Experiments conducted on board the GEOS 1 satellite have revealed the emission of a new type of magnetospheric electrostatic ELF waves, having a wide spectrum between the lower hybrid frequency ($\Omega_{li}$) and the ion plasma frequency ($\Omega_{pi}$) (Cornilleau Wehrlin, 1981). The main characteristics of these waves is that they are strongly modulated in amplitude by a ULF electromagnetic ion cyclotron wave whose frequency is near the helium gyrofrequency. The electromagnetic ULF waves are detected simultaneously in all cases but one. At least two factors seem to control the existence of the ELF waves: the amplitude of the electromagnetic ion cyclotron wave ($E_{IZ}$) and the plasma density ($N$).
The emission mechanism of this ELF electrostatic ion acoustic wave in the presence of a coherent-electromagnetic ion cyclotron wave has been presented recently on the basis of plasma maser theory (Nambu et al., 1986).

In this chapter we investigate the emission of the EM ion cyclotron wave associated with electrostatic wave from a slightly different approach. In our analysis, we study the electrostatic instability driven by this nonlinear force and apply our results to the emission of the electrostatic waves reported by Cornilleau-Wehrlin (1981). In section 4.2, the nonlinear force arising out of the resonant interaction between the electrons and modulated fields caused by coupling between the EM ion cyclotron wave with a test ES ion acoustic wave is calculated. The dispersion relation for the electrostatic wave is obtained using the nonlinear force. Induced bremsstrahlung of ion acoustic waves by electrons scattered on a coherent electromagnetic cyclotron wave is calculated in Section 4.3. The last Section contains the application and discussion of our results.

4.2 Calculation of the nonlinear force:

We consider a homogeneous magnetized plasma in the presence of an enhanced coherent electromagnetic
ion cyclotron wave, propagating obliquely to the external magnetic field $\mathbf{B}_0 = \mathbf{B}_e \mathbf{Z}$. The basic equations governing the interaction of the EM ion cyclotron wave with an ion-acoustic test wave are the Vlasov-Maxwell equations

$$\left\{ \frac{\partial}{\partial t} + \mathbf{v} \cdot \frac{\partial}{\partial \mathbf{r}} - \frac{e}{m} \left[ \mathbf{E} \cdot \mathbf{v} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right] \frac{\partial}{\partial \mathbf{v}} \right\} f_e(\mathbf{r}, \mathbf{v}) = 0 \quad (4.1)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \quad (4.2)$$

$$\nabla \times \mathbf{B} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \mathbf{J}, \quad (4.3)$$

where $f_e$ is the electron distribution function, and other notation is standard.

Since the low frequency fluctuations are present in our systems, the unperturbed electron distribution function $f_{e_0}$ of the background plasma and the electric and magnetic fields are represented by

$$f_{e_0} = f_{e_0} + \varepsilon f_{e_1} + \varepsilon^2 f_{e_2} \quad (4.4)$$

$$\mathbf{E} = \varepsilon \mathbf{E}_0, \quad \mathbf{B} = \mathbf{B}_0 + \varepsilon \mathbf{B}_1 \quad (4.5)$$
where $\mathcal{E}$ is the ordering of the low frequency fluctuations, $f_{\text{oe}}$ is the space and time averaged part of the electron distribution function, $f_{\text{le}}$ and $f_{\text{pe}}$ are the fluctuating parts and $E_l$ and $B_l$ are the electric and magnetic fields of the EM ion cyclotron waves, respectively.

Then to order $\mathcal{E}$ and $\mathcal{E}^2$, the Vlasov-equation reduces, respectively, to

$$\left( \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m} \frac{\vec{v} \times \vec{E}_l}{c} \cdot \frac{\partial}{\partial \vec{v}} \right) f_{\text{le}}(\vec{r}, \vec{v}, t)$$

$$= \frac{e}{m} \left[ \vec{E}_l(\vec{r}, t) + \frac{\vec{v} \times \vec{B}_l(\vec{r}, t)}{c} \right] \cdot \frac{\partial}{\partial \vec{v}} f_{\text{le}}(\vec{r}, \vec{v}, t), \quad (4.6)$$

$$\left( \frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m} \frac{\vec{v} \times \vec{B}_l}{c} \cdot \frac{\partial}{\partial \vec{v}} \right) f_{\text{pe}}(\vec{r}, \vec{v}, t)$$

$$= \frac{e}{m} \left[ \vec{E}_l(\vec{r}, t) + \frac{\vec{v} \times \vec{B}_l(\vec{r}, t)}{c} \right] \cdot \frac{\partial}{\partial \vec{v}} f_{\text{pe}}(\vec{r}, \vec{v}, t), \quad (4.7)$$

Taking Fourier transform of the form

$$\hat{A}(\vec{r}, \omega) = \hat{A}(\vec{r}, \vec{k}, \omega) \exp \left[ i (\vec{k} \cdot \vec{r} - \omega t) \right] \quad (4.8)$$

and integrating over the unperturbed orbits, we
obtain from Eqn. (4.6)

\[
\sum_{\mathbf{k}} (\mathbf{g}) = \frac{e}{m} \int_{-\infty}^{0} \left[ \mathbf{E}_{\mathbf{k}}(\mathbf{g}) + \frac{\mathbf{v} \times \mathbf{B}_{\mathbf{k}}(\mathbf{g})}{\mathbf{c}} \right] \frac{2}{\omega_{ce}^2} d\omega
\]

\[
\times \exp \left[ ik \left( \mathbf{r}' - \mathbf{r} \right) - i\omega t \right] d\omega
\]

(4.9)

where \( \mathbf{r}' - \mathbf{r} = \mathbf{z} \) and \( \mathbf{r}'(t' = t) = \mathbf{r} \), \( \mathbf{r}'(t = t) = \mathbf{u} \).

We now introduce cylindrical coordinates in velocity space, i.e.,

\[
\begin{align*}
\mathbf{v}_x &= \mathbf{u}_x \cos \phi, & \mathbf{v}_y &= \mathbf{u}_y \sin \phi, & \mathbf{v}_z &= \mathbf{u}_z \\
(4.10)
\end{align*}
\]

In terms of these variables, the particle orbits are given by

\[
\begin{align*}
\mathbf{v}'_x &= \mathbf{v}_x \cos (\phi - \omega_0 t), & x' &= x - \frac{v_y}{\omega_0} \sin (\phi - \omega_0 t) \frac{\partial \theta}{\partial z} \\
\mathbf{v}'_y &= \mathbf{v}_y \sin (\phi - \omega_0 t), & y' &= y + \frac{v_x}{\omega_0} \cos (\phi - \omega_0 t) - \frac{v_x}{\omega_0} \cos \phi \\
\mathbf{v}'_z &= \mathbf{v}_z \quad \therefore \quad z' &= v_z t + \mathbf{z}
(4.11)
\end{align*}
\]

where \( \omega_0 = \frac{eB_0}{mc} \) is the electron cyclotron frequency.
Using Eqns. (4.10) and (4.11), we obtain from Eqn. (4.9) (Kennel, 1966),

$$f_{\lambda e}(\mathbf{k}^2 \omega) = -\frac{e}{\pi} \sum_{n} J_{\frac{n}{2}} \left( \frac{k_\perp v_{\perp}}{\alpha_\perp} \right) \omega \left[ \left( \frac{\eta}{2} \right) \chi \right] \times$$

$$\left( \frac{2 z v_{\perp} J_{\frac{n}{2}}(\frac{k_\perp v_{\perp}}{\alpha_\perp}) + \lambda v_{\parallel}(\frac{\eta}{2} J_{\frac{n}{2}}(\frac{k_\perp v_{\perp}}{\alpha_\perp}) - \lambda v_{\parallel}(\frac{\eta}{2} J_{\frac{n}{2}}(\frac{k_\perp v_{\perp}}{\alpha_\perp}))}{\chi (\omega - \lambda \omega - k_{\parallel} v_{\parallel})} \right)$$

(4.12)

where

$$X = E_{z} \frac{2}{\lambda_{\parallel} v_{\parallel}} \sum_{\omega} \frac{v_{\parallel}}{\alpha_\parallel} \left( k_{\parallel} E_{z} - k_{\perp} E_{x} \right) \left( \frac{\partial}{\partial \omega} \frac{\partial}{\partial \omega} - \frac{\partial}{\partial \omega} \frac{\partial}{\partial \omega} \right)$$

$$\tilde{v}_{\omega}$$

(4.13)

Here $J_{\frac{n}{2}}$ and $J_{\frac{\eta}{2}}$ are the Bessel functions of order $n$ and $\frac{\eta}{2}$, respectively. For $E_{\perp} \gg E_{\parallel}$, $E_{\parallel}$, and $\lambda = \lambda_{\parallel} = 0$, we obtain from Eqn. (4.12)

$$f_{\lambda e}(\mathbf{k}^2 \omega) = -\frac{e}{\pi} \frac{E_{\perp}^{2}(\mathbf{k}^2 \omega)}{\chi(\omega - k_{\parallel} v_{\parallel} + \lambda \omega)} \sum_{\omega} \frac{v_{\parallel}}{\alpha_\parallel} \left( k_{\parallel} v_{\parallel} \right)$$

(4.14)

In deriving Eqn. (4.14), we take $\tilde{f}_{\omega e}$ as the Maxwell distribution function. The $\lambda_{\parallel}$ is the small imaginary part.
We now perturb the steady state by a high frequency electrostatic ion acoustic test wave field \( \alpha \vec{E}_{lh} (\alpha \ll \varepsilon) \), propagating along the applied magnetic field. The total perturbed electric and magnetic fields and the electron distribution function are given by

\[
\vec{S}E = \alpha \vec{E}_{lh} + \omega \vec{E} \vec{E}_{lh} + \varepsilon \vec{a} \vec{e},
\]

\[
\vec{S}B = 0 + \omega \varepsilon \vec{S}_{lh} + \alpha \varepsilon \vec{a} \vec{B}
\]

\[
\vec{S}f = \alpha \vec{S}_{lh} + \omega \varepsilon \vec{S}_{lh} + \varepsilon \vec{a} \vec{f}
\]

where \( \vec{S}_{lh}, \vec{S}_{lh}, \vec{A} \vec{E} \) and \( \vec{A} \vec{B} \) are the modulation fields and \( \vec{S}_{lh} \) and \( \vec{A} \vec{f} \) are the perturbed electron distribution functions corresponding to the modulation fields.

We linearize the Vlasov Equation (4.11), for

\[
\vec{E}_L \gg \vec{S} \vec{E} \quad \text{and obtain}
\]

\[
\left( \frac{2}{\beta L} + \frac{\gamma}{2} - \frac{e}{m} \left[ \varepsilon \vec{E}_L + \frac{\vec{v} \times (\vec{B}_L + \varepsilon \vec{B}_L)}{c} \right] \right) \vec{S} \vec{f}
\]

\[
- \frac{e}{m} \left( \vec{S} \vec{E} + \frac{\vec{v} \times \vec{S} \vec{B}}{c} \right) \beta \left( f_{oe} + \varepsilon f_{ke} + \varepsilon f \right) = 0
\]
To order \( \mu , \mu E \) and \( \mu E^2 \), we obtain from Eqn. (4.12),

\[
P \partial_S^f (\rho^h, \mathbf{v}, t) - \frac{e}{m} \mathbf{S}_h (\mathbf{r}, t) \cdot \frac{\partial}{\partial \mathbf{v}} f_{oe} = 0 \quad (4.19)
\]

\[
P \partial_S^f (\mathbf{r}, \mathbf{v}, t) - \frac{e}{m} \left[ \mathbf{E}_h (\mathbf{r}, t) + \frac{\mathbf{v} \times \mathbf{B}_h (\mathbf{r}, t)}{c} \right].
\]

\[
\frac{2}{\partial \mathbf{v}} \partial_S f_{he} (\mathbf{r}, \mathbf{v}, t) - \frac{e}{m} \left[ \mathbf{S}_h (\mathbf{r}, t) + \frac{\mathbf{v} \times \mathbf{B}_h (\mathbf{r}, t)}{c} \right]. \frac{2}{\partial \mathbf{v}} f_{oe} = 0
\]

\[
- \frac{e}{m} \mathbf{S}_h (\mathbf{r}, t) \cdot \frac{2}{\partial \mathbf{v}} f_{le} (\mathbf{r}, \mathbf{v}, t) = 0
\]

\[
(4.20)
\]

and

\[
P \partial_S^f (\rho^h, \mathbf{v}, t) - \frac{e}{m} \left[ \mathbf{E}_h (\mathbf{r}, t) + \frac{\mathbf{v} \times \mathbf{B}_h (\mathbf{r}, t)}{c} \right]. \frac{2}{\partial \mathbf{v}} f_{oe} = 0,
\]

\[
- \frac{e}{m} \left[ \mathbf{S}_h (\mathbf{r}, t) + \frac{\mathbf{v} \times \mathbf{B}_h (\mathbf{r}, t)}{c} \right]. \frac{2}{\partial \mathbf{v}} f_{le} (\mathbf{r}, \mathbf{v}, t) = 0
\]

\[
(4.21)
\]
where

$$\mathcal{P} = \frac{2}{\partial t} + \vec{v} \cdot \nabla - \frac{e}{m} \frac{\vec{v} \times \vec{B}}{C} \cdot \nabla$$

(4.22)

Integrating Eqn. (4.19) over the unperturbed orbits, we obtain

$$Sf_h (\vec{R}, \vec{p}, \omega - \omega) = \frac{e}{\hbar} \frac{2}{\partial v_{||}} \sum_{\gamma_i = 1}^{\infty} \left[ \frac{b_{\gamma} v_{||}}{\omega} \left( J_{i+1} \gamma_i - J_{i-1} \gamma_i \right) \right] f_{E_{||}}$$

(4.23)

where $K$ and $\omega$ are the wave vector and frequency of the ion acoustic wave. From Equation (4.20) we obtain for electrons,

$$Sf_h (\vec{R}, \vec{p}, \omega - \omega) = \frac{e}{\hbar} \frac{2}{\partial v_{||}} \sum_{\gamma_i = 1}^{\infty} \left[ \frac{b_{\gamma} v_{||}}{\omega} \left( J_{i+1} \gamma_i - J_{i-1} \gamma_i \right) \right] f_{E_{||}}$$

(4.24)
where \( \kappa_{\text{II}} = \kappa + \kappa_{\text{II}} \)

As the plasma maser originates because of electron acceleration through the modulation electric field \( \mathbf{S} \mathbf{E}_{\text{th}} \), we have emitted the contribution of \( \mathbf{S} \mathbf{E}_{\text{th}} \) in Eqn. (4.24).

We get the similar expression for ions and then substitute into the Poisson equation for the mixed mode

\[
\nabla \cdot \mathbf{S} \mathbf{E}_{\text{th}}(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) = 4\pi \sum_{j} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) d\mathbf{r} \quad (4.25)
\]

Then, we obtain

\[
\mathbf{S} \mathbf{E}_{\text{th}}(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) = \frac{1}{\mathcal{R}} \left[ -\frac{4\pi e^2}{m} \right] \sum_{j} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) d\mathbf{r} \]

\[
\times \left\{ \exp[i(\mathbf{a} \cdot \mathbf{r})] \right\} \frac{2}{\mathcal{R}_{\text{th}}} \sum_{\mathbf{v}} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) d\mathbf{r} \]

\[
- \frac{4\pi e^2}{m} \sum_{j} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) \sum_{\mathbf{v}} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) \left\{ \exp[i(\mathbf{a} \cdot \mathbf{r})] \right\} \quad (4.26)
\]

where \( \mathcal{R} = (k_{\text{II}}) \sum_{j} \frac{4\pi e^2}{m} \sum_{\mathbf{v}} \mathbf{S} \mathbf{f}_j(\mathbf{k}, \mathbf{b}, \omega, \mathbf{r}) \left\{ \exp[i(\mathbf{a} \cdot \mathbf{r})] \right\} \]

with

\[
\mathbf{A} = \frac{k_{\text{II}} \mathbf{v}_j}{2\omega} \left( \mathbf{J}_{\text{th}, \omega} + \mathbf{J}_{\text{th}, -\omega} \right), \quad \mathbf{B} = -\frac{2k_{\text{II}} \mathbf{v}_j}{\kappa_{\text{II}}} \left( \mathbf{J}_{\text{th}, \omega} - \mathbf{J}_{\text{th}, -\omega} \right),
\]

\[
\mathbf{C} = \mathbf{J}_\omega - \frac{k_{\text{II}} \mathbf{v}_j}{2\omega} \left( \mathbf{J}_{\text{th}, \omega} + \mathbf{J}_{\text{th}, -\omega} \right)
\]
The high frequency nonlinear force $F_N$ acting on unit volume of electrons can be written as,

$$ F_N = e n_0 \int \left[ E_{\perp} \cdot \frac{2}{\hbar c} \nabla_{\perp} \cdot \frac{2}{\hbar c} \nabla_{\parallel} \right] \nabla_{\parallel} v \, dV, \quad (4.28) $$

where $n_0$ is the electron number density. Introducing Fourier transforms in space and time, we obtain from Eqn. (4.28)

$$ F_N(k, \omega) = e n_0 \int \left[ E_{\perp}(k, \omega) \frac{2}{\hbar c} \nabla_{\perp} \cdot \nabla_{\parallel} \right] \nabla_{\parallel} v \, dV \quad (4.29) $$

Assuming that $\omega \gg \omega$, $k \gg k_{\parallel}$, and keeping only $\gamma = 0$ in $\sum_{\gamma} J_{\gamma}(x)$ and because $\omega \ll \omega_e$ and $J_{\gamma}(x) \approx 1$ for $x \approx 0, x = k_{\parallel} u_{\parallel}/\omega_e$, the dominant contribution comes from the direct coupling term. Therefore we can write Eqn. (4.24) as

$$ \sum_{\gamma} J_{\gamma}(k, \omega) \approx \frac{1}{\gamma} \omega \cdot \frac{1}{\omega_e} \frac{S_{E_{\parallel}(k, \omega)}}{\nu_{\parallel}(k, \omega)} \left( \frac{\nu_{\parallel}(k, \omega)}{\nu_{\parallel}(k, \omega)} \right)^{4.30} $$

Using Eqns. (4.6) and (4.30) in Eqn. (4.29) we obtain the imaginary part as,
In obtaining the above equation, we have replaced \((\omega - k_{\parallel}v_e - i\omega)^{-1}\) by \(\pi i \mathcal{S}(\omega - k_{\parallel}v_e)\). We have assumed that \(f_{oe}(v_e) = (m/2\pi T_{\parallel e})^{3/2} \exp(-mv_{\parallel}^2/2T_{\parallel e})\), \(\omega/\pi^2 \gg k_{\parallel}^2/\kappa\) and \(\omega = k_{\parallel}v_{A}\).

Here \(v_e\) and \(v_A\) are respectively the electron thermal velocity and Alfven velocity. \(T_{\parallel e}\) is the electron temperature parallel to the magnetic field.

### 4.3 Growth rate of ion acoustic wave:

We shall now obtain the dispersion relation for the ion acoustic wave and then calculate its growth rate. The electron momentum equation with the nonlinear force term as defined in Eqn. (4.28) can be written as (Jackel et al., 1981)

\[
\mathbf{E} \cdot \mathbf{V} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\mathbf{e}(\mathbf{E} + \mathbf{V} \times \mathbf{B}) - \nabla p + \mathbf{F}(4.32)
\]

Linearizing the above equation, and taking the z component, we have,
\[ -i \Omega m n \nu_i = -e \eta_s E_h - i k T \eta_i + \int \eta S \eta_h \quad (4.33) \]

where \( \eta = F_n(k, \Omega) / \eta_s E_h \) and \( k \) is the Boltzmann constant.

From the linearized ion fluid equation, the ion velocity can be written as

\[ \nu_i = \frac{e \eta_s E_h}{-i \Omega M} \quad (4.34) \]

where \( M \) is the mass of the ion.

Using Eqn. (4.34) in Eqn. (4.33), we obtain

\[ -M \nu_i = e \eta_s E_h / i \Omega + \int \eta S \eta_h / (-\Omega \eta_i) \quad (4.35) \]

From the ion continuity equation and the Boltzmann distribution for electrons, we obtain from Eqn. (4.34), the dispersion relation for ion acoustic wave in the presence of ion cyclotron wave as

\[ \Omega^2 = \frac{k^2 T}{M} - \frac{i \int \eta S}{\eta_i} \frac{k^2 T}{M} \quad (4.36) \]

The above equation shows that the ion acoustic instability occurs due to the second term in Eqn. (4.36).

We put \( \Omega = \Omega_r + i \Omega_i \) where \( \Omega_r \) is the real part
of the frequency and $\gamma$ is the growth rate. Neglecting the nonlinear frequency shift, we get

$$\Omega_{\gamma} = \frac{K \sqrt{\tau}}{T/M}$$

and

$$\gamma = - \frac{f}{\varepsilon e \gamma_0} \frac{K^2}{M}$$

where $k_e$ is the electron Debye wave number.

$$\frac{\gamma}{\Omega} = - \frac{f}{\varepsilon e n_e} \frac{\sqrt{\tau}}{z} \left( \frac{k_e}{k_i} \right) \left( \frac{\nu_e}{\nu_i} \right) \chi \frac{|E_{lx}(\vec{B}_0)|^7 \exp[-\nu_i^2/\nu_e^2]}{4\lambda n_0 T_i}$$

where $k_e$ is the electron Debye wave number.

4.4 Discussion and Conclusion:

We have thus shown that the high frequency nonlinear force as given by Eqn. (4.29) drives the ion acoustic instability in a magnetized plasma in the presence of a coherent ion cyclotron wave. The growth rate of the ion acoustic instability is given by Eqn. (4.38).

A new type of magnetospheric electrostatic wave is reported in this paper. These waves have a wide
spectrum, extending between the lower hybrid frequency ($\omega_{lh}$) and ion plasma frequency ($\omega_{pi}$). At least two factors seem to control the existence of these events, the amplitude of electromagnetic ion cyclotron wave ($E_z$) and the plasma density ($N$). According to observation there exists first a electromagnetic ion cyclotron wave which modulates the amplification condition of electrostatic ion acoustic wave.

We now apply the result of our investigation to the observation of the electrostatic ELF waves. Taking the following plasma parameters (Nambu et al., 1986, Cornilleau-Wehrlin, 1981)

\[
\frac{K_{||}}{K_{\perp}} = 2.5 \times 10^{-7}, \quad \frac{K}{k_e} = 3.3 \times 10^{-3}, \quad E_z = 10 \text{ kV/m}
\]

$\gamma \approx 3 \text{ eV}$ and $\nu_e \approx \nu_A$, the growth rate of ion acoustic wave reduces to, from Eqn. (4.38), $\gamma/\omega \approx 2 \times 10^{-3}$. The growth rate is large enough to explain the observed value. Although we have considered the parallel propagation of ion acoustic waves in our analysis, the wave characteristics as inferred from the observation (Cornilleau-Wehrlin, 1981) is oblique to local magnetic field. The critical amplitude of a coherent quasi-electrostatic ion-cyclotron wave
for isothermal plasma \( T_{\text{He}} = T_{\text{ni}} \)

\[
\left( \frac{E_{\text{LZ}}}{4 \pi N T_{\text{He}}} \right)_{\text{C}} = 1.8 \times 10^{-10} \quad (4.39)
\]

Fig. 4.1 is a plot of the critical amplitude \( E_{\text{LZ}} \), in unit of \( \mu \text{V/m} \) VS electron number density \( N \), in unit of \( \text{cm}^{-3} \). The generations of ion-acoustic wave occurs above the solid line. Equation (4.39) predicts that the strong amplitude of \( E_{\text{LZ}} \) and small value of \( N \) favour the instability which are consistent with the observation (Cornilleau-Wehrlin, 1981). Further the typical values \( E_{\text{LZ}} = 10 \mu \text{V/m} \), \( N = 4 \times 10^{3} \text{cm}^{-3} \), \( T_{\text{e}} = 2 \text{kV} \) satisfy the instability condition.

In Fig. 4.1, we have assumed the ELF waves as the ion-acoustic mode with \( T_{\text{o}} = T_{\text{i}} \). Strictly speaking, the above condition is not valid for a homogeneous plasma considered here. However, the real space plasma is spatially inhomogeneous. In an inhomogeneous plasma, the obliquely propagating ion acoustic mode (drift sound wave) coupled with the inhomogeneity exists even for \( T_{\text{i}} < T_{\text{e}} \). Accordingly we cannot rule out the conditions for \( T_{\text{e}} = T_{\text{i}} \) for ion acoustic mode in space plasma.
The new maser effect predicts (Nambu, 1983) a particular phase relation between a pump field which is in this case is the electromagnetic ion cyclotron wave and the high frequency emission which is in this case is the electrostatic ion acoustic wave. The first two curves in Fig. 4.2 show the electric field \(E_\parallel\) and the potential \(-\varepsilon \mathcal{E}_\parallel\) seen by the electrons. The third and fourth curves are a plot of the non-resonant and resonant electron density perturbation due to a coherent low frequency pump wave \((E_\perp)\). We see easily that the resonant electrons are rich in density for the potential energy minimum. The resonant electrons are necessary to change the Doppler shifted frequency of the high frequency non-resonant field. Accordingly, the expected high frequency bursts due to new maser effect has a close correlation in phase with resonant electrons number density. The bottom curve in figure represents the high frequency radiation field VS phase \((k_x - \omega t)\). The observation by Cornilleau-Wehrlin (1981) also suggest such a phase relation between the low frequency ULF wave and the high frequency ELF wave.

The growth rate is obtained for the Maxwell distribution function of electrons. It suggests that this maser effect is possible without the population
inversion of electrons. If we consider the electron beam along the magnetic field, then the growth rate is much enhanced. Furthermore, this does not contradict with the observation (Norris et al., 1983) that thermal (1 eV) electrons are accelerated along the field lines (upto tens of eV) when intense electromagnetic ion cyclotron waves are simultaneously present. Because, the Landau interaction between thermal electrons and a coherent ion cyclotron wave causes the acceleration of the thermal electrons through the new maser effect. Accordingly, as a result of instability, the electrons are accelerated along the field lines which are observed (Norris et al., 1983).

We therefore, suggest that emission of electrostatic ion acoustic wave in the presence of an electromagnetic ion cyclotron wave takes place because of plasma maser interaction through a high frequency nonlinear force discussed here. Thus, we can say that observations are consistent with the hypothesis that ELF emissions are generated through the Landau interaction between resonant electrons and intense PCI ion cyclotron waves. Finally observation suggest that the low frequency waves are helium ion cyclotron waves. Therefore it is necessary to include the helium component in our calculations.
REFERENCES :


Fig. 4.1 The critical EM ion-cyclotron wave amplitude $E_{zz}$ (μV/m) vs electron number density $N$ (cm$^{-3}$) for $T_e = 3$ eV and $T_e/T_i = 1$. The electrostatic ion-acoustic wave becomes unstable above the solid line.
Fig. 4.2 Phase relation of density for electrons in a low-frequency pump field. $E_l$ and $\varphi_l$ are the electric field and the potential of the low-frequency wave. $n_{\text{nonres}}$ and $n_{\text{res}}$ show the number density for the nonresonant electrons and resonant electrons. $\delta E_h$ is the high frequency radiation field. The high frequency radiation occurs at the particular phase due to the plasma maser instability.