APPENDICES AND BIBLIOGRAPHY
Appendix A

Integrating the equation of continuity (7.1) w.r.t. y over the film thickness 0,h , we get

\[ \int_{0}^{h} \frac{\partial u}{\partial x} \, dy + v \bigg|_{y=0}^{h} = \int_{0}^{h} \frac{\partial u}{\partial y} \, dy + (v_{b} + v_{bh} - v_{so}) = 0 \]

(by boundary conditions (7.7))

or, \[ \int_{0}^{h} \frac{\partial u}{\partial y} \, dy + (v_{b} - \frac{k_{b} \frac{d^2 p}{d \delta^2}}{\mu} \frac{\partial^2 p}{\partial \delta^2} - \frac{k_{s} \frac{d^2 p}{d \delta^2}}{\mu} \frac{\partial^2 p}{\partial \delta^2}) \right|_{y=0}^{h} \]

(A.1)

But \( u(x,y) \) is a function of \( x \) and \( y \), and general \( h \) is a function of \( x \), so by the rule of differentiation under the integral sign with variable limits,

\[ \int_{0}^{h} \frac{\partial u}{\partial y} \, dy = \frac{\partial}{\partial x} \left( \int_{0}^{h} u \, dy \right) - u(x,h) \frac{dh}{dx} + u(x,0) \frac{d(0)}{dx} \]

\[ = \frac{\partial}{\partial x} \left( f_{r} \right) - (u_{b} + u_{bh}) \frac{dh}{dx} \]

(by (7.19) & (7.3) )

(A.2)

Also \((u_{b} + u_{bh})(dh/dx)\) is equivalent to the y-component of the local velocity of the surface \( y= h \) due variation of curvature (Dowson, 1977, p.26). But \( v_{b} \) in the equation (A.1) is sum of y-component of local velocity of the surface \( y= h \) and the squeezing velocity at the least film thickness i.e.

\[ v_{b} = (u_{b} + u_{bh})(dh/dx) + v_{b} \bigg|_{h=h_{0}} \]  

(A.3)

From the relations (A.1) to (A.3) we finally get,

\[ \frac{\partial}{\partial x} \left( f_{r} \right) = - v_{b} \bigg|_{h=h_{0}} + \frac{k_{b}}{\mu} \frac{d^2 p}{d \delta^2} b + \frac{k_{s}}{\mu} \frac{d^2 p}{d \delta^2} s \]
Integrating this w.r.t. $x$, for simplicity denoting $v_b$ for $v_b \big|_{h=h_0}$, we finally get the equation (7.20).

Appendix B:

Purely from the theoretical point of view, the values of $s_1$ and $s_2$ of the equation (1.2.5) are to be considered as $-\infty$ to $\infty$ respectively, for the pressure gradients are produced throughout the lubricant region. But, in practical situation regarding elastohydrodynamic contact, the value of pressure $p$ and pressure gradient $dp/dx$ are negligible outside the Hertzian contact zone and specially in outlet region. Thus the values of $s_1$ and $s_2$ can be set to two finite values $x_{in}$ and $x_{out}$ respectively in the inlet and outlet regions. It is found by experimental observations that it suffices to take the values of $x_{in}$ as 4 - 5 times of the value of the semi-width of the Hertzian contact and the cavitation theory suggests that the value of $x_{out}$ may be taken as equal to or less than the value of the semi-width of the Hertzian contact. Since the value of $p$ is almost zero outside the Hertzian contact zone, the value of $h$ is almost uneffected even when $x_{in}$ is taken as 100 times the value of semi-width of the Hertzian contact. This is also explained in the text in the section 7.3 of Chapter VII.
Appendix C

Case (1) when $\frac{\partial U}{\partial Y} > 0$ i.e. when $Q'Y + C > 0$, the equation (8.26) can be written as

$$\frac{\partial U}{\partial Y} = (Q'Y + C)^{1/n}$$  \hspace{1cm} (C.1)

Integrating with respect to $Y$, we get

$$U = \frac{(Q'Y + C)^{1/n + 1}}{(1/n + 1) Q'} + C_1(X) = \frac{|Q'Y + C|^{1/n + 1}}{(1/n + 1) Q'} + C_1$$  \hspace{1cm} (C.2)

(Since, $Q'Y + C > 0 \Rightarrow Q'Y + C = |Q'Y + C|$)

Case (2) when $\frac{\partial U}{\partial Y} = 0$ i.e. when $Q'Y + C = 0$, on integration with respect to $Y$, it gives

$$U = C_1(X), \text{ which can also be put in the form}$$

$$U = \frac{|Q'Y + C|^{1/n + 1}}{(1/n + 1) Q'} + C_1, \hspace{1cm} (\because Q'Y + C = 0)$$  \hspace{1cm} (C.3)

Case (3) when $\frac{\partial U}{\partial Y} < 0$ i.e. when $Q'Y + C < 0$, let us put $Q'Y + C = -f(Y)$, then $f(Y) > 0$ and

$$|Q'Y + C| = -(Q'Y + C) = f(Y) = |f(Y)|.$$  

Then the equation (8.26) takes the form

$$\frac{\partial U}{\partial Y} = |Q'Y + C|^{1/n - 1}(Q'Y + C) = |f(Y)|^{1/n - 1}[-f(Y)] = -|f(Y)|^{1/n}$$

Integrating this with respect to $Y$ and noting that $f'(Y) = -Q'$, we finally get

$$U = \frac{|f(Y)|^{1/n + 1}}{(1/n + 1) Q'} + C_1(X) = \frac{|Q'Y + C|^{1/n + 1}}{(1/n + 1) Q'} + C_1$$  \hspace{1cm} (C.4)

Thus from relations (C.2) to (C.4), we can conclude that for all the three cases the equation (8.28) is established.
Appendix D

Integrating the equation of continuity (8.22) w.r.t. Y over the film thickness, we get

\[ \int_0^H \frac{\partial u}{\partial X} \, dY + V \bigg|_{Y=0}^H = 0 \]

or, \[ \int_0^H \frac{\partial u}{\partial X} \, dY + V_2 = 0 \] (by boundary conditions (8.27)) \hspace{1cm} (D.1)

But, \( U(X,Y) \) is a function of \( X \) and \( Y \), and \( H \) is a function of \( X \), by the rule of differentiation under integral sign with variable limits,

\[ \int_0^H \frac{\partial u}{\partial X} \, dY = \frac{\partial}{\partial X} \int U dY - U(X,H) \frac{dH}{dX} + U(X,0) \frac{d(0)}{dX} \]

\[ = \frac{\partial}{\partial X} (F_r) - U_2 \frac{dH}{dX} \] (by (8.31) & (8.27)) \hspace{1cm} (D.2)

Also \( U_2 \frac{dH}{dX} \approx V_2'(X) \), where \( V_2' \) is the \( Y \)-component of local velocity of the surface \( Y = H \), and \( V_2 = V_2'(X) + V_{20} \) where \( V_{20} \) is the \( Y \)-component of velocity at the least film thickness \( Y = H_0 \) at \( X = 0 \).

Hence from (D.1) and (D.2) we get

\[ \frac{\partial}{\partial X} (F_r) - V_2' + V_2' + V_{20} = 0 \]

or, \[ \frac{\partial}{\partial X} (F_r) = - V_{20} \]

Integrating this w.r.t. \( X \) we get, \( F_r = - V_{20} X + C_2 \)

For simplicity, we write \( V_2 \) for \( V_{20} \) in the text.