CHAPTER - I
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1. BASIC CONCEPTS OF RELIABILITY THEORY

1.1 RELIABILITY : AN INTRODUCTION

Reliability was first recognized during World War II. The mathematicians like Gnedenko et al. [30], Polovoko, [42], Barlow and Proschan [3] have contributed for the development of the theory of reliability. The last four decades have seen remarkable progress in the application of reliability principles in industries and Government departments in almost all developed and developing countries.
The development of reliability evaluation techniques initially associated with the aerospace industry and military applications. These developments were followed by applications in the nuclear industry which is now under extreme measure to ensure safe and reliable nuclear reactors; in the electricity supply, which is expected to supply energy on demand without local failures or large scale blackouts and in continuous process plants such as steel plants and chemical plants, which can suffer large scale losses if system failure errors as well as causing deaths and environmental pollution. In recent times, all of these areas have suffered severely which include aerospace (Challenger space shuttle (1986) and several commercial aircraft accidents), nuclear (Three Mile Island (1979) and Chernobyl (1986)), electricity supply (New york blackout (1977)), Process plant (Flix boroush (1974), Seveso (1976) and Bhopal (1984) and many incidents in which failures have resulted in severe social and environmental consequences and many more deaths.

The National Aeronautics and Space Administration (NASA) defines reliability as the probability of a device performing its function adequately for the period of time intended under the operating conditions encountered. Reliability as a function of time, leads to the failure rate. Reliability characteristics, such as mean time to failure, availability, probability of survival, mean down time and frequency of failures are the measures of system effectiveness.
1.2 FAILURES AND FAILURE MODES

The word “Failure” acted as main role in the reliability theory and it is defined as the termination of the ability of an item to perform its intended function. Failures are classified into different ways like inherent weakness failure, sudden failure, and gradual failure. Failure is giving the partial change in those properties of system that its functioning completely stopped. For example switches and electric bulbs have well defined failures. The concept of failure and their details help in the evaluation of the quantitative reliability of a device failures model are related to life test result and probability theory. The first step in a failure model is to locate plan a test on parts substantially the same as those to be used, when we arrange large collection of units into operation, then there are large number of failures initially. These failures are called initial failures or infant mortality.

These failures are primarily due to manufacturing defects, after initial failures fewer failures are reported but, it’s determine is difficult to their causes. They occur due to the sharp change in parameters in either case. It is difficult to predict the amplitude of stress variations and their time of occurrence, during this period are called failures. The units get out worm and begin to deteriorate a gradual change of the parameters to estimate the performance of the unit result is called the wear-out region.
A curve is shown in fig. 1 and it’s called bathtub curve in the reliability theory.

Fig: 1.1 Failure Rate Curve or Mortality Rate Curve

--- Operating Stress $S_1$  ―――――――― Operating Stress $S_2$, $(S_2 > S_1)$

1.3 ROLE OF PROBABILITY

Reliability is based on the concept of Probability theory. Probability theory is necessary for any real application of reliability. Probability distributions are described reliability evaluation with some parameters. Some of the continuous distributions like exponential, weibul, gamma, normal and log normal are useful in the theory of reliability and also discrete distributions namely Binomial and poisson are useful in reliability theory.
Mostly reliability learns on probability for its underlying support fail at different duration of time and the failure phenomenon can only be described in probability terms. Failure model and hazard rate are effectively used in reliability theory.

If $T$ is the time to failure of the unit ($T$ itself is a variable), then the probability that it will not fail in a given environment before time $t$.

$$R(t) = P(T > t)$$

Since it is a probability, its numerical value is always between one and zero. That is, $R(0) = 1$, $R(\infty) = 0$ and $R(t)$ is a non increasing function between these limits.

### 1.4 RELIABILITY FUNCTION

Consider a test in which a set of $N$ components in operation from $t=0$. After time $t$ the number of surviving components are $N_s(t)$ and the failed components are $N_f(t)$

$$F(t) = 1 - R(t)$$

$R(t)$ is the probability of success.

That is,

$$R(t) = \frac{N_s(t)}{N} = 1 - \frac{N_f(t)}{N}$$

General Reliability function is

$$R(t) = \exp \left[ - \int_0^t \lambda(t) \, dt \right]$$
1.5 RELIABILITY ANALYSIS WITH SOME CONFIGURATIONS

The system may be complex system involving thousands of components associated systematically by a specific configuration. The system is generally analysed by decomposing the system into smaller subsystems and estimating each of them. The configuration in which the components are connected to form the system represented by a network diagram. The following configurations are discussed in the reliability theory.

(i) Series system
(ii) Parallel system
(iii) Series – Parallel system
(iv) Parallel – Series system

1.5.1 RELIABILITY – SERIES SYSTEM

The series configuration, also called a chain structure, is the simplest and commonly used in the reliability theory. In the series configuration, any system which is success depends on the success of all its components and only one component need to fail for the entire system to fail. Assuming ‘n’ components are in the system, which are connected in series and given in the figure (1.2)
If the system consists of 'n' units which are dependent then the reliability of the system is,

\[ R(t) = P(s) = P(X_1) P(X_2/X_1) \cdots P(X_n/X_1 \text{ and } X_2 \cdots \text{and } X_{n-1}) \]

If the 'n' units are independent then we have,

\[ R(t) = P(t) = P(X_1) P(X_1) \cdots P(X_n) \]

\[ = \prod_{i=1}^{n} P(X_i) \]

If the units are identical then,

\[ R(t) = [R(t)]^n \]

Time to failure follow exponential distribution then

\[ R(t) = \exp\left[-\sum_{i=1}^{n} \lambda_i \cdot t\right] \]
Mean time to failure of the system is,

\[
MTTF = \int_{0}^{\infty} R(t) \, dt
\]

\[
= \frac{1}{\sum_{i=1}^{n} \lambda_i}
\]

1.5.2 RELIABILITY – PARALLEL SYSTEM

Assume that ‘n’ components in the system will survive for operating the system. If a system of ‘n’ components can function properly when only one of the components is good, a parallel configuration is indicated in which successful operation depends on the satisfactory functioning of any one of their ‘n’ sub systems. The system fails when all the components fail. The block diagram representing a parallel configuration which is shown in fig.1.3

![Block Diagram](image)
If the units are dependent then the reliability of the system is given by,

\[ R(t) = P(s) = 1 - P(f) = 1 - P[X_1] P[X_2/X_1] \ldots P[X_n/X_1 X_2 \ldots X_{n-1}] \]

If the units are independent then,

\[ R(t) = P(s) = 1 - P(f) = 1 - P[X_1] P[X_2] \ldots P[X_n] \]

If the units are constant and identical

\[ R(t) = 1 - \left(1 - \exp(-\lambda t)\right)^n \]

Then mean time to failure is,

\[ \text{MTTF} = \int_0^\infty R(t) \, dt = \frac{1}{\lambda} \sum_{i=1}^n \frac{1}{i} \]

if the units are non-identical then,

\[ \text{MTTF} = \sum_{i=1}^n \frac{1}{\lambda_i} - \sum \sum \frac{1}{i} \left(\sum_{j=1}^i \left(\lambda_i + \lambda_j\right) + \sum \sum \sum \frac{1}{\lambda_i + \lambda_j + \lambda_k} \right) + \ldots + (-1)^{n-1} \left(\frac{1}{\lambda_1 + \lambda_2 + \ldots + \lambda_n}\right) \]
1.5.3 RELIABILITY – SERIES-PARALLEL SYSTEM

The block diagram representing a Series – Parallel system and shown in Fig. (1.4).

The reliability of the system for series – parallel is given by,

\[
R(s) = R_1 R_2 \cdots R_k = \prod_{i=1}^{k} \left( \prod_{j=1}^{n_i} \left( 1 - P(x_{ij}) \right) \right)
\]
1.5.4 RELIABILITY – PARALLEL-SERIES SYSTEM

A parallel series system consists of $k$ disjoint branches that are connected in parallel and branch $i$ for $1 \leq i \leq k$ consists of $n_i$ components that are connected in series. The block diagram of a parallel – series system is given in Fig.(1.5)

![Fig. (1.5) A Block Diagram for Parallel – Series system](image)

The reliability of a parallel – series system is given by

$$R(s) = 1 - \prod_{i=1}^{k} \left[ 1 - \prod_{j=1}^{n_i} \pi(X_{ij}) \right]$$


1.6 CONCEPT OF MAINTAINABILITY AND AVAILABILITY

Maintainability is one of the affective ways of increasing the system reliability. Repair maintenance is considered to be beneficial when the repair cost in terms of time and money spent is considerably low compared to the cost of the equipment.

The word 'Maintainability' is defined as the probability that the equipment will be restored to operational effectiveness within a specified time when the repair is performed in accordance with the prescribed conditions. The maintainability is clearly a function of repair time \( M(t) \).

If \( T \) is a random variable representing the repair time, then maintainability is defined as

\[
M(t) = \Pr ( T \leq t )
\]

If the repair time is exponentially distributed with the parameter \( \mu \), then the maintainability equation is

\[
M(t) = 1 - \exp \left[ - \mu t \right]
\]

The term 'Availability' function \( A(t) \) is defined as the probability that the equipment is operating at time \( t \). Although this definition appears to be very similar to the reliability function \( R(t) \), the two have different meanings. While Reliability places emphasis on trouble-free operation up to time \( t \), where as availability is
concerned with the status of the equipment at time $t$. The availability function does not say anything about the number of failures that occurred during time $t$.

Availability is always associated with the concept of maintainability. Availability depends up on both failure and repair rates. The long-run availability is defined as

$$\text{Availability} = \frac{\text{Up-time}}{\text{Up-time} + \text{Down time}}$$