6.0 MODEL COMPUTATION OF IRREGULARITY PARAMETERS FROM SCINTILLATION OBSERVATIONS:

6.1 Introduction:

The proper interpretation of radio wave scintillation observations require deduction of irregularity structure in the scattering or diffraction medium from the observed amplitude fluctuations at the observing plane. Over the past few decades, a detailed theory has been developed by introducing an equivalent phase screen model which successfully reproduces the observed scintillation characteristics. This is particularly true for weak \((S_i < 0.5)\) scintillations. Phase screen models have been used since the early development of the theory (Booker et al., 1950; Bramely, 1954; Mercier, 1962; Briggs and Farkin, 1963; Saito, 1967; Singleton, 1970; Uscinski et al., 1981). Employing spatial correlation approach, Briggs and Farkin (1965) developed a diffraction based theory assuming a gaussian spectrum of the irregularities. But the theory was found inadequate to explain the observed scintillations over a wide range of frequency. In subsequent years through multi-technique observations including in-situ measurements by satellite and rocket flown probes, the power-law nature of the irregularity spectrum has been established. The irregularity and scintillation models (Fremouw and Mino, 1973; Iope, 1974) based on a gaussian irregularity spectrum were suitable for estimating the r.m.s. level in received signal strength at meter wavelengths only but misleading results were obtained when it was extended to shorter wavelengths. A more realistic approach using a three dimensional power-law description of the irregularities (Rufenach, 1975) produced good results. Even then to explain
the unexpectedly strong scintillations observed at microwave frequencies (Taur, 1973; Aarons et al., 1983) in the equatorial region, certain adjustments like spectral hump near the Fresnel dimension (Wernik and Liu, 1974) or assuming the outer scale dimension very close to the Fresnel dimension (Rufenach, 1975) were suggested. Recently Rino (1982) brought in excellent agreement between derived scintillation features and experimental data from the wideband satellite campaign (Fremouw et al., 1978) over several coherent frequencies and he suggested a varying power-law index. This means multiple power-law regimes rather than rigid cut-offs encountered in constant index form of irregularity spectrum.

In this chapter an attempt is made to derive irregularity parameters from the observed scintillation characteristics within the framework of phase screen theory with a three dimensional power-law irregularity description.

6.2 Phase screen theory: Preliminaries

The irregular plasma structure in the scattering medium imposes a random phase on the incident plane wave and that produces an interference pattern as the wave travels away from the irregular layer. This interference pattern is usually observed at a single fixed point on the observer's plane as a temporal fluctuation in intensity since a component of the irregularity motion is transverse to the raypath. Here it is assumed that an irregular structure moves as a rigid formation.

A typical scintillation geometry may be visualised as shown in Fig. 47a (Rufenach, 1975). An equivalent thin phase screen can represent the situation by assuming a negligible thickness which may be depicted as in Fig. 47b. This assumption amounts to projec-
Fig. 47: Geometry of a scintillation observation (a) and concept of a thin phase screen (b).
Fig. 48: Model of irregularity spectral form.
ting the irregularities onto the thin screen.

To derive the irregularity parameters, a power-law spectral form as shown in Fig. 48 is assumed in preference to a Gaussian form. The power-law controlled irregularity description has been successfully used by many workers (Rufenach, 1975; Basu et al., 1976; Yeh et al., 1975; Kino, 1982) to explain the observed scintillation characteristics throughout a wide frequency range. Again the Gaussian form can explain only VHF scintillations at meter wavelengths i.e. corresponding to a Fresnel dimension of about one Km (Ratcliffe, 1956; Briggs and Parkin, 1963). Fig. 48 depicts the model three dimensional power-law irregularity form based on in-situ measurements (Dyson et al., 1974; Phelps and Sagalyn, 1976). Here a three dimensional constant spectral index has been assumed. However, recent in-situ measurements as well as wideband scintillation observations suggest a varying spectral index leading to multiple power-law regimes (Livingston et al., 1981). The spatial wave number $K_r$ is resultant of three components $K_x, K_y$ and $K_z$ along the three respective directions. The spatial period or irregularity dimension is given by $L_r = 2\pi/K_r$. From the figure it is seen that for the region $K_r < K_0$, spectral density $P_N(K_r)$ is constant but it starts falling monotonically through

$$P_N(K_r) \propto K_r^{-p} \quad \text{for} \quad K_r > K_0,$$

where $K_0$ is the outer scale wave number. Spectral index $p$ determines the roll off rate in the region from $K_0$ to $K_i$ (the intrinsic inner scale wave number). For $K_r > K_i$ the density falls more rapidly as shown in the figure by broken line but this region can be ignored considering the fact that $K_f$, the Fresnel wave number is much smaller than $K_i$ or alternately, Fresnel dimension $L_f$ is
orders of magnitude greater than the inner scale dimension $L_i$ even in the GHz range (Yeh et al., 1975). Thus ignoring the inner scale dimension in scintillation studies particularly in the VHF region as in the present case is not going to influence the result to significant level. It is recalled that most contribution to scintillations come from irregularities having dimension close to $L_f$. So spectral density function follows the model form shown in the figure (Fig. 48) as a solid line.

Rufenach (1975) has shown that, for such an irregularity spectrum, scintillation index may be given by

$$S_h = 2^\frac{1}{2} \varphi_0 F(f(\beta)) \text{ for } S_h < 0.5 \ldots (1)$$

where $\varphi_0 = \text{phase deviation}$

$$\varphi_0 = \left[ \pi^\frac{1}{2}(r_e \lambda) N_0 (L_e \alpha)^\frac{1}{2} \right] (\text{Sec } \chi)^\frac{1}{2}$$

$r_e$ = classical electron radius

$\lambda$ = radio wavelength

$L_e$ = thickness of the irregularity layer

$\chi$ = ionospheric zenith angle

$F$ = power-law Fresnel filter function for isotropic irregularities

$$F = \left[ 1 - \exp(-u) \right]^{\frac{1}{2}}$$

$u = (\lambda Z/2 \pi)K_0^2 = 2K_0^2/K_f^2$

$K_0$ = outer scale wave number

$K_f$ = Fresnel wave number

$Z$ = mean height of the irregularity layer from earth surface
\[ f(\beta) = \text{geometric factor for anisotropic irregularities} \]
\[ = \frac{1}{2} (2)^{\frac{1}{2}} (3\beta^4 + 2\beta^2 + 3)^{\frac{1}{2}} \]
\[ \beta = \text{effective axial ratio of the irregularities} \]
\[ = (\cos^2 \gamma + \alpha^2 \sin^2 \gamma)^{\frac{1}{2}} \]
\[ \alpha = \text{axial ratio of the irregularities} \]
\[ \gamma = \text{angle between the raypath and magnetic field} \]
\[ N_0 = \text{r.m.s. electron density deviation.} \]

For \( K_0 \ll K_f/2^{\frac{1}{2}} \), it has been shown by Hufenach (1975) that \( f(\beta) \) becomes independent of \( \beta \) provided \( \beta > 5 \), but significant filtering effect may be observed when \( \beta < 5 \). In the model computations it has been assumed that \( \alpha = 5 \) i.e. moderately elongated irregularities. It is believed to be reasonable considering available reports (Sinclair and Kelleher, 1969; Bhar et al., 1970; Weber et al., 1980; Tsunoda, 1980b). Due to relatively low angle between the raypath and magnetic field lines for the present configuration effective axial ratio is considerably reduced. But even then geomagnetic factor becomes significant in equation (1) as will be seen.

6.3 Model computations:

In order to have an idea about the magnitude of the irregularities over this region a sample data from in-situ measurements made by OGO-6 is considered here. RPA probe onboard OGO-6 showed on 19 November, 1969, an electron density deviation greater than \( 10^8 \text{ el. m}^{-3} \) while the satellite was passing over this region (orbit 2414/5, geog. long. 91^\circ E) at about 2100 h.t. Background density was about \( 10^{10} \text{ el. m}^{-3} \) (Basu et al., 1976) at that time.
It is to be noted that the sampling was done in winter i.e. low scintillation activity period and solar activity was also relatively low compared to present observation period. So higher irregularity amplitude may also be considered for computations considering the higher background ionization density during present observation period. With that typical irregularity amplitude $N_0$

Table 2: Model computations with $\lambda = 2.2$ m, $z = 350$ Km, $\alpha = 5$, $\gamma = 44^0$, $L_e = 100$ Km. Asterik mark indicates Fresnel dimension and Fresnel wave number.

<table>
<thead>
<tr>
<th>$L_0$ Km</th>
<th>$K_0$ Km$^{-1}$</th>
<th>$N_0$ el.m$^{-3}$</th>
<th>$\phi_0$ rad</th>
<th>$F$</th>
<th>$S_4$ fade depth peak to peak dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.062</td>
<td>$10^8$</td>
<td>0.009</td>
<td>0.021</td>
<td>0.008 -</td>
</tr>
<tr>
<td>12.6</td>
<td>0.05</td>
<td>$10^6$</td>
<td>0.032</td>
<td>0.173</td>
<td>0.023 -</td>
</tr>
<tr>
<td>100</td>
<td>0.062</td>
<td>$10^9$</td>
<td>0.91</td>
<td>0.021</td>
<td>0.08 1.1</td>
</tr>
<tr>
<td>20</td>
<td>0.31</td>
<td>$10^9$</td>
<td>0.41</td>
<td>0.107</td>
<td>0.18 3.0</td>
</tr>
<tr>
<td>12.6</td>
<td>0.50</td>
<td>$10^9$</td>
<td>0.32</td>
<td>0.173</td>
<td>0.23 4.3</td>
</tr>
<tr>
<td>6.8</td>
<td>1.00</td>
<td>$10^9$</td>
<td>0.22</td>
<td>0.338</td>
<td>0.31 6.2</td>
</tr>
<tr>
<td>4.2</td>
<td>1.50</td>
<td>$10^9$</td>
<td>0.18</td>
<td>0.489</td>
<td>0.37 7.8</td>
</tr>
<tr>
<td>3.14</td>
<td>2.00</td>
<td>$10^9$</td>
<td>0.16</td>
<td>0.621</td>
<td>0.42 8.8</td>
</tr>
<tr>
<td>2.51</td>
<td>2.50</td>
<td>$10^9$</td>
<td>0.14</td>
<td>0.730</td>
<td>0.43 9.1</td>
</tr>
<tr>
<td>2.09</td>
<td>3.00</td>
<td>$10^9$</td>
<td>0.13</td>
<td>0.816</td>
<td>0.45 9.5</td>
</tr>
<tr>
<td>1.55</td>
<td>4.05*</td>
<td>$10^9$</td>
<td>0.11</td>
<td>0.907</td>
<td>0.42 8.8</td>
</tr>
</tbody>
</table>

$= 10^8$ el.m$^{-3}$ the observed level of scintillation (at 136 MHz) will be $S_4 = 0.008$ as may be seen from Table 2 when outer scale dimension is 100 Km. This level of scintillation is barely detectable. Whereas reducing the outer scale dimension to a more reasonable
value, $L_0 = 12.6 \text{ km}$ (which is still very much larger than $L_f$) can produce scintillation level $S_4 = 0.02$, a typical weak scintillation event observed in a November evening. In these computations Fresnel dimension corresponds to a mean height of 350 km with irregularity layer thickness $L_e = 100 \text{ km}$. A realistic irregularity amplitude $h_0 = 10^9 \text{ el. m}^{-3}$ can produce almost all scintillation levels which were observed during present investigation once a suitable outer scale dimension is selected. From the table it is observed that with the same electron density deviation ($10^9 \text{ el. m}^{-3}$) scintillation levels from $S_4 = 0.08$ to $S_4 = 0.45$ can be explained with a range of outer scale dimension, from $L_0 = 100 \text{ km}$ to $2.09 \text{ km}$. But it must be pointed out that choosing outer scale dimension so close to the Fresnel dimension $L_f$ (about 1.5 km) to explain the relatively strong scintillations ($S_4 > 0.30$) with peak to peak fade depth about 6 dB, essentially indicates a Gaussian character of the irregularity spectrum. In such cases the difference between the scintillation levels predicted by a Gaussian spectrum becomes very small with that predicted by a power-law controlled spectrum for a given configuration. It is recalled that at this station strong scintillations with $S_4 > 0.30$ are rarely observed, thus it is possible to accommodate the value of $L_0$ in the range 10-20 km which is well within in-situ measurement's value (Dyson et al., 1974; Phelps and Sagalyn, 1974; Livingston et al., 1981).

A study of Table 2 shows that the most probable value of $L_0$ is about 12 km which can explain the most frequently observed scintillations in summer with $S_4 = 0.20$ (i.e. S.I. about 40 p.c.) or peak to peak fade depth about 4 dB, over this region. This is noted from Table 2 that $N_0$ remaining constant, the r.m.s. phase deviation $Q_0$ decreases along with outer scale dimension but the Fresnel
filter factor $F$ increases much more rapidly so much so that the net effect is increase in $S_4$. In the present configuration the geometric factor $f(\beta)$ is about 3 for $\alpha = 5$ and $\gamma = 4^0$. Thus it has a significant value compared to an assumption $\alpha = 1$ i.e. isotropic irregularities. It is highly improbable to expect the irregularities to be isotropic with axial ratio unity keeping in view of the various multitechnique observations made by many workers as already mentioned. To evaluate the significance of $f(\beta)$, let $\alpha$ be unity. For such an irregularity to produce a scintillation level $S_4 = 0.20$ the electron density deviation will have to be $3.2 \times 10^9$ el m$^{-3}$ for an outer scale dimension $L_o = 20$ Km. This amounts to as high as 30 p.c. of the background value which is unrealistic. But for $\alpha = 5$ a very probable value, to produce the same level of scintillation only about $10^9$ el. m$^{-3}$ is predicted in the same configuration. This amount of electron density deviation is more realistic as it normally constitutes about 10 p.c. of the ambient value. Again an irregularity amplitude corresponding to $10^{10}$ el. m$^{-3}$ which is typical in the African and Atlantic sector is supposed to give rise to $S_4 > 1.0$ i.e. peak to peak fade depth greater than 20 dB in this present configuration. But observations seldom showed any scintillation greater than 9.5 dB fade level. Thus such high irregularity amplitude is unrealistic over this region, a fact in conformity with OGO-6 in-situ measurements.

In the model computations no gaussian spectrum is considered for deriving irregularity parameters as it led to inconsistent results. For example the stronger Fresnel filtering function in a gaussian spectrum (Rufenach, 1975) predicts unrealistically higher electron density deviations compared to a power-law form.

* For $S_4 > 0.5$ thin phase screen theory is invalid.
for a given level of scintillation. Quantitatively a gaussian spectrum predicts a deviation $N_0 = 7 \times 10^9 \text{el. m}^{-3}$ against a corresponding power-law projection of about $10^9 \text{el. m}^{-3}$ for $L_0 = 20 \text{Km}$ and $S_4 = 0.20$. Likewise it has been observed that a three dimensional power-law form of irregularity spectrum can successfully explain the scintillations observed most of the time within the framework of phase screen theory accommodating a realistic set of irregularity parameters. With these prior observational facts about typical irregularities over this region, expected levels of scintillations are predicted in two higher frequencies - in UHF and GHz range keeping the other parameters similar to the present one (Table 3).

Table 3: Predicted scintillation levels for 300 MHz and 1 GHz radio links as a function of outer scale dimension.

Other parameters are $Z = 350 \text{Km}$, $L_e = 100 \text{Km}$, $\chi = 44^\circ$, $\alpha = 5$, $\phi = 52^\circ$, $N_0 = 10^9 \text{el. m}^{-3}$.

<table>
<thead>
<tr>
<th>$L_0$ (Km)</th>
<th>$K_0$ (Km$^{-1}$)</th>
<th>$\Phi_0$ (rad)</th>
<th>300 MHz</th>
<th>1 GHz</th>
<th>300 MHz</th>
<th>1 GHz</th>
<th>$S_4$ (300 MHz)</th>
<th>$S_4$ (1 GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>0.62</td>
<td>0.12</td>
<td>0.038</td>
<td></td>
<td>0.155</td>
<td>0.085</td>
<td>0.06</td>
<td>0.013</td>
</tr>
<tr>
<td>6.28</td>
<td>1.0</td>
<td>0.10</td>
<td>0.030</td>
<td></td>
<td>0.24</td>
<td>0.137</td>
<td>0.10</td>
<td>0.017</td>
</tr>
<tr>
<td>4.2</td>
<td>1.5</td>
<td>0.08</td>
<td>0.024</td>
<td></td>
<td>0.36</td>
<td>0.204</td>
<td>0.12</td>
<td>0.020</td>
</tr>
<tr>
<td>3.14</td>
<td>2.0</td>
<td>0.07</td>
<td>0.021</td>
<td></td>
<td>0.47</td>
<td>0.270</td>
<td>0.14</td>
<td>0.024</td>
</tr>
<tr>
<td>2.51</td>
<td>2.5</td>
<td>0.06</td>
<td>0.018</td>
<td></td>
<td>0.57</td>
<td>0.334</td>
<td>0.145</td>
<td>0.025</td>
</tr>
<tr>
<td>2.09</td>
<td>3.0</td>
<td>0.05</td>
<td>0.017</td>
<td></td>
<td>0.66</td>
<td>0.396</td>
<td>0.13</td>
<td>0.028</td>
</tr>
</tbody>
</table>

In these computations the mean irregularity height is assumed well above the average peak density F-layer height (about 300 Km). This is considered reasonable since it has already
been established that most of the irregularities which give rise to scintillations lie above the peak density F-region. A study of Table 3 shows that even reasonably high electron density deviation \( N_0 = 10^9 \text{ el. m}^{-3} \) and at oblique incidence (\( \chi = 52^\circ \)), will produce weak or moderate scintillations with \( S_4 < 0.20 \) for outer scale dimension only 2 Km. But choosing \( L_0 \) so close to the Fresnel dimension (1.12 Km for 300 KHz) is contrary to a wide band power-law description of the irregularities with a constant spectral index. Most of the in-situ measurements put the outer scale dimension in the range 10 to 100 Km. Thus in all probability even the highest scintillation levels (which are only \( S_4 \leq 0.15 \)) predicted will not be observed leaving out the extraordinary exceptions (e.g. multiple scattering, multiple power-law regimes). The predicted scintillation levels at 1 GHz will be very difficult to observe as fade depth corresponding to \( S_4 = 0.01 \) can just be detected above the system noise level. It is most unlikely that scintillation level at GHz range will ever rise to the extent of degrading the satellite - ground communication links over this region. A note of caution may however be raised here about the possibility of multiple power-law regimes dictated by a varying spectral index. Such a situation might reverse the normal frequency law \( S_4 \propto f^{-n} \) giving rise to unexpectedly high scintillation levels at higher frequencies. Such cases have been reported from the equatorial region (Deshpande et al., 1978). Due to single frequency observation in the present study, it was not possible to examine whether such frequency reversal of scintillation level is a possibility at all over this region.
6.4 Discussion:

It has been noted that the model computations based on thin phase screen theory with a three dimensional power-law description of the irregularity spectrum can successfully explain the observed scintillation characteristics most of the time. This has largely been possible due to weak to moderate scintillations observed. Except for few rare occasions (e.g. during great magnetic storms) when scintillations with $S_\gamma > 0.45$ (about 9.5 dB) were observed, thin phase screen model is applicable in this region. However, in absence of any simultaneous in-situ measurements along with the ground based scintillation data, it has not been possible to ascertain if assumed irregularity parameters are realistic.

The assumed irregularity amplitudes are based on in-situ measurements by OGO-6 (Basu et al., 1976), during a high solar activity period similar to the present study. OGO-6 probes showed presence of irregularities with amplitudes as high as $3 \times 10^{11}$ el. m$^{-3}$ rather frequently over African and Atlantic sectors while in the Indian sector such high deviations were barely observed. The present study also does not indicate the presence of such irregularity at least over this region.

Recent campaigns to measure the spectral density function (SDF) by rocket flown probes (Szuszczewicz, et al., 1980; Rino et al., 1981; Kelley et al., 1982) indicate the presence of irregularities having outer scale dimension about 50 Km and inner scale dimension about 500 m. This is indeed a widespread characteristic of the irregularity spectrum with a low one dimensional SDF ($p \approx 1.5$). But spectral analysis of ground based scintillation data (Rufenach, 1975) indicated a small outer scale dimension (about 3 Km) which is believed to be an artifact of strong Fresnel filtering. In any case
outer scale dimension cannot be measured directly from such single
frequency ground based scintillation data as outer scale dimension
is orders of magnitude greater than the Fresnel dimension, since
most of the contributions to scintillations come from irregularit-
ies having dimensions close to Fresnel dimension (Wernik and Liu,
1974). Livingston et al. (1981) have observed from satellite(AE-E)
data a varying SDF of the irregularities. The physical implicati-
ons of such a varying SDF will be multiple power-law regimes rath-
er than rigid cut offs contrary to a constant index form. Such a
situation manifests itself in high level of scintillations even
in GHz range (Rino, 1982). In such cases the prediction of scinti-
lation levels as shown in Table 3, particularly in the GHz range
will have to be suitably modified. Even though in the model compu-
tations a gaussian spectrum is avoided in preference to power-law
one (for being more realistic), this is not to deny the possibili-
ty of a gaussian character of the irregularities over this region.
This is interesting to note that, to explain the occasional observ-
ation of strong scintillations ($S_4 > 0.4$) at 136 MHz with a feas-
ible irregularity amplitude ($\approx 10^9$ el. m$^{-3}$), outer scale dimensions
have to be selected very close to the Fresnel dimension. This fact
may be a pointer towards the presence of a gaussian irregularity
spectrum on such occasions unless a varying spectral index is int-
roduced retaining the power-law characteristics.