1

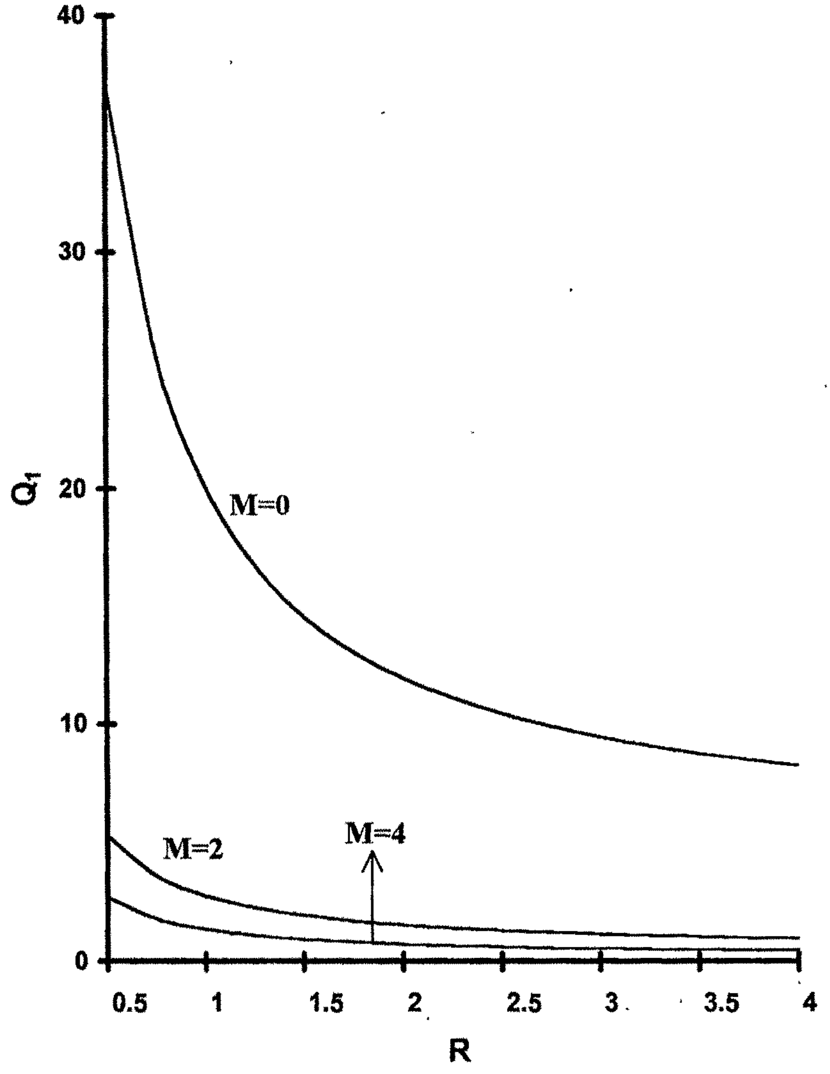

Fig 10.4: The amplitude Q_1 of the first order skin friction for P=7.0 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15, S =1

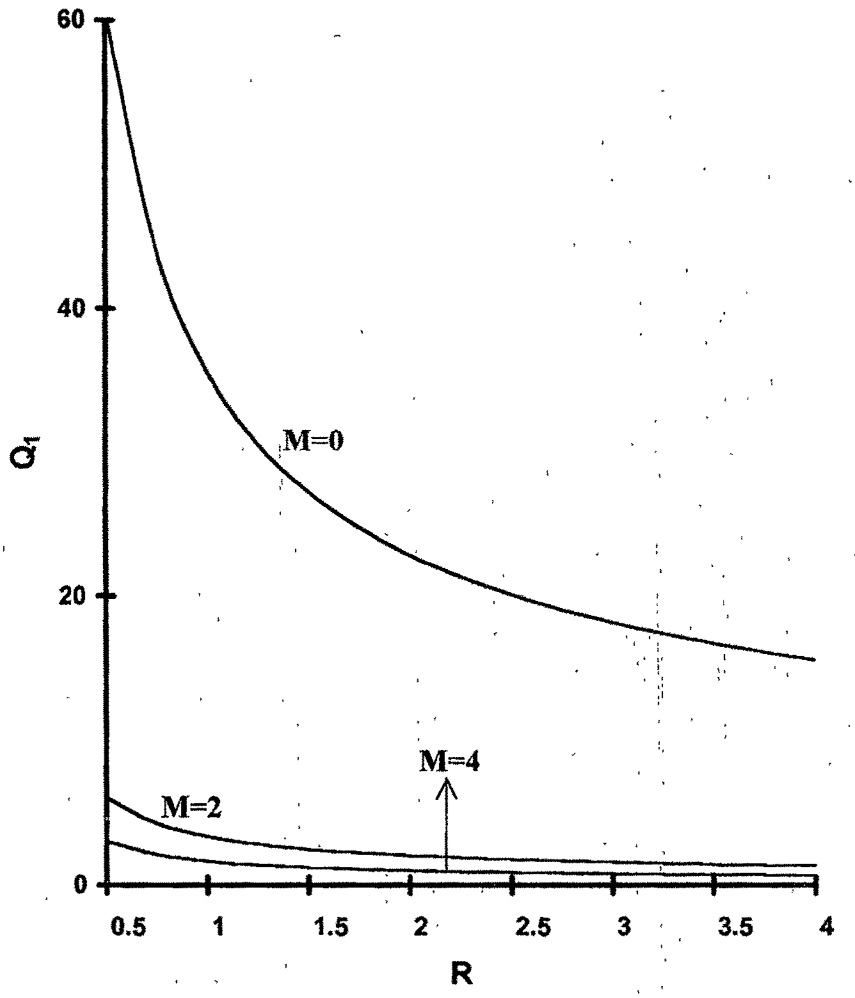

Fig 10.5: The amplitude Q_2 of the first order Nusselt Number for P=0.7 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15 S =1

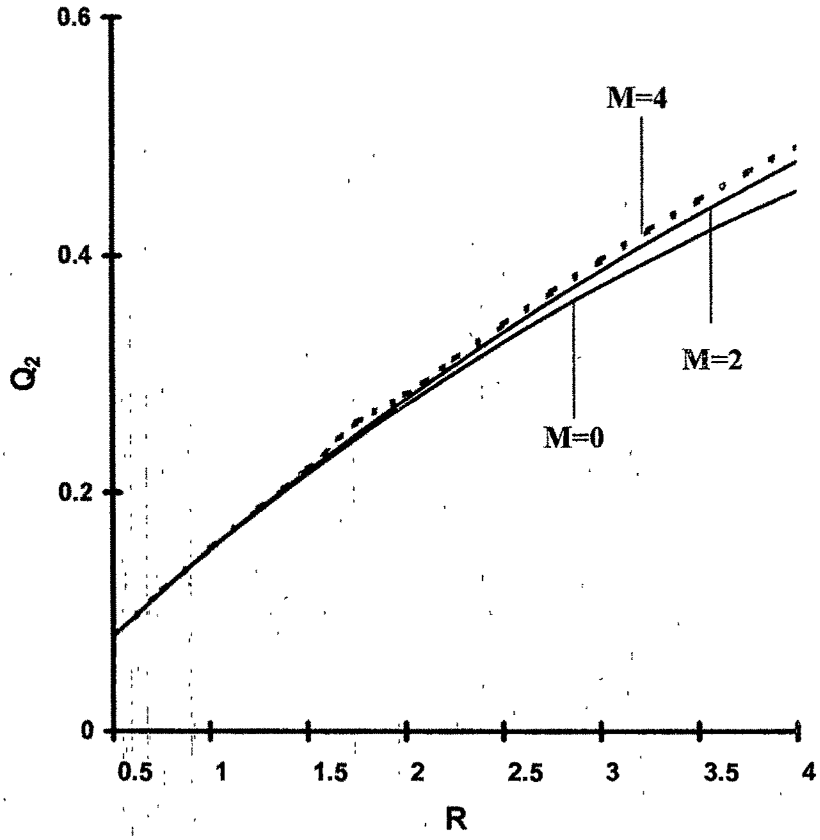

Fig 10.6: The amplitude Q_2 of the first order Nusselt Number for P=7.0 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15 S =1

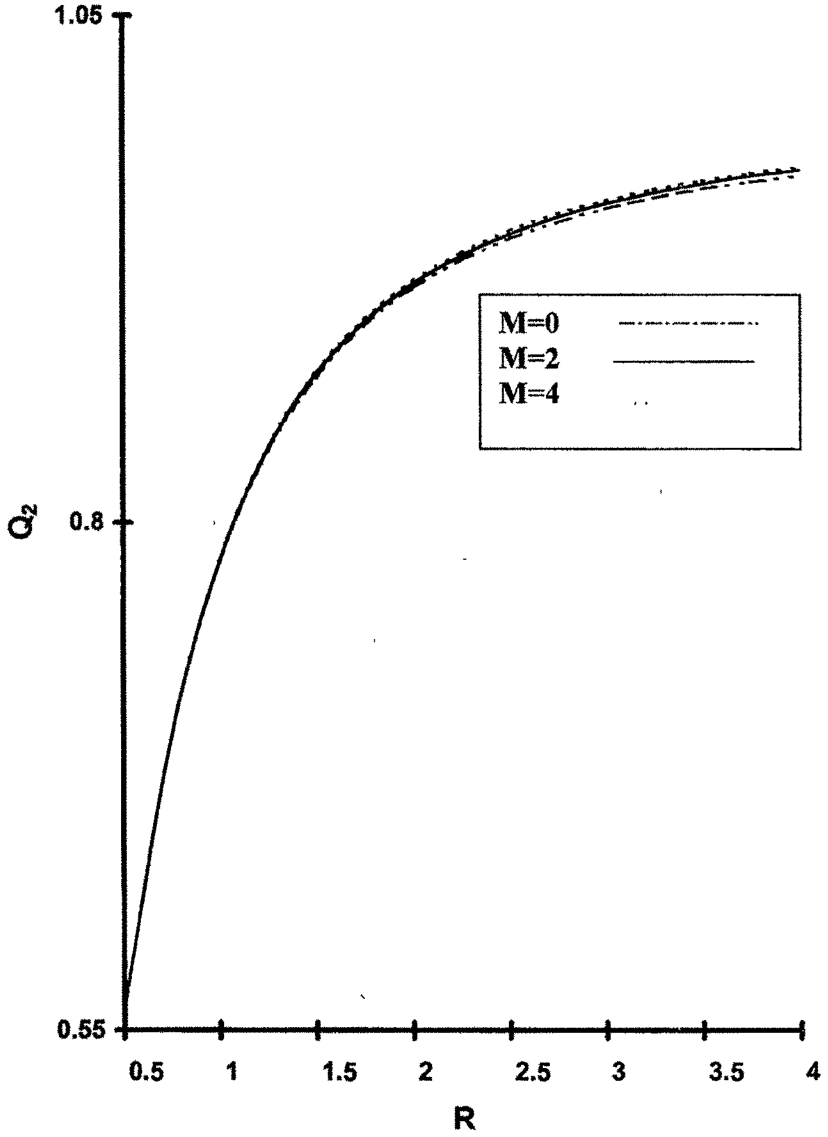

Fig 10.7: The amplitude Q_3 of the first order Sherwood Number for P=0.7 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15 M=1

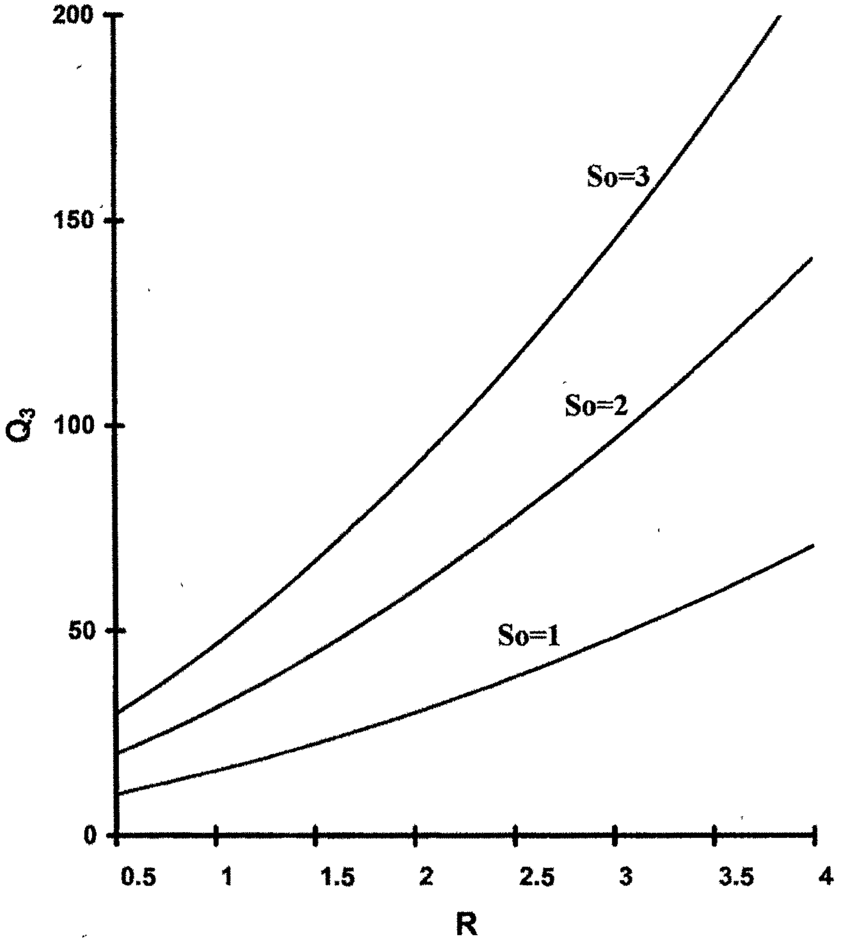

Fig 10.8: The amplitude Q_3 of the first order Sherwood Number for P=7.0 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15 M=1

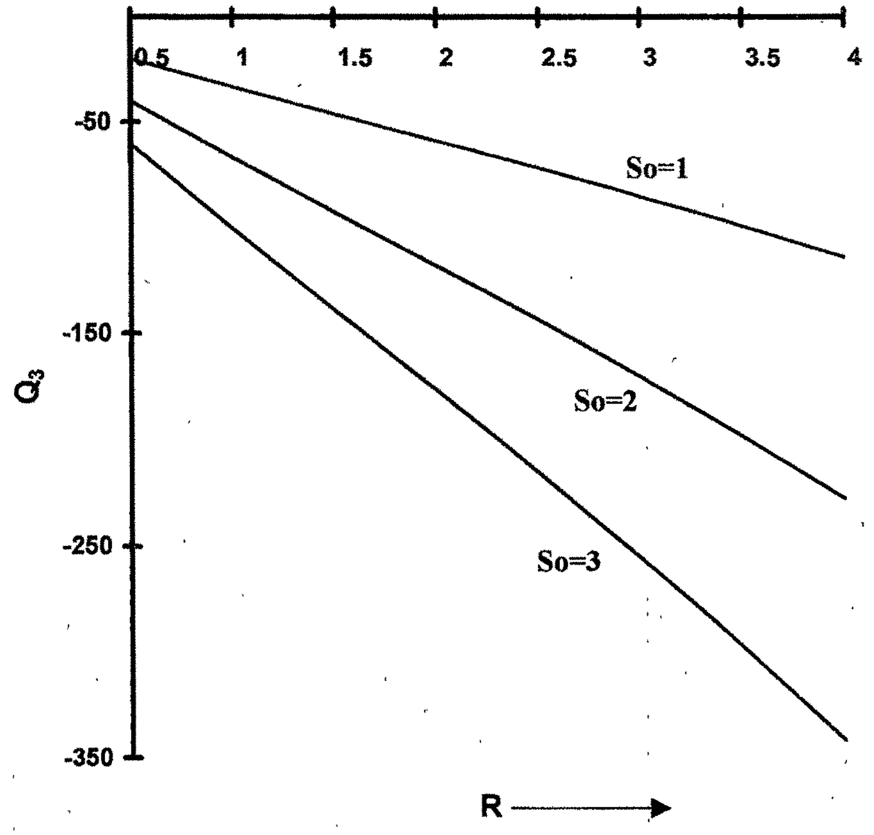

Fig 10.9: The amplitude Q_3 of the first order Sherwood Number for P=0.7 against Reynolds Number R when Sc=0.6, Gr=10, Gm=15 S =1

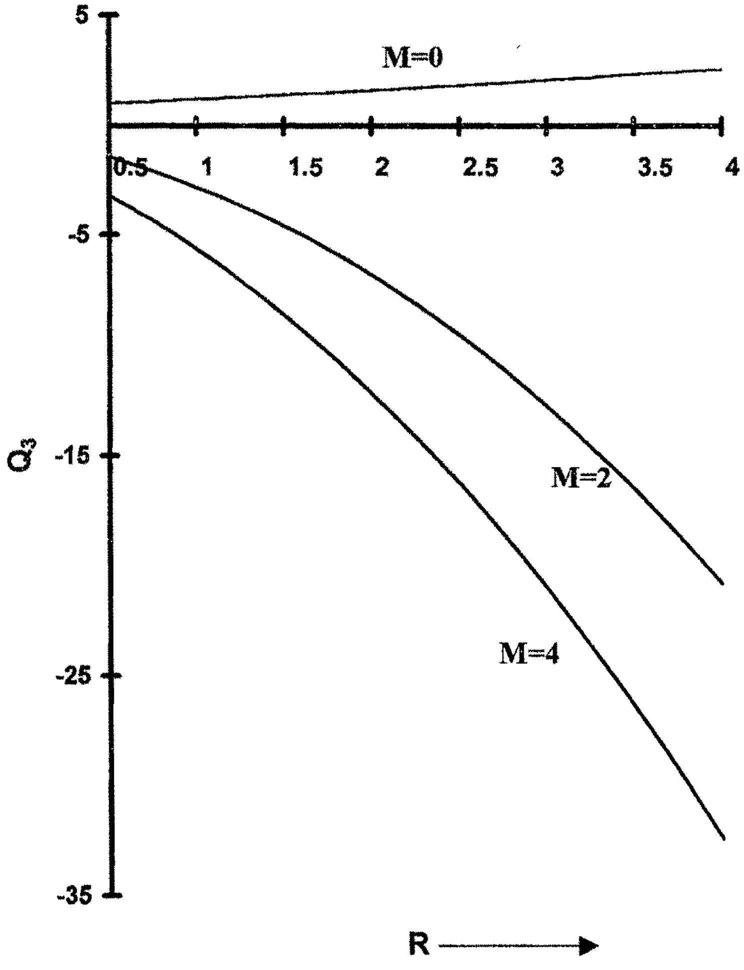

Fig 10.10: The amplitude Q_3 of the first order Sherwood Number for P=7.0 against Reynolds Number R when Sc=0.6, Gr=10 Gm=15 S =1

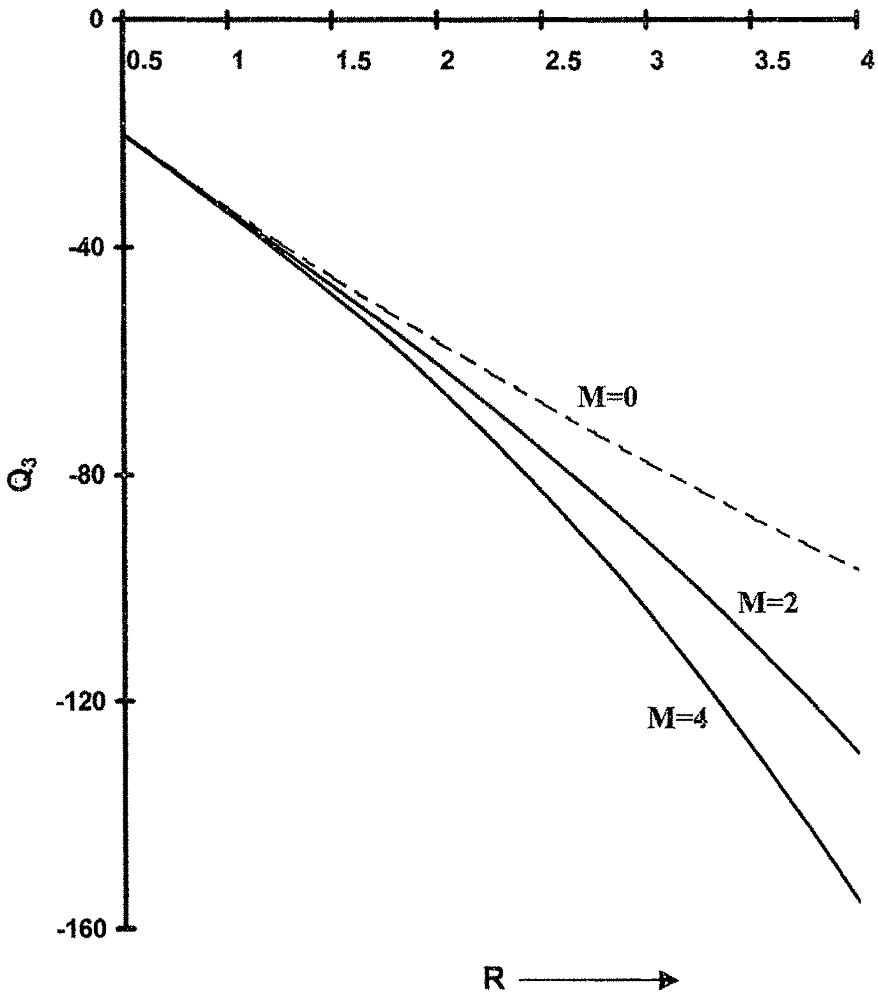

Fig 10.11: The zeroth order skin friction τ_0 at the plate against Reynolds Number R for P=0.7 when Sc=0.6, Gr=10, Gm=15, M=1

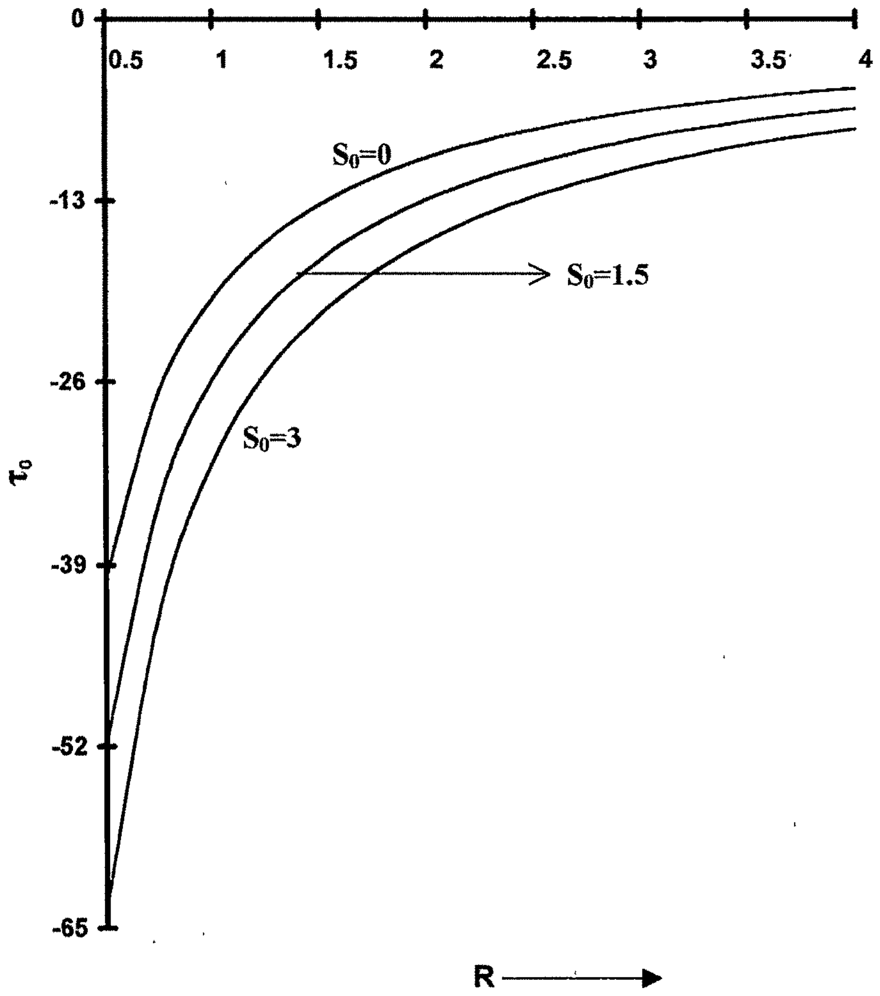

Fig 10.12: The zeroth order skin friction τ_0 at the plate against Reynolds Number R for P=7.0 when Sc=0.6, Gr=10, Gm=15, M=1

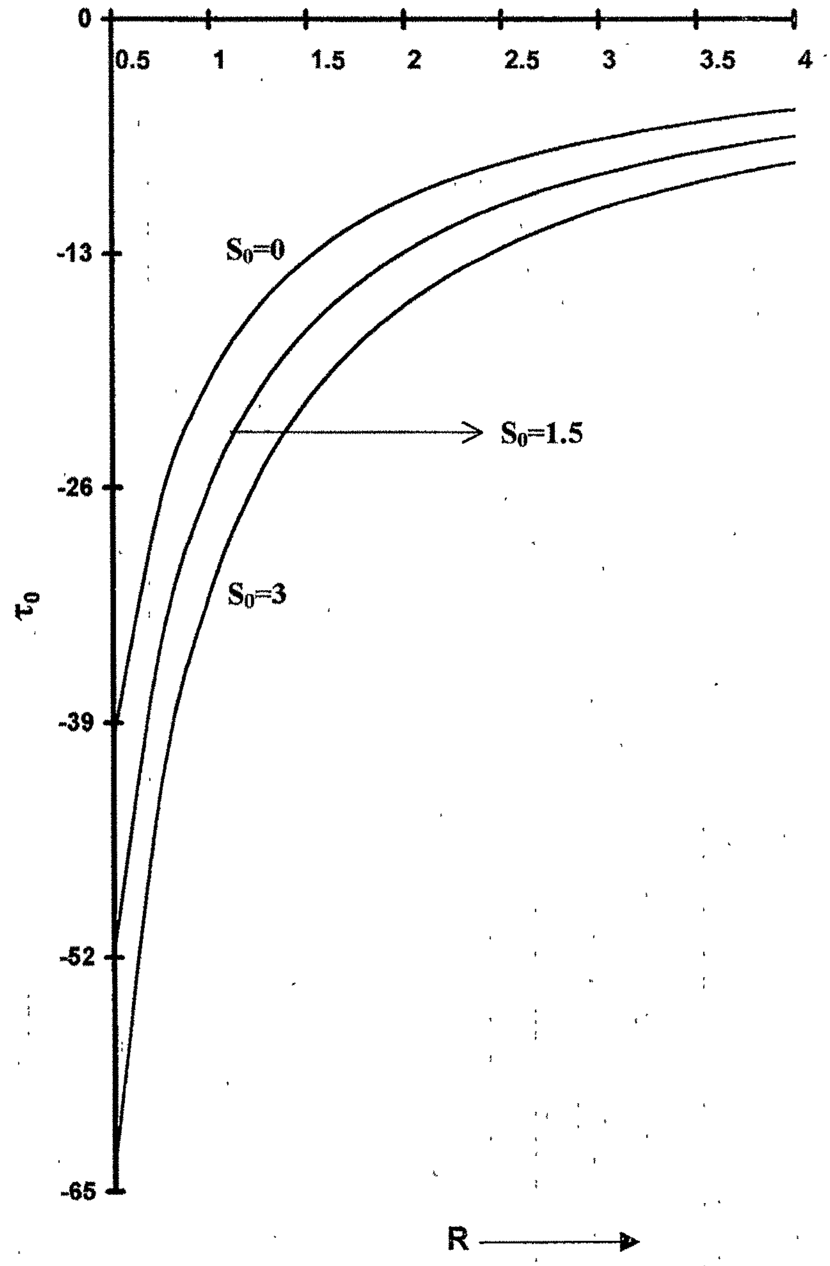

Fig 10.13: The zeroth order skin friction τ_0 at the plate against Reynolds Number R for P=0.7 when Sc=0.6, Gr=10, Gm=15, S =1

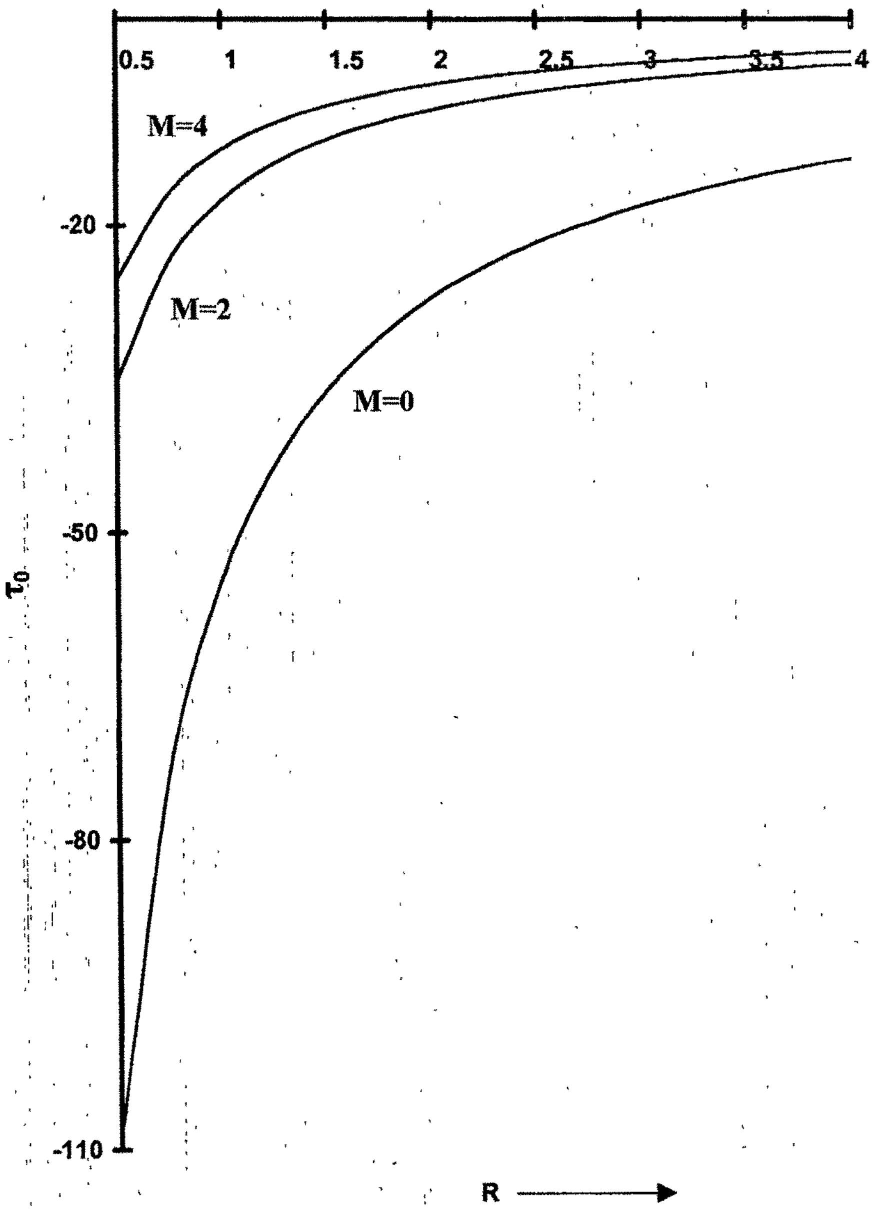

Fig 10.14: The zeroth order skin friction τ_0 at the plate against Reynolds Number R for P=7.0 when Sc=0.6, Gr=10, Gm=15, S =1