CHAPTER IX

LAMINAR BOUNDARY LAYERS IN MHD OSCILLATORY FLOW PAST A POROUS PLATE WITH CONSTANT SUCTION IN ROTATING SYSTEM

9.1:INTRODUCTION

Many natural phenomena and technological problems are susceptible to MHD analysis Geophysics encounters MHD characteristics in the interactions of conducting fluids and magnetic fields. Engineers employ MHD principle, in the design of heat exchangers pumps and flow meters, in space vehicle propulsion, thermal protection, braking, control and re-entry, in creating novel power generating systems etc. The study of MHD is quite important in the field of aerodynamics, since the temperature that occurs in such flight speeds are sufficient to dissociate or even ionize the air appreciably and the motion of this ionized air may be controlled by applying a magnetic field suitably. The study of MHD is also relevant in medical science. For instance there have been researches on Arteriosclerosis (the cause of a cardiac arrest) where the effect of externally applied transverse magnetic field on a pulsalite flow in constricted arteries (tubes) is considered. When an electrically conducting fluid flows past a flat plate, its motion can be retarded by applying a transverse magnetic field as in such a case the Lorentz force that comes due to the interaction of the magnetic field and the fluid velocity acts as a resistance force in the direction opposite to the direction of the fluid velocity. Due to this the skin friction at the plate is reduced and hence the transition from laminar to the turbulent flow may be prevented. In other words the boundary flow may be controlled by transverse magnetic field.

The geophysical importance of the flows in rotating frame of reference has attracted the attention of a number of scholars. Many have done model studies in this literature. Some of them are Vidyanidhy and Nigam (1967), Jana and Datta (1977). The effects of uniform transverse magnetic field with or without suction was investigated by Gupta (1972), Soundalgekar and Pop (1973) and Mazumdar *et al.* (1976). The similarity solutions of the unsteady Navier-Stokes equations in a rotating frame of reference has been obtained by Gupta (2000). Singh *et al.* (1994) studied the unsteady MHD couette flow of electrically conducting fluid in a rotating system. Recently Singh *et al.* (2005) has studied a periodic solution of oscillatory couette flow in rotating system through porous medium. The object of the present work is to investigate the effects of the transverse magnetic field and rotation parameter in an oscillatory flow past a horizontal porous plate with constant suction because of the importance of such problems in industry as well as in aerodynamics.

9.2:MATHEMATICAL FORMULATION

Consider an unsteady flow of a viscous and incompressible fluid past a horizontal porous plate with constant suction $-w_0$ (say). Choose the origin on the plate and the X-axis parallel to the direction of the flow. The Z-axis taken perpendicular to the plate is the axis of rotation about which the entire system is rotating with angular velocity $\vec{\Omega}=\left(0,0,\overline{\Omega}\right)$. A uniform magnetic field is applied in the transverse direction of flow. Since the plate is infinite in extent, all physical quantities, except the pressure p, depend $\overline{z}$ and $\overline{t}$ only. Let $\left(\overline{u},\overline{v},\overline{w}\right)$ be the fluid velocity at a point $\left(\overline{x},\overline{y},\overline{z}\right)$.

The equation of continuity gives $\dfrac{\partial\overline{w}}{\partial\overline{z}}=0$,

Which holds $\overline{w}=-w_0$ = a constant = suction velocity (9.2.1)

With the foregoing assumptions and under the usual boundary layer approximations, the equations governing the flow and heat transfer are

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \upsilon \frac{\partial^2 \bar{u}}{\partial \bar{z}^2} + \frac{\partial \bar{U}}{\partial \bar{t}} + \overline{\Omega} \bar{v} + w_0 \frac{\partial \bar{u}}{\partial \bar{z}} + \frac{\sigma B_0^2}{\rho}\left(\bar{U} - \bar{u}\right) \tag{9.2.2}$$

$$\frac{\partial \bar{v}}{\partial \bar{t}} = \upsilon \frac{\partial^2 \bar{v}}{\partial \bar{z}^2} + \overline{\Omega}\left(\bar{U} - \bar{u}\right) + w_0 \frac{\partial \bar{v}}{\partial \bar{z}} - \frac{\sigma B_0^2 \bar{v}}{\rho} \tag{9.2.3}$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} = \alpha \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} + w_0 \frac{\partial \bar{T}}{\partial \bar{z}} + \frac{\upsilon}{c_p}\left\{\left(\frac{\partial \bar{u}}{\partial \bar{z}}\right)^2 + \left(\frac{\partial \bar{v}}{\partial \bar{z}}\right)^2\right\} + \frac{\sigma B_0^2}{\rho c_p}\left\{\left(\bar{U} - \bar{u}\right)^2 + \left(\bar{v}\right)^2\right\} \tag{9.2.4}$$

where, υ is the kinematic viscosity, ρ is the density, σ is the electrical conductivity, α is the thermal diffusivity, c_p is the specific heat at constant pressure, B_0 is the applied magnetic field, $\bar{U}$ is the free stream velocity, $\bar{T}$ is the temperature and the other symbols have their usual meanings

The relevant boundary conditions are

$$\text{at } \bar{z} = 0 \quad : \quad \bar{u} = 0, \bar{v} = 0, \bar{T} = \bar{T}_w + \varepsilon\left(\bar{T}_w - \bar{T}_\infty\right)e^{i\bar{\omega}\bar{t}} \tag{9.2.5}$$

$$\text{at } \bar{z} \to \infty \quad : \quad \bar{u} = \bar{U} = U_0\left(1 + \varepsilon e^{i\bar{\omega}\bar{t}}\right), \bar{v} = 0, \bar{T} = \bar{T}_\infty \tag{9.2.6}$$

We introduce the following non-dimensional quantities

$$u = \frac{\bar{u}}{U_0},\ v = \frac{\bar{v}}{U_0},\ U = \frac{\bar{U}}{U_0},\ t = \frac{\bar{t}\omega_0^2}{\upsilon},\ \omega = \frac{\upsilon\bar{\omega}}{\omega_0^2},\ \Omega = \frac{\overline{\Omega}\upsilon}{w_0^2},\ z = \frac{\bar{z}w_0}{\upsilon},\ M = \frac{\sigma B_0^2 \nu}{\rho w_0^2},$$

$$P = \frac{\upsilon}{\alpha},\ T = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty},\ E = \frac{U_0^2}{c_p\left(\bar{T}_w - \bar{T}_\infty\right)}.$$

Where, ω is the frequency of oscillation, M is the Hartmann number, P is the Prandtl number, E is the Eckert number and U_0 is mean free stream velocity $\bar{T}_w$ is the mean temperature at the plate and $\bar{T}_\infty$ is the free stream temperature.

The non-dimensional equations and boundary conditions are

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial z^2}+\frac{\partial U}{\partial t}+\Omega v+\frac{\partial u}{\partial z}+M(U-u) \tag{9.2.7}$$

$$\frac{\partial v}{\partial t}=\frac{\partial^2 v}{\partial z^2}+\Omega(U-u)+\frac{\partial v}{\partial z}-Mv \tag{9.2.8}$$

$$P\frac{\partial T}{\partial t}-p\frac{\partial T}{\partial z}=\frac{\partial^2 T}{\partial z^2}+EP\left\{\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right\}+MEP\left\{(U-u)^2+v^2\right\} \tag{9.2.9}$$

subject to the boundary conditions

$$\left.\begin{array}{l} z=0; u=0, v=0, T=1+\varepsilon e^{i\omega t} \\ z\to\infty; u=U=1+\varepsilon e^{i\omega t}, v=0, T=0 \end{array}\right\} \tag{9.2.10}$$

9.3:SOLUTION OF THE PROBLEM

We introduce the complex variable q defined by $q = u + i\,v$ (9.3.1)

where $i^2 = -1$

The non-dimensional forms of the equation governing the flow can be rewritten as follows:

$$\frac{\partial q}{\partial t}=\frac{\partial^2 q}{\partial z^2}+\frac{\partial U}{\partial t}+(M+i\Omega)(U-q)+\frac{\partial q}{\partial z} \tag{9.3.2}$$

$$P\frac{\partial T}{\partial t}=P\frac{\partial T}{\partial z}+\frac{\partial^2 T}{\partial z^2}+EP\left|\frac{\partial q}{\partial z}\right|^2+MEP(U-q)(U-\bar{q}) \tag{9.3.3}$$

where, $\bar{q}=u-iv$

The boundary conditions are

$$\left.\begin{array}{ll} z=0 & ;q=0, T=1+\varepsilon e^{i\omega t} \\ z\to\infty & ;q=1+\varepsilon e^{i\omega t}=U, T=0 \end{array}\right\} \tag{9.3.4}$$

Assuming the small amplitude oscillation (ε <<1), we represent the velocity q and temperature T as follows

$$q = q_0(z) + \varepsilon e^{i\omega t} q_1(z) + O(\varepsilon^2) \tag{9.3.5}$$

$$T = T_0(z) + \varepsilon e^{i\omega t} T_1(z) + O(\varepsilon^2) \tag{9.3.6}$$

Substituting (9.3.5) and (9.3.6) in (9.3.2) and (9.3.3) and by equating the harmonic terms and neglecting ε^2 the following differential equations are obtained.

$$q_0'' + q_0' - (M + i\Omega) q_0 = -(M + i\Omega) \tag{9.3.7}$$

$$q_1'' + q_1' - (M + i\Omega + i\omega) q_1 = -(M + i\Omega + i\omega) \tag{9.3.8}$$

$$T_0'' + PT_0' = -EP\left|\frac{dq_0}{dz}\right|^2 - MEP(1 - q_0)(1 - \bar{q}_0) \tag{9.3.9}$$

$$T_1'' + PT_1' - Pi\omega T_1 = -EP\left\{\frac{dq_1}{dz}\bar{q}_0'(z) + \frac{d\bar{q}_1}{dz}q_0'(z)\right\} - 2MEP\left\{(1 - \bar{q}_0)(1 - q_1) + (1 - q_0)(1 - \bar{q}_1)\right\} \tag{9.3.10}$$

subject to the boundary conditions

$$\left.\begin{array}{ll} z = 0 & ; q_0 = 0,\ q_1 = 0,\ T_0 = 1,\ T_1 = 1 \\ z \to \infty & ; q_0 = 1,\ q_1 = 1,\ T_0 = 0,\ T_1 = 0 \end{array}\right\} \tag{9.3.11}$$

Here $\bar{q}_0, \bar{q}_1$ indicate the conjugates of the complex numbers q_0 and q_1 respectively.

The solutions of the equations (9.3.7), (9.3.8), (9.3 9) and (9.3.10) subject to the boundary conditions (9.3.11) are

$$q_0(z) = 1 - e^{-\lambda_1 z} \tag{9.3.12}$$

$$q_1(z) = 1 - e^{-\lambda_2 z} \tag{9.3.13}$$

$$T_0(z) = (1 + EPL_3)e^{-Pz} - EPL_3 e^{-(\lambda_1+\bar{\lambda}_1)z} \tag{9.3.14}$$

$$T_1(z) = (1 + L_8 + L_9)e^{-\lambda_3 z} - L_8 e^{-(\lambda_1+\bar{\lambda}_2)z} - L_9 e^{-(\lambda_2+\bar{\lambda}_1)z} \tag{9.3.15}$$

Where,

$$\lambda_1 = \frac{1+\sqrt{1+4(M+i\Omega)}}{2},\ \lambda_2 = \frac{1+\sqrt{1+4(M+i\Omega+i\omega)}}{2},$$

$$\lambda_3 = \frac{P+\sqrt{P^2+4iP\omega}}{2},\ L_1 = \frac{\lambda_1\bar{\lambda}_1}{(\lambda_1+\bar{\lambda}_1)^2 - P(\lambda_1+\bar{\lambda}_1)},$$

$$L_2 = \frac{1}{(\lambda_1+\bar{\lambda}_1)^2 - P(\lambda_1+\bar{\lambda}_1)},\ L_3 = L_1 + ML_2,$$

$$L_4 = \frac{\lambda_1\bar{\lambda}_2}{(\lambda_1+\bar{\lambda}_2)^2 - P(\lambda_1+\bar{\lambda}_2) - Pi\omega},\ L_5 = \frac{\lambda_2\bar{\lambda}_1}{(\lambda_2+\bar{\lambda}_1)^2 - P(\lambda_2+\bar{\lambda}_1) - Pi\omega},$$

$$L_6 = \frac{1}{(\lambda_2+\bar{\lambda}_1)^2 - P(\lambda_2+\bar{\lambda}_1) - Pi\omega},\ L_7 = \frac{1}{(\lambda_1+\bar{\lambda}_2)^2 - P(\lambda_1+\bar{\lambda}_2) - Pi\omega},$$

$$L_8 = EP(L_4 + ML_7), L_9 = EP(L_5 + ML_6).$$

Here the constants L_1, L_2, L_3 are real and the others are complex, whose real and imaginary parts are shown in the Appendix.

9.4:VELOCITY AND TEMPERATURE FIELD

The non-dimensional velocity field is given by

$$q = q_0(z) + \varepsilon e^{i\omega t} q_1(z) \tag{9.4 1}$$

By splitting it into real and imaginary parts the primary and secondary velocity components are derived as given bellow:

$$u = u_0 + \varepsilon|A|\cos(\omega t + \alpha) \tag{9.4.2}$$

$$v = v_0 + \varepsilon|A|\sin(\omega t + \alpha) \tag{9.4.3}$$

where, $u_0 + iv_0 = q_0$ (9.4.4)

$$|A| = |q_1| \tag{9.4.5}$$

$$\alpha = \arg(q_1) \tag{9.4.6}$$

The temperature in the non-dimensional form is given by

$$T = T_0(z) + \text{Real part of}\left\{\varepsilon e^{i\omega t} T_1(z)\right\}$$

$$= T_0(z) + \varepsilon|B|\cos(\omega t + \beta) \tag{9.4.7}$$

where, $|B| = |T_1(z)|$ (9.4.8)

and $\beta = \arg T_1(z)$ (9.4.9)

9.5: COEFFICIENT OF SKIN-FRICTION

The skin frictions at the plate in the direction of primary and secondary velocities are respectively given by

$$\tau_x = \left.\frac{du}{dz}\right]_{z=0} = \tau_{x0} + \varepsilon|G|\cos(\omega t + \gamma) \tag{9.5.1}$$

$$\tau_y = \left.\frac{dv}{dz}\right]_{z=0} = \tau_{y0} + \varepsilon|G|\sin(\omega t + \gamma) \tag{9.5.2}$$

where, $|G| = |q_1'(0)|$ (9.5 3)

$$\gamma = \arg\left\{q_1'(0)\right\} \tag{9.5.4}$$

$$\tau_{x0} = u_0'(0) \tag{9.5.5}$$

$$\tau_{y0} = v_0'(0) \tag{9.5.6}$$

9.6:RATE OF HEAT TRANSFER

The non-dimensional rate of heat transfer in terms of Nusselt number from the plate to the fluid is given by

$$N_u = -\text{Real part of}\left(\frac{\partial T}{\partial z}\right)_{z=0}$$

$$= -\text{Real part of } \left\{T_0'(0) + \varepsilon e^{i\omega t} T_1'(0)\right\}$$

$$= -T_0'(0) - \varepsilon \text{Real part of } \left\{e^{i\omega t} T_1'(0)\right\}$$

$$= N_{u_0} + \varepsilon |H| \cos(\omega t + \delta)$$

where, $|H| = \left|T_1'(0)\right|$ (9.6.1)

$$\delta = \text{argument of } T_1'(0) \tag{9.6.2}$$

9.7:RESULTS AND DISCUSSION

In order to study the effects of magnetic field and the rotation parameter Ω on the flow and heat transfer characteristics, we have plotted the values of |G| (skin friction amplitude), |H| (Heat transfer amplitude), tan γ (skin friction phase), tan δ (Heat transfer phase), τ_x (skin friction at the plate due to primary velocity),

τ_y (skin friction at the plate due to secondary velocity) against M (Hartmann Number) for different values of Ω. Throughout our investigation the Prandtl number P is taken to be equal to 0 .7, which corresponds to the air at 298^0 K and 1 atmospheric pressure and the Eckert number E is assumed to be 0.05, the frequency of oscillation ω is assumed to be 1 and the value of ωt is taken as $\pi/2$ The values of the Hartmann number M and rotation parameter Ω are chosen arbitrary.

The variation of the amplitude |G| of skin friction versus Hartmann number M and rotation parameter Ω are shown in figure 9.1. Here we observe that |G| increases due to the application of the transverse magnetic field as well as due to rotation of the system. This figure also indicates that, the effect of M on |G| is negligible for large values of Ω. Figure 9.2 exhibits the variation of phase tan γ of skin friction against M and Ω. It is clear from this figure that there is a steady growth in $\tan\gamma$ when the angular velocity of the system is increased, but magnetic field effect causes tan γ to decrease slowly. In this figure also, we notice that the effect of M on $\tan\gamma$ is negligible for large Ω.

Figures 9.3 and 9.4 demonstrate the change of behaviour of skin frictions τ_x and τ_y due to primary and secondary velocity respectively versus Hartmann M under the rotational effect. From these two figures we observe that τ_x and τ_y both increase due to rotation of the system. It is further noticed from these figures that τ_x is increased whereas τ_y is decreased due increase in the Hartmann number. That is say that the rotation of the system causes the drag on the plate along the primary and secondary direction to increase and on the other hand the drag on the plate is found to be increased along the primary velocity direction and decreased along the secondary velocity direction under the influence of the transverse magnetic field.

Figure 9.5 and 9.6 depict the variation of heat transfer amplitude |H| and heat transfer phase $\tan\delta$ against M and Ω respectively. It is inferred from these two figures that |H| falls sharply under the rotational effect and slowely under the magnetic field effect, but $\tan\delta$ increases due to increase of Ω and M.

Finally figure 9.7 exhibits the behaviour of Nusselt number Nu against M and Ω. From this figure it is seen that Nu decreases when the angular velocity Ω and the intensity of the magnetic field are increased.

9.8:CONCLUSIONS

Our investigation leads to the following conclusions

1 The rotation of the system causes the drag on the plate along the primary and secondary direction to increase.

2. The drag on the plate is found to be increased along the along the primary velocity direction and decreased along the secondary velocity direction under the influence of the transverse magnetic field.

3. |G|, the amplitude of the fluctuating part of the skin friction at the plate increases due to the application of the transverse magnetic as well as due to rotation of the system.

4. |H|, the amplitude of the fluctuating part of the rate of heat transfer at the plate increases falls under the rotational and magnetic field effects.

5. Nu, the heat transfer coefficient at the plate decreases when the angular velocity Ω and the intensity of the magnetic field are increased.

Figure 9.1: The skin friction amplitude |G| versus Hartmann number M at the plate y=0

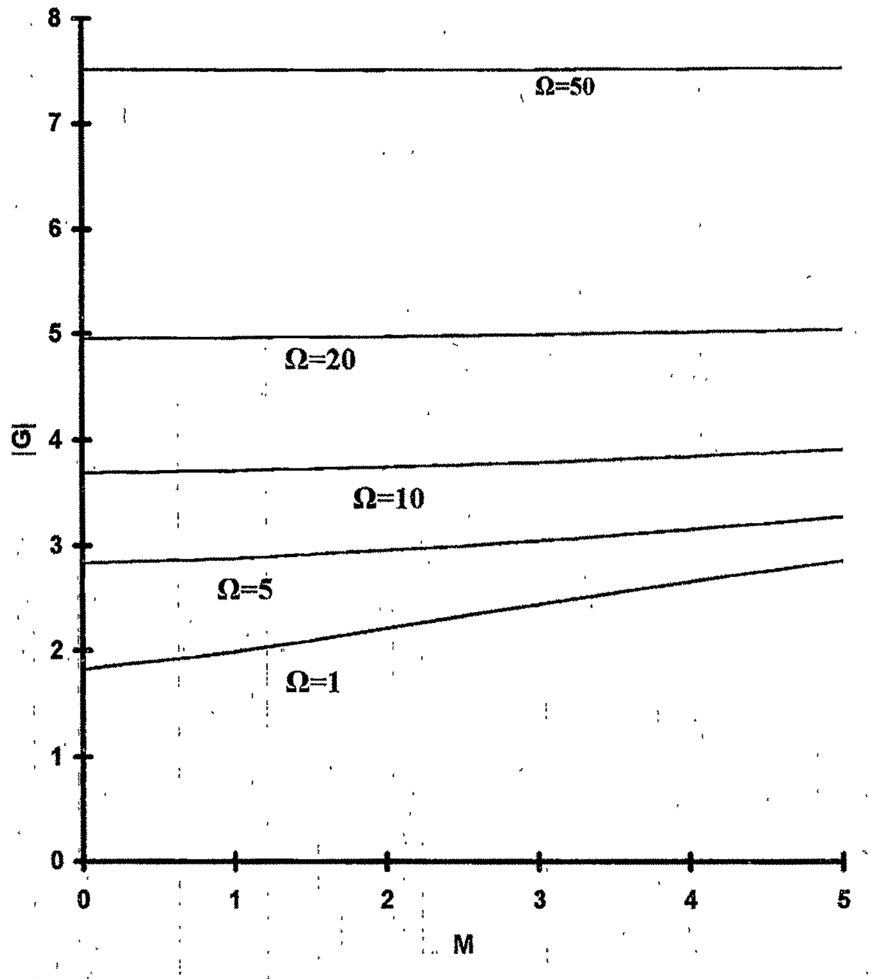


Figure 9.2: The skin friction phase tan γ versus Hartmann number M at the plate y=0

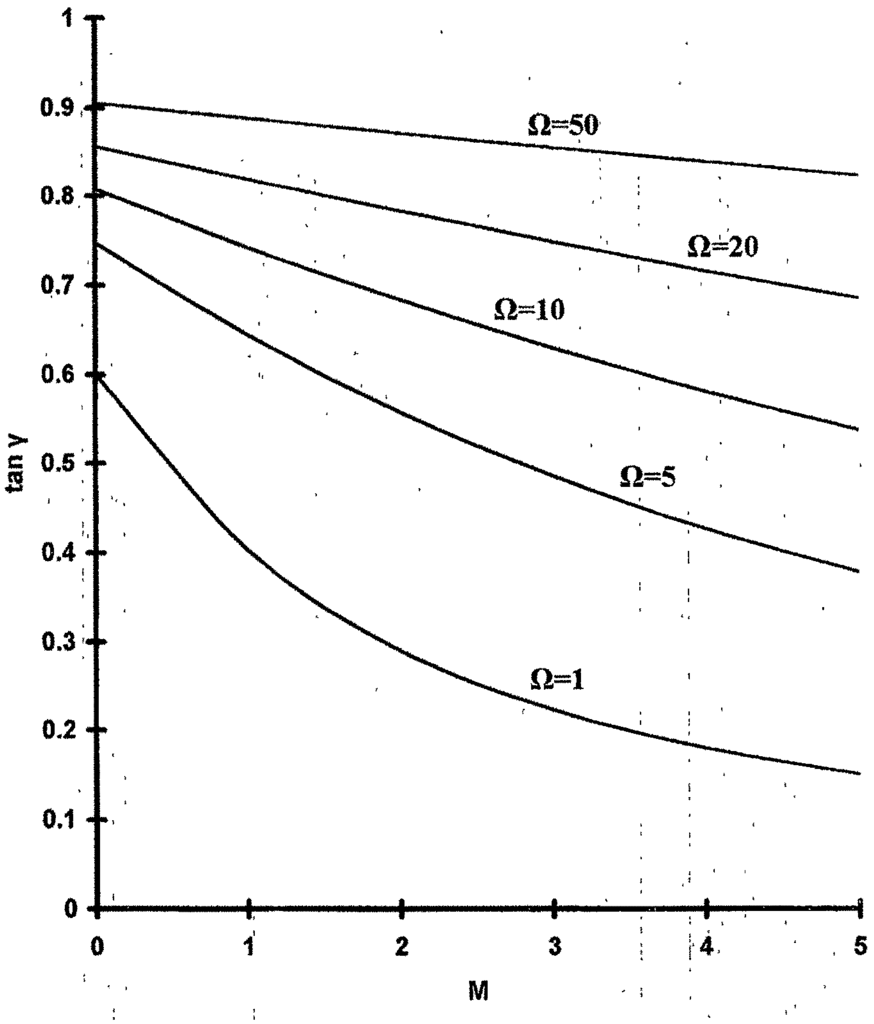


Figure 9.3: The skin friction τ_x due to primary velocity versus Hartmann number M at the plate y=0

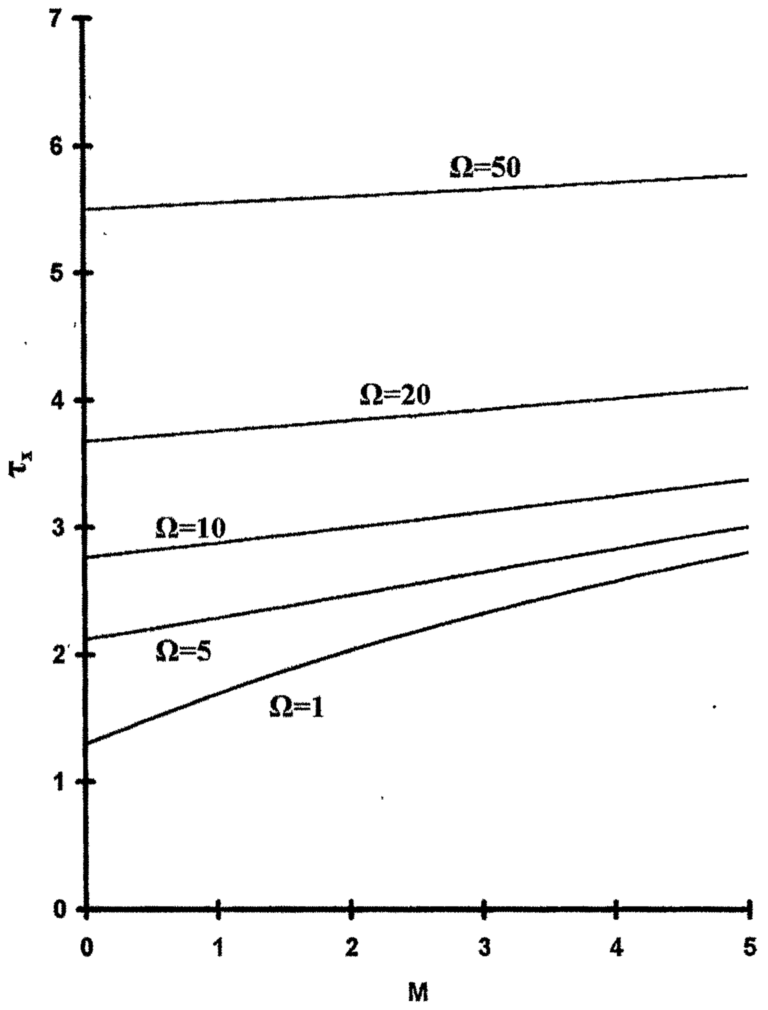


Figure 9.4: The skin friction τ_y due to secondary velocity versus Hartmann number M at the plate y=0

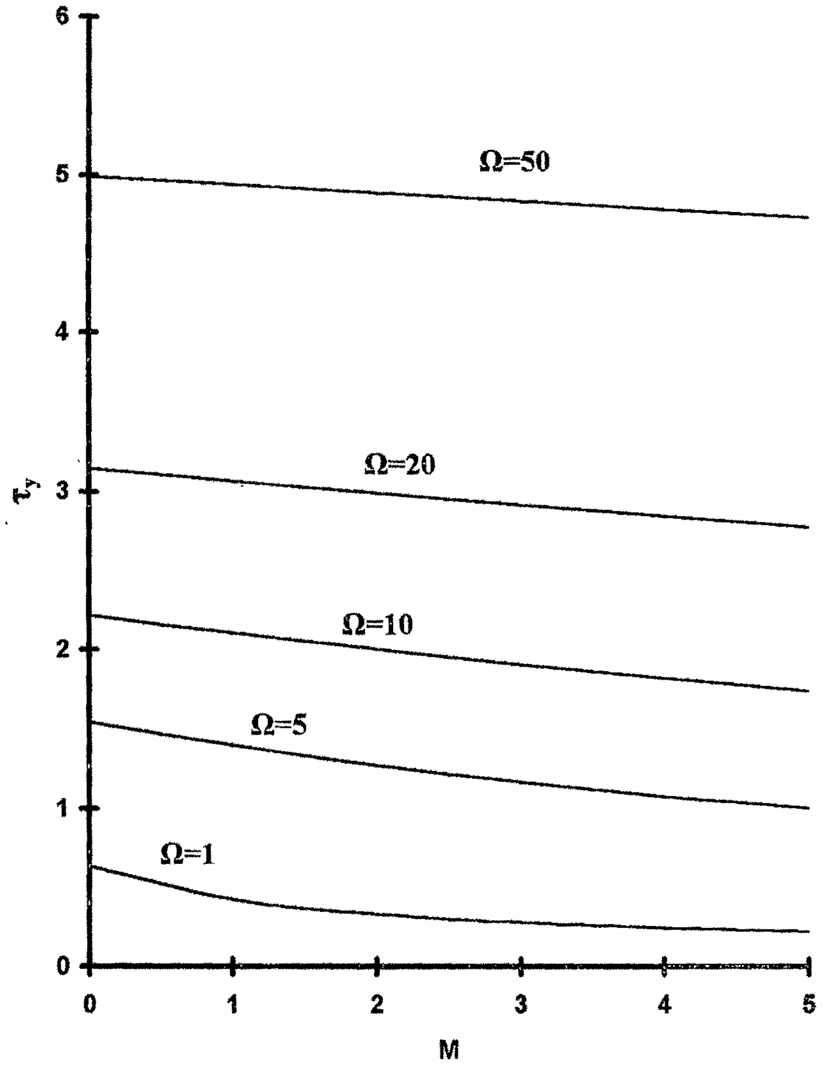


Figure 9.5: The Heat transfer amplitude |H| against Hartmann number M at the plate y=0

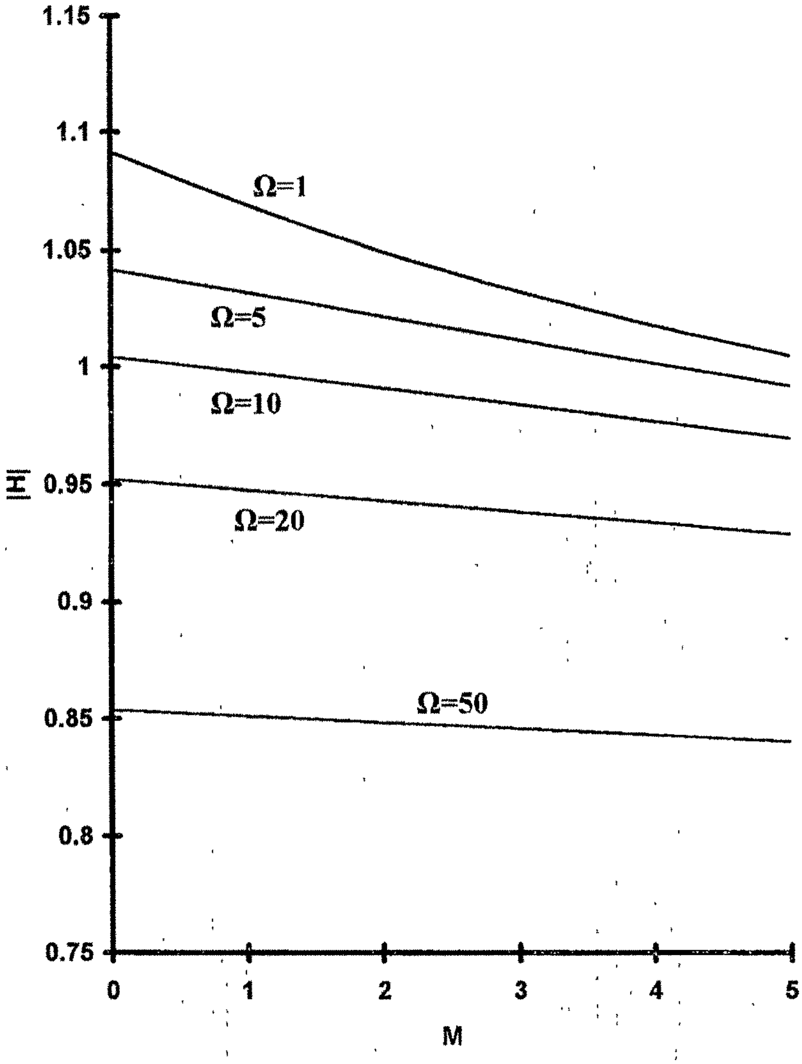


Figure 9.6: The Heat transfer phase tanδ against Hartmann number M at the late =0

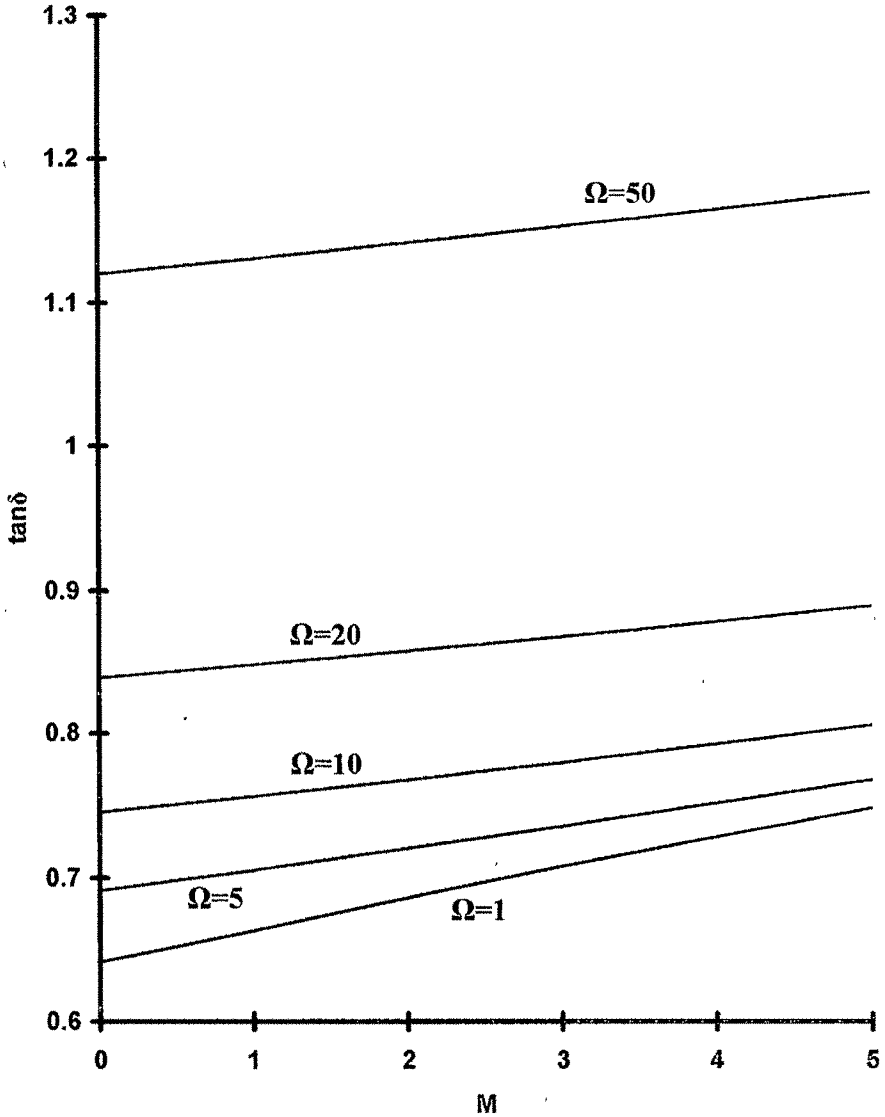
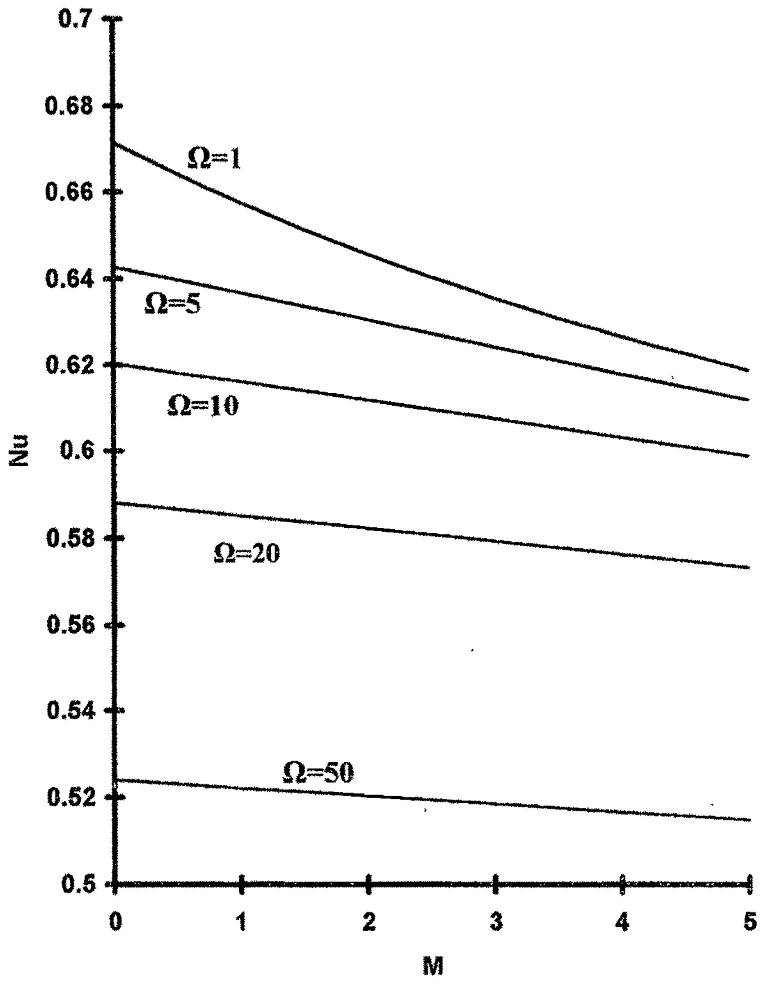


Figure 9.7: The Nusselt number Nu against Hartmann number M at the plate y=0



APPENDIX

$$\lambda_1 = A_1 + iB_1,\ A_1 = \frac{1+X_1}{2},\ B_1 = \frac{Y_1}{2},\ X_1 = \sqrt{\left\{\frac{(1+4M)+\sqrt{16\Omega^2+(1+4M)^2}}{2}\right\}},$$

$$Y_1 = \sqrt{\left\{\frac{\sqrt{16\Omega^2+(1+4M)^2}-(1+4M)}{2}\right\}},\ \lambda_2 = A_2 + iB_2,\ A_2 = \frac{1+X_2}{2},$$

$$B_2 = \frac{Y_2}{2},\ X_2 = \sqrt{\left\{\frac{(1+4M)+\sqrt{16(\Omega+\omega)^2+(1+4M)^2}}{2}\right\}},$$

$$Y_2 = \sqrt{\left\{\frac{\sqrt{16(\Omega+\omega)^2+(1+4M)^2}-(1+4M)}{2}\right\}},\ \lambda_3 = X_4 + iY_4,\ X_4 = \frac{P+X_3}{2},$$

$$Y_4 = \frac{Y_3}{2},\ X_3 = \sqrt{\left\{\frac{P^2+\sqrt{P^4+16P^2\omega^2}}{2}\right\}},\ Y_3 = \sqrt{\left\{\frac{\sqrt{P^4+16P^2\omega^2}-P^2}{2}\right\}},$$

$$L_4 = X_9 + iY_9,\ X_5 = (A_1A_2 + B_1B_2),\ Y_5 = (A_2B_1 - A_1B_2),\ X_6 = (A_1 + A_2),$$

$$Y_6 = (B_1 - B_2),\ X_7 = -P(A_1 + A_2),\ Y_7 = -P(B_1 - B_2 + \omega),$$

$$X_8 = X_6^2 - Y_6^2 + X_7,\ Y_8 = 2X_6Y_6 + Y_7,\ X_9 = \frac{X_5X_8 + Y_5Y_8}{X_8^2 + Y_8^2},$$

$$Y_9 = \frac{X_5Y_5 - X_5Y_8}{X_8^2 + Y_8^2},\ L_5 = X_{12} + iY_{12},\ X_{10} = -P(A_1 + A_2),$$

$$Y_{10} = P(B_1 - B_2) - P\omega,\ X_{11} = X_6^2 + Y_6^2 + X_{10},\ Y_{11} = -2X_6Y_6 + Y_{10},$$

$$X_{12} = \frac{X_5X_{11} - Y_5Y_{11}}{X_{11}^2 + Y_{11}^2},\ Y_{12} = \frac{-X_5Y_{11} - Y_5X_{11}}{X_{11}^2 + Y_{11}^2},\ L_6 = X_{13} + iY_{13},$$

$$X_{13}=\frac{X_{11}}{{X_{11}}^2+{Y_{11}}^2},\quad X_{13}=\frac{-Y_{11}}{{X_{11}}^2+{Y_{11}}^2},\quad L_7=X_{14}+iY_{14},\quad X_{14}=\frac{X_8}{{X_8}^2+{Y_8}^2},$$

$$Y_{14}=\frac{-Y_8}{{X_8}^2+{Y_8}^2},\quad L_8=X_{15}+iY_{15},\quad X_{15}=EP\left(X_9+MX_{14}\right),$$

$$Y_{15}=EP\left(Y_9+MY_{14}\right),\quad L_9=X_{16}+iY_{16},\quad X_{16}=EP\left(X_{12}+MX_{13}\right),$$

$$Y_{16}=EP\left(Y_{12}+MY_{13}\right).$$