CHAPTER-II

THEORY

BACKGROUND

Theoretical studies of Stokes (1849) showed that compressional and shear waves could be propagated through a homogeneous, isotropic solid medium, subsequently, Rayleigh (1885) showed that waves may be propagated along the surface of such an isotropic medium. These Rayleigh waves are vertically polarized, propagating along the free surface with amplitudes falling off exponentially with distance from the free surface.

Although the first recordings of distant earthquakes were made in 1889, it was not until a decade later that Oldham (1900) recognized that recordings of teleseisms consisted of three types of waves - compressional, shear and surface waves - as predicted by theory.

It was observed, however, that some of the surface waves on the seismograms were not of the Rayleigh type. These waves had a horizontally polarised shear vibration, the same had not been explained by theory. Love (1911) therefore investigated the nature of this wave theoretically by introducing a homogeneous, isotropic (solid) low velocity layer over a half space and showed that surface waves of the observed type could be propagated along such a system. Unlike classical Rayleigh waves, these Love waves are dispersive. In the same investigation, Love showed that dispersive waves with a Rayleigh type (Vertical) motion will also be
propagated where a low-velocity surface layer overlies a half space.

Later investigators studied the dispersion of surface waves for the cases of two and three layers overlying a half space (Jeffreys, 1925, 1935; Sezawa, 1927; Stonely and Tillotson, 1928; Sezawa and Kanai, 1935; Stonely, 1937; Bateman, 1938). A brief resume of the development of the period equation is included so as to lead to the matrices of Thomson (1950) and Haskell (1953), which are the basic equations of this study.

GENERAL EQUATIONS OF DISPLACEMENT

From classical theory, two displacement potentials, may be defined such that the displacement components in a plane wave are given by

\[ u = \frac{\partial \phi}{\partial \chi} + \frac{\partial \psi}{\partial z} \]  
\[ w = \frac{\partial \phi}{\partial z} - \frac{\partial \psi}{\partial \chi} \]

where \( u \) and \( w \) are the particle displacements in the \( \chi \) and \( z \) directions respectively. By definition, the dilatation is

\[ \Delta = \frac{\partial u}{\partial \chi} + \frac{\partial w}{\partial z} \]

and the rotation

\[ 2\omega = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial \chi} \]

It can readily be shown that \( \nabla^2 \phi = \Delta \), and \( \nabla^2 \psi = 2\omega \).

Therefore, \( \phi \) is associated with dilatation and \( \psi \) a rotation or curl.

The displacements \( u \) and \( w \) must satisfy the equations of motion

\[ \rho \frac{\partial (u, w)}{\partial t^2} = \left\{ \left( \lambda + \mu \right) \right\} [\partial \Delta /\partial (\chi, z)] + \mu \nabla^2 (u, w) \]
where \( p \) is the density of the medium, and \( \lambda \) and \( \mu \) are Lame's constants. Substituting equations (1) and (2) into (3) shows that \( \phi \) and \( \psi \) must satisfy the following wave equations:

\[
\frac{\partial^2 \phi}{\partial t^2} = \alpha^2 \nabla^2 \phi \tag{4a}
\]

and

\[
\frac{\partial^2 \psi}{\partial t^2} = \beta^2 \nabla^2 \psi \tag{4b}
\]

where \( \alpha = \left[ \frac{(\lambda + 2\mu)}{\rho} \right]^{1/2} \) is the compressional-wave velocity, and

\( \beta = \left( \frac{\mu}{\rho} \right)^{1/2} \) the shear-wave velocity. Assuming \( \phi \) has a solution of the form

\( \phi = f(z) \exp[ik(\chi - ct)] \)

and substituting this into equation (4a), \( \phi \) is given by

\[
\phi = \{A'\exp[-kz(1-c^2/\alpha^2)^{1/2} + A''\exp[kz(1-c^2/\alpha^2)^{1/2}]\exp[ik(\chi - ct)]\} \tag{5a}
\]

Similarly for \( \psi \):

\[
\psi = \{B'\exp[-kz(1-c^2/\beta^2)^{1/2}] + B''\exp[kz(1-c^2/\beta^2)^{1/2}]\exp[ik(\chi - ct)]\} \tag{5b}
\]

where \( C \) is the phase velocity; \( k \) the wave number, \( \frac{2\pi}{L} \);

\( L \) the wave length; and \( A', A'', B' \) and \( B'' \) are constants (primes and double primes denoting the incident and reflected waves, respectively).

\[
\Delta = \nabla^2 \phi \quad \text{and} \quad \frac{\partial^2 \phi}{\partial t^2} = \alpha^2 \nabla^2 \phi, \quad \text{it follows that}
\]

\[
\phi = - \left( \frac{\alpha^2}{\zeta^2} \right) \Delta \tag{6a}
\]

and

\[
\psi = 2 \left( \frac{\beta^2}{\zeta^2} \right) \omega \tag{6b}
\]
where $\zeta = k_c$ is the circular frequency.

The solution for the displacements $U$ and $W$ in the $m$th layer of an $n$-layered system may be written in terms of $\Delta$ and $\omega$ (Haskell, 1953) by using equations (1), (6a), and (6b). Then

$$u = -\left[ (\alpha_m/\zeta)^2 \partial\Delta_m/\partial\chi + 2 (\beta_m/\zeta)^2 \partial\omega_m/\partial z \right]$$

$$w = -\left[ (\alpha_m/\zeta)^2 \partial\Delta_m/\partial z + 2 (\beta_m/\zeta)^2 \partial\omega_m/\partial\chi \right]$$

From the theory of elasticity, the equations for normal ($\sigma$) and tangential ($\tau$) stresses in terms of displacement are

$$\sigma = \lambda \Delta + 2\mu \partial w/\partial z = \lambda \Delta + 2\mu (\partial^2 \phi/\partial z^2 - \partial^2 \psi/\partial\chi \partial z)$$  \hspace{1cm} (8a)

$$\tau = \mu (\partial U/\partial z + \partial W/\partial\chi) = \mu \left[ (2\partial^2 \phi/\partial\chi \partial z) + (\partial^2 \psi/\partial z^2) - (\partial^2 \psi/\partial\chi^2) \right]$$  \hspace{1cm} (8b)

By substituting equations (6a) and (6b) into (8a) and (8b), Haskell (1953) obtained

$$\sigma = \rho_m \left\{ (\alpha_m^2 - 2\beta_m^2)\Delta_m + 2\beta_m^2[-(\alpha_m/\zeta)^2 (\partial^2 \Delta_m/\partial z^2) + 2(\beta_m/\zeta)^2 (\partial^2 \omega_m/\partial\chi \partial z)] \right\}$$ \hspace{1cm} (9a)

and

$$\tau = 2\rho_m\beta_m^2 \left\{ - (\alpha_m/\zeta)^2 (\partial^2 \Delta_m/\partial\chi \partial z) + (\beta_m/\zeta)^2 [(\partial^2 \omega_m/\partial\chi^2) - (\partial^2 \omega_m/\partial z^2)] \right\}$$ \hspace{1cm} (9b)

The interfaces between the layers must remain in welded contact during the passage of a disturbance; this requires continuity of stress and amplitude across an interface at all times.

**Matrix Formulation**

For the $M$th layer, harmonic solutions of the elastic equations of motion may be obtained with dilatational solutions of the form
\[ \Delta_m = (\partial u / \partial \chi) + (\partial w / \partial z) = \{ \Delta'_m \exp[-i \gamma_{am} z] + \Delta''_m \exp[i \zeta t - k\chi] \} \exp[i (\zeta t - k\chi)], \] (10a)

and with rotational solutions of the form
\[ w_m = 1/2[(\partial u / \partial z) - (\partial w / \partial \chi)] = \{ w'_m \exp[-i \gamma_{pm} z] + w''_m \exp[i \zeta t - k\chi] \} \exp[i (\zeta t - k\chi)], \] (10b)

where \( \gamma_{am} = -i [1 - (c/\alpha_m)^2]^{1/2} \) for \( c > \alpha_m \),
\[ \gamma_{pm} = -i [1 - (c/\beta_m)^2]^{1/2} \) for \( c < \beta_m \), and
\[ \Delta'_m, \Delta''_m, w'_m, w''_m \] are constants. Continuity of displacements across an interface is assured if the corresponding velocity components \( \dot{u} \) and \( \dot{w} \) (time derivatives of displacements) are made continuous. As \( C \), the velocity of propagation of any phase in the surface-wave train, is the same in all layers, Haskell (1953) took the dimensionless quantities \( \dot{u}/C \) and \( \dot{w}/C \) to be continuous.

Setting \( Z = d_m \), the thickness of the \( m \)th layer, and then at the origin setting \( Z = 0 \) and \( \chi = 0 \), Haskell (1953) obtained the following matrices:

\[
\begin{pmatrix}
\dot{\Upsilon}_m / C \\
\dot{\Theta}_m / C \\
\sigma_m \\
\tau_m
\end{pmatrix}
= D_m
\begin{pmatrix}
\Delta'_m + \Delta''_m \\
\Delta'_m - \Delta''_m \\
w'_m - w''_m \\
w'_m + w''_m
\end{pmatrix}
\] (11)
where

\[
D_m = \begin{bmatrix}
-(\alpha_m/c)^2 \cos \alpha_m & i(\alpha_m/c)^2 \sin \alpha_m \\
 i(\alpha_m/c)^2 \gamma_{am} \sin \alpha_m & -(\alpha_m/c)^2 \gamma_{am} \cos \alpha_m \\
-\rho_m \alpha_m^2 (\gamma_m-1) \cos \alpha_m & i \rho_m \alpha_m^2 (\gamma_m-1) \sin \alpha_m \\
-\rho_m \alpha_m^2 \gamma_{am} \sin \alpha_m & \rho_m \alpha_m^2 \gamma_{am} \cos \alpha_m
\end{bmatrix}
\]

and \( \rho_m = k \gamma_{am} d_m \), \( \gamma_m = \gamma_{bm} d_m \),

\( \gamma_m = 2(\beta_m/c)^2 \)

At the \((m-1)\) interface,

\[
\begin{pmatrix}
\dot{U}_{m-1}/c \\
\dot{W}_{m-1}/c \\
\sigma_{m-1} \\
\tau_{m-1}
\end{pmatrix}
= E_m
\begin{pmatrix}
\dot{\Delta}_m + \Delta_m^{/}/c \\
\dot{\Delta}_m^{/}/c - \Delta_m^{/}/c \\
\omega_m - \omega_m^{/}/c \\
\omega_m^{/}/c + \omega_m^{/}/c
\end{pmatrix}, \quad (13)
\]

where

\[
E_m = \begin{bmatrix}
-(\alpha_m/c)^2 & 0 & -\gamma_m \gamma_{bm} & 0 \\
0 & -(\alpha_m/c)^2 \gamma_{am} & 0 & \gamma_m \\
-\rho_m \alpha_m^2 (\gamma_m-1) & 0 & -\rho_m \gamma_m \gamma_{bm} & 0 \\
0 & \rho_m \alpha_m^2 \gamma_{am} & 0 & -\rho_m \gamma_m \gamma_{bm} (\gamma_m-1)
\end{bmatrix}
\]

Equation (11) may then be written

\[
\begin{pmatrix}
\dot{U}_m/c \\
\dot{W}_m/c \\
\sigma_m \\
\tau_m
\end{pmatrix}
= \begin{pmatrix}
\dot{U}_{m-1}/c \\
\dot{W}_{m-1}/c \\
\sigma_{m-1} \\
\tau_{m-1}
\end{pmatrix}
\begin{pmatrix}
\Delta_m^{-1} E_m
\end{pmatrix}^{-1}, \quad (14)
\]
where $E_m^{-1}$ is the inverse of $E_m$, i.e.

$$
E_m^{-1} = \begin{bmatrix}
-2(\beta_m/\alpha_m)^2 & 0 & (\rho_m\alpha_m^2)^{-1} & 0 \\
0 & c^2(\gamma_m - 1)/\alpha_m^2 \gamma_m & 0 & (\rho_m\alpha_m^2\gamma_m)^{-1} \\
(\gamma_m - 1)/\gamma_m \beta_m & 0 & -\rho_m c^2 \gamma_m \beta_m)^{-1} & 0 \\
0 & 1 & 0 & (\rho_m c^2 \gamma_m)^{-1}
\end{bmatrix}
$$

(15)

The boundary conditions require that the values of $\dot{U}/C$, $\dot{W}/C$, $\sigma$ and $\tau$ at the top of the $m$th layer be the same as those at the bottom of the $(m-1)$ layer. Therefore,

$$
\begin{pmatrix}
\dot{U}_m/C \\
\dot{W}_m/C \\
\sigma_m \\
\tau_m
\end{pmatrix}
= (D_mE_m^{-1})(D_{m-1}E_{m-1}^{-1})
\begin{pmatrix}
\dot{U}_{m-2}/C \\
\dot{W}_{m-2}/C \\
\sigma_{m-2} \\
\tau_{m-2}
\end{pmatrix}
$$

(16)

By iteration

$$
\begin{pmatrix}
\dot{U}_{n-1}/C \\
\dot{W}_{n-1}/C \\
\sigma_{n-1} \\
\tau_{n-1}
\end{pmatrix}
= a_{n-1} a_{n-2} \ldots a_1 \begin{pmatrix}
\dot{U}_0/C \\
\dot{W}_0/C \\
\sigma_0 \\
\tau_0
\end{pmatrix}
$$

(17)
where \( a = \text{DE}^{-1} \). Applying the inverse of equation (13) for the \( n \)th layer results in

\[
\begin{bmatrix}
\Delta'_n + \Delta''_n \\
\Delta'_n - \Delta''_n \\
\omega'_n - \omega''_n \\
\omega'_n + \omega''_n
\end{bmatrix} = E^{-1} a_{n-1} a_{n-2} \ldots a_1 
\begin{bmatrix}
\dot{u}_0/c \\
\dot{w}_0/c \\
\sigma_0 \\
\tau_0
\end{bmatrix}
\] (18)

The development thus far has been general and equation (18) is applicable to surface waves or to any plane waves propagating through a layered medium. An additional boundary condition is that there are no stresses across the free surface.

Then \( \sigma_0 = \tau_0 = 0 \). Also, as there can be no sources at infinity, \( \Delta'_n = \omega''_n = 0 \).

Letting \( J = E^{-1} a_{n-2} \ldots a \), Haskell obtained

\[
\begin{bmatrix}
\Delta'_n \\
\Delta'_n \\
\omega'_n \\
\omega'_n
\end{bmatrix} = J 
\begin{bmatrix}
\dot{u}_0/c \\
\dot{w}_0/c \\
0 \\
0
\end{bmatrix} = 
\begin{bmatrix}
J_{11} & J_{12} \\
J_{21} & J_{22} \\
J_{31} & J_{32} \\
J_{41} & J_{42}
\end{bmatrix} 
\begin{bmatrix}
\dot{u}_0/c \\
\dot{w}_0/c
\end{bmatrix}
\] (19)

Finally, eliminating \( \Delta'_n \) and \( \omega'_n \), Haskell (1953) obtained the phase-velocity relationship as follows:

\[
\dot{u}_0/\dot{w}_0 = (J_{22} - J_{12})/(J_{11} - J_{21}) = (J_{42} - J_{32})/(J_{31} - J_{41})
\] (20)

where the \( J \)'s are functions of \( C \) and \( k \), the phase velocity and wave number respectively of the disturbance. The group velocity is obtained by differentiating phase
velocity, i.e. Group velocity \( u = \frac{d\omega}{dk} = \frac{d(ck)}{dk} \).

It can be written according to Brune (1964),

\[
u = \frac{d\omega}{dk} = \Delta \left[ t - \frac{d(\phi_2(T) - \phi_1(T))}{d\omega} \right] \text{I=const} \equiv \Delta \left( t - \phi' \right) \tag{21}
\]

where \( u \) = group velocity

\[\omega = \text{angular frequency, } 2\pi / T\]

\( \Delta = \text{distance between two stations or between epicentre and station}\)

\( T = \text{period corresponding to phase velocity } d\Delta / dt\)

\( I = \text{mode number; for fundamental mode } I = 1.\)

\( \phi' \) is a total derivative of \((\phi_2(T) - \phi_1(T))\) with respect to \( \omega \) along the curve \( I = \text{constant in the phase velocity } C \text{ vs. } \omega \text{ plane.} \)

\[
\phi' = \left\{ \frac{d(\phi_2(T) - \phi_1(T))}{d\omega} \right\} \text{I=const}
\]

It can be expanded into two partial derivatives:

\[
\phi' = \left\{ \frac{\partial(\phi_2(T) - \phi_1(T))}{\partial \omega} + \frac{\partial(\phi_2(T) - \phi_1(T))}{\partial C} \right\} \left( \frac{dc}{d\omega} \right)
\]

**NUMERICAL METHODS**

The matrix iteration method of Haskell (1953) was first programmed for numerical computation by Dorman, Ewing and Oliver (1960). Modifications and additions have been made by Dorman and Prentiss (1960), Press, Hakrider and Seafeldt (1961), Dorman (1962), Hakrider and Anderson (1962), Hakrider (1963, 1964), and others. With a more advanced computer, greatly improved the speed of computation, Randall (1967) later applied Knopoff's (1964) method to this problem and reported a further improvement in speed for the Rayleigh-wave case. Then, Schwab (1970) and Schwab & Knopoff (1970, 1971, 1972, 1973) improved the
optimization, for computer application, of both the Thomson-Haskell technique and Knopoff's method for flat, multi-layered media.

Prior to these computer technique developments, only a few theoretical dispersion points could be calculated for comparison with observational data because of the complexity in hand computations required for models consisting of more than one or two layers. The computer programs, however, permit rapid computations of Love- and Rayleigh-wave dispersions, including higher modes of both types, for models consisting of a large number of horizontal layers. Computation of this formulation for an arbitrary number of surface layers requires little more work than computation for a single layer. Hence, structural models can be made with sufficiently complex distributions of velocity as a function of depth to provide agreement with observed dispersion data within the precision of the data.

THEORETICAL DISPERSION

Seismic waves depend both on source properties and on the structure of the media traversed by the waves. In surface wave seismology, the observed record is expressed in spectrum but the observed record is as the product of the spectra of the source-time function, the effect of finiteness, propagation, attenuation and recording.

Signal Processing involves the measurement of spectra. The vast application of spectral methods in geophysics is mainly due to development of electronic computers in last 3 to 4 decades. Spectral methods working with frequency as independent parameter are much more powerful. The phenomena mentioned depend on frequencies and not on time. For example, a given body or a given layered structure favours certain frequencies and
suppresses others by its own free oscillation modes or reverberations. Likewise, attenuation in a body is a phenomenon which is generally assumed to be frequency-dependent. All observed phenomena are studied in relation to the frequency of the waves. Earlier in seismology, most phenomena were studied only in time domain, i.e. directly from seismograms, then using period as measured directly as discriminant.

Surface waves appear on records of long period seismographs as the most prominent group. Both Rayleigh and Love waves are dispersive, that is, their velocities are dependent on the period. This effect is due to greater penetration of the longer period waves. The longer period waves therefore travel faster and appear earlier in seismograms. Dispersion studies of these waves provide a very important method for investigating the elastic properties of the wave guide. Experimentally obtained dispersion curves based on assumed earth models to give the elastic parameters of the layers and the structure of the wave guide.

Before the era of electronic computers, the calculation of dispersion curves for earth models was so difficult and time-consuming that it was only possible to carry out the numerical calculations for very simple models. Thomson (1950) introduced a matrix method for determining transmission and reflection coefficients for plane body waves through a stratified solid medium. Haskell (1953) applied Thomson’s matrix formalism to the surface wave problem and developed a convenient method for computing dispersion on multi-layered media composed of any number of plane-parallel isotropic layers. This is known as Thomson-Haskell method. The method is described in General Equations of Displacement and Matrix formulation. It was extensively applied by Dorman, Ewing and Oliver (1960). The method has been generalized later
for anisotropic media and spherical shells (Anderson, 1965).

LITERATURE

Tams (1921) and others observed that the average velocity of surface waves is different for continental and oceanic crusts. Gutenberg (1924) was the first to attempt to use the observed dispersion of surface waves for investigating crustal structures; he found that group velocities of Love and Rayleigh waves with periods shorter than about 30 seconds were greater in the crust under Eurasia, although the velocities of the longer period surface waves were nearly the same in all continental and oceanic provinces.

With improved theoretical relations and with increased observations, numerous analysis have been made on the dispersion of surface waves and their implications on crustal and sub-crustal structures in various regions. Wilson & Baykal (1948) studied Rayleigh-wave group velocity dispersion across the Atlantic ocean and were the first to make corrections for the continental portion of the path traversed. A single surface layer was assumed for the crustal model in attempting to explain the observed dispersion. Ewing and Press (1950) reinterpreted the data of Wilson & Baykal by assuming that the crust consisted of a layer of water overlying a layer of sediments and a relatively thick section of basic rocks. The agreement between the theoretical curves for this model and the observed group velocity dispersion was a considerable improvement over the range of observed periods.

Ewing and Press (1952) were able to produce additional evidence that the observed dispersion of Rayleigh waves across oceanic paths is controlled
primarily by the presence of a water layer overlying a basaltic layer in the crust.

Evernden (1953,1954) investigated the technique of using tripartite arrays to determine phase-velocity dispersion of Rayleigh waves. Although primarily concerned with the directions of approach of the surface waves, his results were consistent with known structural relationships in the area of study. He showed that the phase-velocity measurements could be obtained when separate phases in a Rayleigh-wave train are correlated across a tripartite arrays.

Brilliant and Ewing (1954) devised a technique for studying continental structure by using group velocities, that eliminated the effects of oceanic paths and errors in epicentral locations and origin times. Using only stations which were suitably located along great circle paths from the epicentre, phase-period as a function of arrival time was plotted for each station. Consistent dispersion data were obtained from these plots.

Press (1956) pioneered the use of tripartite arrays to determine crustal structures from phase-velocity dispersion. He measured such dispersion in Southern California and in the San Francisco area.

Investigations of surface-wave dispersion have since been made in many other regions especially by applying the matrix formulation of Haskell (Dorman et al., 1960). Quite highly resolved crustal and sub-crustal models have been hypothesized which have not only satisfied the observed dispersion but have provided additional and otherwise not known velocity layering.

Most of these studies, especially the earlier ones, were made without the use of any spectral analysis. However, spectral techniques have now been used extensively in dispersion investigation.
Group-velocity can be determined from records at only one station and it is found by dividing epicentral distance by the respective travel times. Group-velocity can also be obtained by differentiating phase-velocity as given below (Bath, 1982) and it is applied by Toksöz and Ben-Menahem (1963), Brune (1965) and Dewart & Toksöz (1965).

\[
C(\omega) = \omega \frac{(r_2 - r_1)}{[\omega(t_{02} - t_{01}) + [\phi_2(\omega) - \phi_1(\omega)]} + \frac{m\pi}{2} + 2n\pi
\]

\[
u(\omega) = \frac{d\omega}{dk} = c / \left[ 1 + \frac{T}{c} \right] \frac{dc}{dT}
\]

\[
= \frac{(r_2 - r_1)}{[\omega(t_{02} - t_{01}) + (\frac{d\phi_2(\omega)}{d\omega}) - (\frac{d\phi_1(\omega)}{d\omega})]
\]

\[C = \text{phase velocity}
\]

\[c = \omega / k ;
\]

\[(r_2 - r_1) = \text{distance between two stations or distance between epicentre and station.}
\]

Ewing et al. (1959) used an electrical analogue spectrograph which displays the spectral variation of the analyzed seismogram with time. Their method gives a means of directly recording group velocities as a function of frequency. Iyer (1964) and Jacob et al. (1972) used moving window analysis method for finding group velocities from Rayleigh waves. In the moving window analysis method, intervals of suitable length along the record are chosen and each interval is Fourier analyzed. In mathematical form, the method consists in calculating the so-called moving Fourier amplitude \(F(\omega, \tau)\) from (Bath, 1982)

\[
F(\omega, \tau) = \pi \int_{-\infty}^{\infty} f(t) w(t-\tau) e^{-i\omega t} dt
\]
Where $\tau$ is the time lag of the moving window $W(t-T)$. For any given, constant frequency $\omega$ or period $T$, the calculation of $F(\omega, \tau)$ is done for a series of $\tau$ values, so that the variation of $F$ with $\tau$ can be easily calculated.

The use of Fourier Analysis in surface wave studies was introduced by Valle (1949) and Sato (1955, 1956 a & b, 1958, 1960) using the phase shift method of phase velocity. Particularly for crustal structure studies, the phase-shift method was used in a simplified manner by Berckhemer et al. (1961), also by McEvilly (1964) for structural studies of the U.S.A. using fundamental mode Rayleigh and Love waves and Wickens and Pec (1968) applied the method to segmental phase-velocity determination of Love waves in North America combined with structural interpretations. Tarr (1969) used the method of FFT calculation and got group velocity dispersion from differential phase velocities for the purpose of ray-path network technique. Phase-velocity is determined by using spectral technique of different methods in different regions of the world by number of authors Pilant & Knopoff (1964), Knopoff et al. (1966), Knopoff & Pilant (1966).

Improved accuracy is achieved in finding phase and group velocities by applying filtering technique before the Fourier Analysis by which disturbances due to noise, surface waves travelling over longer paths than the great circle arc and surface waves of other modes are mostly eliminated. Toksöz and Ben-Menahem used low-pass filtering before Fourier-analyzing Love & Rayleigh mantle wave records and Noponen (1969) combined band-pass filtering with the peak and trough method. Even more efficient are time-variable filters as applied by Landisman et al. (1969) and Starovoit (1971).