CHAPTER - II

INTERDEPENDENCE OF SECTORAL OUTPUT BETWEEN ASSAM AND THE REST OF INDIA - THE MODEL.

2.1 Introduction:

It appears from the last two sections of the previous chapter that there exists a close trading inter-dependence as well as interdependence of prices between Assam and the rest of India. As has been mentioned in the preface that the main thrust of this thesis is to investigate the nature and magnitude of the interdependence of sectoral output and prices between Assam and the rest of India with the help of two separate models based on inter-regional input-output system. In this chapter, we make an attempt to build up the theoretical model of interdependence of sectoral output.

The trading interdependence between these two regions is such that a change in production level in a particular sector in one region will have direct and indirect effects in the activity levels of the other. Or an increase in the final demand of a particular commodity in one region will produce some direct and indirect effects on the different activity levels of the other. These effects have been shown here with the help of the model originally developed by R.J. Wonnacott and has been suitably modified in our study to analyse the inter-dependence of sectoral output between Assam and the rest of India.

The study includes how the trade flow of a particular commodity in one of the regions affects the production levels and other activities both in Assam and rest of India. Suppose the export of a commodity of Assam to the rest of India increases. This increase in export will create some direct and indirect effects both in Assam and rest of India. The direct impact is that output and employment of this sector will expand i.e. investment will rise. But there will have some indirect effects also. The expansion of this sector will draw additional supplies of goods from other sectors from both Assam and the rest of India due to the existence of feedback effects in an inter-regional input-output system. Thus, expansion of output of a particular sector of Assam is transmitted throughout the entire Indian economy by an increase in the output of all the industries which either directly or indirectly supply it.

2.2. Methodology of Inter-regional Input-Output Model

As mentioned earlier, our analysis of interdependence of sectoral output between Assam and the rest of India is based on input-output system i.e. the study is based on inter-regional input-output model. Inter-regional flow studies attempt to show the structural relations between regions in a quantifiable manner. The effects of an autonomous shock in a particular sector in one region may be traced to and through different sectors of the different regions under consideration.

2. Here 'export' means outflow of goods from Assam to the rest of India as this is an inter-industry flow study of Assam and the rest of India.
The basic objective of an inter-regional input-output analysis is nothing but a matrix of input-output coefficients related not only to industry but also to regions. The inter-regional input-output model was first suggested by W.W. Leontief, and later extended by Isaïd, Moses, Tiebout and others.

Leontief assumed 'n' regions and each region produces 'm' goods of two types - the first 'h' being regional and the last '(n - h)' being national. The former are consumed within the region and the latter are traded between the regions. In such a 'n' regional model, the structure of the system is determined by two sets of parameters such as -

\[ a_{ij} = \frac{\text{Consumption of good } i \text{ by industry } j}{\text{Total output of } j\text{th industry}} \]

\( (i, j = 1, 2 \ldots m) \)


which gives a matrix of technical input coefficients describing production in each region and \([f_{1g}]\) which is the proportion of national commodity 'g' produced by region 'f' where \((f = 1, 2 \ldots n, g = h + 1, h + 2 \ldots m)\) with a given final demand in each region \(f_{2i}\)

where \((f = 1, 2 \ldots n, i = 1, 2 \ldots m)\)

The determination of output level of both national and regional commodities in each region requires the construction of total demand by nation as a whole for sector \(i\) such that

\[
z_i = \sum_{f=1}^{n} f_{2i} \quad \ldots \ldots \ldots \ldots (1)
\]

\((\text{where } i = 1, 2 \ldots m)\)

Now, with the help of input coefficient matrix \([a_{ij}]\) we get 'm' equations

\[
x_i = \sum_{j=1}^{m} a_{ij} \cdot x_j = z_i \quad \ldots \ldots \ldots \ldots (2)
\]

\((i = 1, 2 \ldots m)\)

where '\(x_i\)' denotes output level of good 'i' for the nation as a whole. Thus total output of both types of goods - 'regional' and 'national' can now be determined by equation (2).

The trade coefficient determines the output of national good 'g' in region 'f' such that,

\[
f_{xy} = f_{1g} \cdot x_g \quad \ldots \ldots \ldots \ldots \ldots \ldots (3)
\]

\((\text{where, } f = 1, 2 \ldots n,\)

\(g = h + 1, h + 2 \ldots m)\)
where \([f_x g]\) denotes output of national goods 'g' produced in region 'f'. Thus regional production of national output is also known.

To solve the system of equations showing inter-regional dependence we are to finally determine regional outputs of regional goods. With the technical input coefficient matrix, the output of regional goods in the 'f'th region may be derived by the set of equations given by,

\[
f^k = \sum_{i=1}^{m} a_{ki} \cdot f^i = f^2_k \quad \cdots \cdots \cdots \quad (4)
\]

\(k = 1, 2, \ldots, n\)

where \([f^k x\)] is the production of regional good 'g' in region 'f'.

Thus the given structure of the system as defined by technical input coefficients \([a_{ij}]\) and the trade coefficients \([f^i g]\)
all output levels in the system are uniquely determined by the final demand of all the regions for all goods \([f^i z]\). Moreover, since the trade pattern of national goods is known, the balance of trade of a region and the rest of the nation is easily derived. Thus, the impact of changes in final demand on output of every producing sector in each region and on balance of trade of each region can be computed.

2.3 ISARD'S MODEL...

ISARD also did pioneering work in this field. If there are 'n' sectors and 'k' regions, it is ideal to have data in such a way that the deliveries of a given sector in a given region to each of the sectors in each of the regions could be obtained. Thus each of the 'n' sectors in each of the 'k' regions becomes a sector.
in its own right. This requires calculation of \(\binom{n+k}{2}\) elements showing inter-sectoral relationships. Isard developed a full fledged inter-regional model involving \(\binom{n+k}{2}\) inter-sectoral and intra-sectoral relationships. This model requires the most complete data and for the same reason it is the most difficult to empirically work out such a model in our situation where the data position is staggeringly poor.

2.4 Theoretical model of two regions - Assam and the rest of India

In this two region model, both the regions are divided into 'n' sectors or activities. A system of account is set up to show the flow of output from each sector to every other sector in both the regions. We define for theoretical formulation.

\[
\begin{align*}
aa_{ij} &= \text{value of output of sector 'i' of Assam consumed by sector 'j' of Assam.} \\
rr_{ij} &= \text{value of output of sector 'i' of the rest of India consumed by sector 'j' of the rest of India.} \\
rar_{ij} &= \text{value of output of sector 'i' of the rest of India consumed by sector 'j' of Assam.} \\
ar_{ij} &= \text{value of output of sector 'i' of Assam consumed by sector 'j' of the rest of India.} \\
rv_i &= \text{value of output of sector 'i' of Assam consumed by final demand sector of Assam.} \\
rv_i &= \text{value of output of sector 'i' of Assam consumed by final demand sector of the rest of India.} \\
rvi &= \text{value of output of sector 'i' of the rest of India consumed by final demand sector of Assam.} \\
rvi &= \text{value of output of sector 'i' of the rest of India consumed by final demand sector of the rest of India.}
\end{align*}
\]
The inter-industry flow of output between Assam and the rest of India has been shown by the flow table given below:

<table>
<thead>
<tr>
<th>Producing sectors of Assam</th>
<th>Consuming sectors in Assam</th>
<th>Consuming sectors in the rest of India</th>
<th>Assam's final demand</th>
<th>Final demand of the rest of India</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ $a_r^{x2}$ ... $a_r^{xn}$</td>
<td>$a_a^{v1}$</td>
<td>$a_r^{v1}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ $a_r^{x2}$ ... $a_r^{xn}$</td>
<td>$a_a^{v2}$</td>
<td>$a_r^{v2}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
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<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
</tr>
<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
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<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
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<tr>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_a^{x1}$ $a_a^{x2}$ ... $a_a^{xn}$</td>
<td>$a_r^{x1}$ ... $a_r^{nn}$</td>
<td>$a_a^{v}$</td>
<td>$a_r^{v}$</td>
</tr>
</tbody>
</table>
If the total output of \( \text{th} \) sector of Assam's industry is denoted by \( a_i \) \((i = 1, 2 \ldots n)\) and that rest of India is denoted by \( r_i \) \((i = 1, 2 \ldots n)\), then the accounting data of the above table can be expressed in the form of a set of '2n' equations:

\[
\begin{align*}
\begin{array}{cccccc}
a_1 & a_11 & a_12 & \cdots & a_1n & - a_1n1 & - a_1n2 & \cdots & - c_r1n = a_v1 + a_r1 \\
a_2 & a_21 & a_22 & \cdots & a_2n & - a_2n1 & - a_2n2 & \cdots & - c_r2n = a_v2 + a_r2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_n & a_n1 & a_n2 & \cdots & a_nn & - a_nn1 & - a_nn2 & \cdots & - a_rnn = a_vn + a_rn \\
r_1 & r_11 & r_12 & \cdots & r_1n & - r_1n1 & - r_1n2 & \cdots & - r_r1n = r_v1 + r_r1 \\
r_2 & r_21 & r_22 & \cdots & r_2n & - r_2n1 & - r_2n2 & \cdots & - r_r2n = r_v2 + r_r2 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\
r_n & r_n1 & r_n2 & \cdots & r_nn & - r_nn1 & - r_nn2 & \cdots & - r_rnn = r_vn + r_rn \\
\end{array}
\end{align*}
\]
Now defining,

\[ a_a^{ij} = \frac{a^{x_{ij}}}{a^{x_j}} = \text{Input of Assam's good 'i' consumed by Assam's industry 'j'} \]

\[ r_a^{ij} = \frac{r^{x_{ij}}}{r^{x_j}} = \text{Total value of output of Assam's industry 'j'} \]

\[ a_r^{ij} = \frac{a^{r_{x_{ij}}}}{r^{x_j}} = \text{Input of good 'i' in the rest of India consumed by the industry 'j' of the rest of India.} \]

\[ r_r^{ij} = \frac{r^{r_{x_{ij}}}}{r^{x_j}} = \text{Total value of output of industry 'j' in the rest of India} \]

\[ a_r^{ij} = \frac{a^{r_{x_{ij}}}}{a^{x_j}} = \text{Input of good 'i' produced in Assam used by industry 'j' of the rest of India} \]

\[ r_r^{ij} = \frac{r^{r_{x_{ij}}}}{a^{x_j}} = \text{Total value of output of industry 'j' in the rest of India} \]

\[ a_r^{ij} = \frac{r^{r_{x_{ij}}}}{a^{x_j}} = \text{Input of good 'i' produced in the rest of India used by industry 'j' of Assam} \]

\[ r_r^{ij} = \frac{a^{r_{x_{ij}}}}{a^{x_j}} = \text{Total value of output of industry 'j' of Assam} \]

\[ \text{........................................... (t)} \]
Using (6), the set of 2n equations given by (5) becomes:

\[
\begin{align*}
\begin{array}{c}
(a_{11} a_1 - a_1 a_{12} x_2 - \cdots - a_{1n} a_n - a_{11} r_1 - a_{12} r_2 - \cdots - a_{12} r_2 - \cdots - a_{2n} r_n = a_1 + a_1 v_1 \\
- a_{21} a_1 + (1 - a_{22}) a_2 - \cdots - a_{2n} a_n - a_{21} r_1 - a_{22} r_2 - \cdots - a_{22} r_2 - \cdots - a_{2n} r_n = a_2 + a_2 v_2 \\
\vdots \\
- a_{nn} a_n - a_{n1} a_1 - a_{n2} a_2 - \cdots - (1 - a_{nn} a_n - a_{nn} r_1 - a_{nn} r_2 - \cdots - a_{nn} r_n = a_n + a_n v_n \\
- r_{11} a_1 - r_{12} a_2 - \cdots - r_{1n} a_n - (1 - r_{11}) r_1 - r_{12} r_2 - \cdots - r_{12} r_2 - \cdots - r_{1n} r_n = r_1 + r_1 v_1 \\
- r_{21} a_1 - r_{22} a_2 - \cdots - r_{2n} a_n - r_{21} r_1 - (1 - r_{22}) r_2 - \cdots - r_{22} r_2 - \cdots - r_{2n} r_n = r_2 + r_2 v_2 \\
\vdots \\
- r_{nn} a_n - r_{nn} a_n - r_{nn} r_1 - r_{nn} r_2 - \cdots - (1 - r_{nn} r_n = r_n + r_n v_n \\
\end{array}
\end{align*}
\]
In matrix notation, the above system of equations can be written as,

\[
\begin{pmatrix}
(I-[a_a^{aij}]) & -[ar^{aij}]
\end{pmatrix}
\begin{pmatrix}
[a_x^j]
\end{pmatrix}
= 
\begin{pmatrix}
[a_a^{i1}] + [r_r^{i1}]
\end{pmatrix}
\]

\[
\begin{pmatrix}
-[r_a^{aij}]
\end{pmatrix}
\begin{pmatrix}
(I-[r_r^{aij}])
\end{pmatrix}
\begin{pmatrix}
[r_x^j]
\end{pmatrix}
= 
\begin{pmatrix}
[r_a^{i1}] + [r_r^{i1}]
\end{pmatrix}
\]

and solution is given by

\[
\begin{pmatrix}
[a_x^j]
\end{pmatrix}
= 
\begin{pmatrix}
(I-[a_a^{aij}]) & -[ar^{aij}]
\end{pmatrix}^{-1}
\begin{pmatrix}
[a_a^{i1}] + [r_r^{i1}]
\end{pmatrix}
\]

\[
\begin{pmatrix}
[r_x^j]
\end{pmatrix}
= 
\begin{pmatrix}
-[r_a^{aij}]
\end{pmatrix}
\begin{pmatrix}
(I-[r_r^{aij}])
\end{pmatrix}
\begin{pmatrix}
[r_a^{i1}] + [r_r^{i1}]
\end{pmatrix}
\]

The solution is further simplified by converting 
\([r_a^{aij}]'s\) and \([ar^{aij}]'s\) into the coefficients \([a_a^{aij}]'s\) and 
\([r_r^{aij}]'s\) by assuming new trace coefficients in the form of...
Total import of good 'i' of Assam from the rest of India

\[ a^i \] = \frac{\text{Total production of good 'i' of Assam}}{\text{Total import of the rest of India of good 'i' from Assam}}

\[ r^i \] = \frac{\text{Total production of good 'i' of the rest of India}}{\text{Total import of good 'i' of Assam}}

With these \( n + n = 2n \) trace coefficients we have,

\[
\begin{bmatrix}
a^{\alpha ij}
\end{bmatrix}
= \begin{bmatrix}
r^i \cdot r^\alpha ij
\end{bmatrix}
\]

\[
\begin{bmatrix}
r^\alpha ij
\end{bmatrix}
= \begin{bmatrix}
\delta^i \cdot a^{\alpha ij}
\end{bmatrix}
\]

\(( i = 1, 2 \ldots n \) \)

\(( j = 1, 2 \ldots n \) \)
Thus (9) becomes,

\[
\begin{bmatrix}
a_x^j \\
r_x^j
\end{bmatrix} = \begin{bmatrix}
(1 - [aa_{ij}]) & -[r_i \cdot r_{a_{ij}}] \\
-[a_{ij} \cdot aa_{ij}] & (1 - [rr_{a_{ij}}])
\end{bmatrix}^{-1} \begin{bmatrix}
 v_i + v_{1} \\
v_i - v_{1}
\end{bmatrix}
\]