INTRODUCTION

1.1 A history of development of the subject:

Hydromagnetic flow and heat transfer problems.

(a) Development of boundary layer theory:

Towards the end of the nineteenth century the science of fluid mechanics began to develop in two directions; one was theoretical hydrodynamics and the other was the science of hydraulics, which had practically no points in common. The theoretical hydrodynamics developed from solutions of Euler’s equation of motion along with the equation of continuity for various flow configurations of frictionless or non-viscous fluid (incompressible) flow past obstacles like plates, cylinders, spheres and through pipes and channels and against disks. However, the results of such studies did not agree with the experimental observations as regards to the pressure losses in pipes and channels, as well as with regard to the drag of the body which moves through a mass of fluid. The most glaring departure of the result of this subject from reality is that leading to d'Alembert's paradox, that is, to the statement that a body which moves uniformly through a fluid which extends to infinity experiences no drag whereas a body experiences a drag in moving through any real fluid. On the other hand, the science of hydraulics was mainly developed by the practical engineers by the need to solve some important problems arising out of progress in technology. It was based on a large number of experimental data and differed greatly in its methods and objects from theoretical hydrodynamics.
The equation of motion of a viscous or real fluid was established in the first half of the nineteenth century by Navier (1823), Poisson (1831), Saint-venant (1843) and Stokes (1845) and its components are known as Navier-Stokes equations. But these being non-linear partial differential equations, there exist only a few exact solutions of these equations for the cases where either the non-linear terms vanish automatically or when the equations can be reduced to ordinary differential equations by taking recourse to Laplace transform or some suitable similarity transformations. Stokes (1851) investigated the case of parallel flow past a sphere for the limiting case when the viscous forces are considerably greater than the inertia forces and so the non-linear terms in the Navier-Stokes equations are neglected. Oseen (1910) gave an improvement on the Stokes' solution by taking partly into account the inertia terms in these equations. However these types of solutions are valid for small Reynolds number which corresponds to slow motion. Such motions are often called creeping motions and do not occur often in practical applications. As a result, there was not much progress, till the beginning of twentieth century in dealing with the flow problems of real fluids by considering the full Navier-Stokes equations along with the no slip condition at a solid wall.

Prandtl, L (1904) propounded the boundary layer theory of fluid flows for large Reynolds number or small viscosity. This theory unified the two divergent branches of fluid dynamics, namely inviscid hydrodynamics and hydraulics; and gave quite agreeable results for drag on the solid body around which the
fluid moves as compared to the results obtained by stokes' method of neglecting the non-linear inertia terms in the Navier-Stokes equations. He achieved a high degree of correlation between theory and experiment and paved the way to the remarkably successful development of modern fluid mechanics. He established through theoretical considerations and several simple experiments that the flow about a solid body can be divided into two regions: a very thin layer in the neighbourhood of the body called the boundary layer where friction, that is, the viscous force plays an essential part and the remaining region outside the boundary layer, where friction may be neglected and the flow there may be regarded as inviscid and irrotational. Thus the tangential (shearing) stress and the condition of no slip at solid walls which distinguishes a real fluid from a perfect (real or non-viscous) fluid are to be taken into account only in the boundary layer.

Thus, in the case of motion of a real fluid past a solid body, the fluid adheres to the solid wall. It means that in the thin boundary layer adjacent to the solid body or the wall, the frictional force retards the motion of the fluid near the wall and the velocity of the fluid increases from its zero value at the solid wall to its full value which corresponds to the external potential flow at the periphery of the boundary layer. The smaller the viscosity, thinner is the transition layer. But the steep velocity gradient, in spite of the small viscosity, produces marked effects which are comparable in magnitude with those due to inertia force if the thickness of the transition layer is proportional to the square root of the kinematic viscosity.
The decelerated fluid particles in the boundary layer do not in all cases remain in the thin layer which adheres to the solid body along the whole wetted length of the wall. In some cases the boundary layer thickness increases considerably in the downstream direction and the flow in the boundary layer becomes reversed. This causes the decelerated fluid particles to be forced outwards which means that the boundary layer is separated from the wall. This is called boundary layer separation. This phenomenon is always associated with the formation of vortices and with large energy losses in the wake of the body. It occurs primarily near blunt bodies, such as circular cylinders and spheres.

A review of the development of boundary layer theory which stresses the mutual cross-fertilization between the theory and experiment is contained in an article by A. Betz (1949). The simplest example of the application of the boundary layer equations is afforded by the flow along a very thin semi-infinite flat plate. Historically this was the first example illustrating the application of Prandtl's boundary layer theory; it was discussed by Blasius (1908) and is often referred to as Blasius problem. Subsequently Bairstow (1925) and Goldstein (1980) solved the same equation with the aid of a slightly modified procedure. Somewhat earlier, Toepfer (1912) solved the Blasius equation numerically by Runge-Kutta method. The same equation was again solved by Howarth (1938) with increased accuracy.

(b) Role of suction in boundary layer control.

The problem of boundary layer control is very important in some fields, in particular in the field of aeronautical engineering. In actual applications it is often necessary to
prevent separation in order to reduce drag and to attain high lift. Several methods have been developed for the purpose of artificially controlling the behaviour of the boundary layer in order to affect the whole flow in a desired direction. One important method of controlling the boundary layer and hence of shifting the separation which reduces the drag is by applying suction at the solid boundary. The treatise entitled "Boundary-Layer and Flow" control by Lachmann (1961).

The effect of suction consists in removing the decelerated fluid particles from the boundary layer before they cause separation. A new boundary layer which is again capable of overcoming a certain adverse pressure gradient is allowed to form in the region behind the slit. With a suitable arrangement of slits and under favourable conditions separation of suction which was first tried by Prandtl (1904) 1935) was later widely used in the design of aircraft wings. By applying suction, considerably greater pressure increases on the upper side of the aerofoil are obtained at large angles of incidence, and, consequently, much larger maximum lift values. Schrenk (1941) investigated a large number of different arrangements of suction slits and their effect on maximum lift.

Subsequently, suction was also applied to reduce drag. By the use of suitable arrangements of suction slits it is possible to shift the point of transition in the boundary layer in the downstream direction. This causes the drag coefficient to
decrease, because laminar drag is substantially smaller than the turbulent drag.

(c) **Heat transfer in fluids and thermal boundary layers.**

It is well known that the heat transfer considerations are often of crucial importance in modern engineering design. Equipment size in power production and chemical processing may be determined primarily by the attainable heat transfer rates. A considerable fraction of the cost of many devices—for example, air-conditioning and refrigeration systems—is due to heat exchangers. In many types of equipments a successful design is possible only if provision is made to maintain reasonable temperatures by adequate heat transfer. Among such modern devices are rocket nozzles, compact electronic components, high speed air-craft, and atmosphere re-entry vehicles.

The study of heat transfer includes the physical processes whereby thermal energy is transferred as a result of a difference, or gradient, of temperature. The information generally desired is the way in which the rate of heat transfer depends upon the various features of the process. The transfer of heat between a solid body and a liquid or gaseous flow is a problem whose consideration involves the science of fluid motion. On the physical motion of the fluid there is superimposed the flow of heat by conduction and convection and generally speaking the two fields interact. In order to determine the temperature distribution it is necessary to combine the equation of motion with those of heat conduction.
It is intuitively evident that the temperature distribution around a hot body in a fluid stream with a different level of temperature has the same character as the velocity distribution in boundary layer flow. If the body is heated so that its temperature is maintained above that of the surroundings then the temperature of the stream will increase only over a thin layer in the immediate neighbourhood of the body and over a narrow wake behind it. The major part of the transition from the temperature of the hot body to that of the colder surroundings takes place in a thin layer near the body which in analogy with flow phenomena, is called the thermal boundary layer. It is evident that flow and thermal phenomena interact to a high degree. To investigate such phenomena it is necessary to establish the energy balance for a fluid element in motion and to consider it in addition to the equation of motion. For an incompressible fluid the energy balance is determined by the internal energy, the conduction of heat, the convection of heat with the stream and the generation of heat due to friction. The resulting equation is referred to as the energy equation.

In motions where temperature differences bring about the differences in density, it is necessary to include buoyancy forces in the equations of motion of a viscous fluid and to treat them as impressed body forces. The buoyancy forces are caused by changes in volume which are associated with temperature differences.
There are three distinct modes of heat transmission, namely, conduction, convection and radiation. Conduction is the process in which heat is transferred from regions of higher temperature to regions of lower temperature within a system or between two systems which are in contact physically without any relative motion of the different parts of the system or systems. In fact, energy is conducted through a material in which a temperature gradient exists by the thermal motion of various of the microscopic particles of which the material is composed. Radiation is an energy transport from material into surrounding space by electromagnetic waves. Radiant emission is also due to the thermal motion of microscopic particles, but the energy is transmitted electromagnetically. A heat (or mass) transfer process whose rate is directly influenced by fluid motion is called a convective process. The heat may be finally transferred through the flowing material by conduction, but the conduction process is basically altered by relative motion of the macroscopic particles in the fluid. Thermal energy and mass are convected in the fluid. Thermal energy and mass are convected about the flow region by the motion of the fluid.

It may be emphasized that in most of the situations occurring in nature, heat flows by more than one of these processes acting simultaneously. The phenomena of conduction and convection are affected primarily by temperature difference, and very little by temperature levels whereas radiation interchange increases rapidly with increase in the temperature levels.
At low temperatures, conduction and convection are the major contributors to the total heat transfer whereas at very high temperatures, radiation is the controlling factor.

In a system of fluid motion heat is transferred by conduction and convection. This is quite evident from the method of heat transfer from a surface to the surrounding fluid. At the surface, heat is first transferred by conduction to the adjacent fluid elements which in turn move to regions of lower temperature (thus convecting heat) and impart heat to the neighbouring fluid particles by conduction as well. Of course, it is virtually impossible to observe pure heat conduction in a fluid because, as soon as a temperature difference is imposed on a fluid, natural convection currents ensure due to resulting density differences. Thus convective process dominates a heat transfer phenomenon in fluid mechanics. As the convective heat transfer process and the motion of the fluid are inseparable, a study of hydrodynamic behaviour of the fluid is necessary in order to understand heat transfer taking place within a moving fluid.

Convection heat transfer is classified, according to the modes of motivating flow, into forced and free convections. If the flow is imposed by some external agency, such as a pressure gradient or a pump or blower the process is called 'forced convection'. In this case the velocity field is independent of the temperature field though the temperature field is dependent on the velocity field. Mathematically, the problem
reduces to finding the temperature field due to heated or cooled boundaries in a given velocity field. On the other hand, when the mining motion takes place merely as a result of the buoyancy forces, the process is called free or natural convection. This case occurs at very small velocities of motion in presence of large temperature differences. The state of motion which accompanies natural convection is evoked by buoyancy forces in the gravitational field of earth, the latter being due to density differences and gradients. For example, the fluid motion which exists outside a vertical hot plate belongs to this class.

Since in free convection, temperature difference is the cause of the fluid motion which in turn changes the rate of heat transfer, the temperature and velocity fields are interdependent here. This coupling between heat transfer and fluid motion causes natural convection process more difficult to analyse than similar forced convection arrangements. Because, here the momentum equation and the energy equation become coupled for velocity and temperature fields. If both the convective process are equally important in a flow system then it is called combined forced and free convective process.

(d) **Magnetohydrodynamics (MHD)**

In the last few decades, much work has been done on the generalisation of viscous flow and heat transfer solutions to take account of the additional effects of a magnetic field when the fluid involved is electrically conducting. It was known from Faraday's (1832) time that a solid or a fluid material moving in a magnetic field experiences an electromotive force (e.m.f.). If the material
is electrically conducting and a current path is available, electric currents ensure. Also, currents may be induced by change of the magnetic field with time. There are two basic consequences:

I. An induced magnetic field associated with these currents appears, perturbing the original magnetic field.

II. An electromotive force due to the interaction of currents and field appears, perturbing the original motion.

These are the two basic effects of magnetohydrodynamics (MHD) or hydrodynamics, the science of motion of electrically conducting fluids under magnetic fields. The situation is essentially one of mutual interaction between the fluid velocity field and the electromagnetic field. The motion affects the magnetic field by carrying the magnetic fields lines partially (depending upon the electrical conductivity of the fluid) along with it and the magnetic field affects the motion by producing a mechanical force, namely, the Lorentz force $\mathbf{3} \times \mathbf{6}$, where $\mathbf{3}$ is the electric current density and $\mathbf{6}$ the magnetic induction vector in the fluid region.

The study of MHD is quite important in the field of aeronautics, especially missile aerodynamics, since the temperature that occur in such flight speeds are sufficient to dissociate or even ionize the air appreciably. In such high speed flights Joule heating due to electric currents plays a very important role. For example, when a high speed missile re-enters the earth's atmosphere, a very large amount of heat is generated due to the
friction of air molecules and this viscous heating may sometimes be so considerable as to ionize the air near the forward stagnation point. Again, for the flow past a body at sufficiently large Mach numbers, the leading edge shock wave lies close to the surface of the body and the temperature rise due to compression along the shock in such hypersonic flows may be sufficiently high to ionize the air. Since the ionized air in this stagnation region is electrically conducting, a magnetic field may be applied to it so as to induce electromotive forces in the air which in turn will be retarded. As a result, the velocity gradient decreases near the wall, implying a reduction in the skin friction. If, as is usually the case, a reduction in the skin friction implies a reduction in heat transfer in the ordinary hydrodynamic case, decrease in the skin friction in MHD should also have a similar favourable effect on the heat transfer. However, this analogy is not to be pushed very far in MHD for, in this case Joule heating may be significantly considerable to offset such thermo-mechanical analogy.

Application of MHD to natural events received a belated stimulus when astrophysicists came to realize how prevalent throughout the universe are conducting, ionized gases (plasmas) and significantly strong magnetic fields. In the interwar period the astrophysicists, notably Cowling (1934) and Ferraro (1937) began to explore the formal theory of MHD and its applications, while other scientists and engineers such as William (1930) and
Hartmann (1937) performed simple experiments on the flow of conducting liquids in the laboratory.

Alfvén first published the classical paper on MHD in 1942. He explained in his paper that if a highly conducting fluid moves in a magnetic field, the induced currents will tend, in some sense, to inhibit relative motion of the fluid and field. So that is convected by the fluid. The boom in the past-war applied science soon affected MHD. Electromagnetic pumping of liquid metal coolants in nuclear reactor became standard practice and electromagnetic pumping, stirring and levitation were exploited in the metallurgical industries.

The continuum approximation is made in MHD, just as in ordinary hydrodynamics. One postulates that the fluid may be treated as continuous and describable in terms of local properties such as pressure and velocity.

MHD differs from ordinary hydrodynamics in that the fluid is electrically conducting. It is not magnetic; it affects a magnetic field not by its mere presence but only by virtue of electric currents, flowing in it. In magneto-hydrodynamic heat transfer problems, the additional body force term, viz, the Lorentz force comes into play in the momentum equation and the term corresponding to Joule heating appears in the energy equation. In a forced convection system, the energy equation remains uncoupled from Maxwell's equations and Navier-Stokes equations. Thus, the electromagnetic and velocity fields can be determined independently of the temperature field. However, when natural convection
forces are present the Navier-Stokes equation become coupled with the energy equation, and simultaneous solution is required. If a magnetohydrodynamic device is to be intelligently designed information should be available concerning the effects of interactions of the electromagnetic, velocity and temperature fields. In forced convection flows, induced magnetic forces may modify the inviscid free stream which in turn may change the external gradient or the free stream velocity which is imposed on the boundary layer. Thus for a complete solution in this case, one should also solve the inviscid free stream problem in determining the boundary layer characteristics. On the other hand in free convection problems, the velocity being zero in the free stream, the induced magnetic field do not exist there. Thus the influence of the magnetic field on the boundary layer is exerted through the Lorentz force confined to the boundary layer only, with no additional effects arising out of the free stream pressure gradient. Thus the free convection MHD problems can be formulated in a much simpler way than the corresponding forced convection problems.

(e) **Unsteady flow and heat transfer**:

The study of unsteady boundary layer flow has achieved importance in the view of its application in practical fields. The common examples of unsteady boundary layers occur when the motion is started from rest or when it is periodic. There are two main subdivisions of periodic flows to consider, on the one hand periodic boundary layers in the absence of an imposed mean flow, and on the other periodic boundary layers with an imposed mean flow.
In the case of a periodic boundary layer with an imposed mean flow, the velocity at the edge of the layer fluctuates about a non-zero mean. For small fluctuations, the mean flow in the boundary layer is unaffected by the Reynolds stress of the oscillation and is given by the steady boundary layer equation. The oscillatory flow is almost exactly like a periodic boundary layer in the absence of an impressed mean flow, and may conveniently be referred to as a secondary boundary layer. Periodic boundary layers occur when either the body or a periodic motion in a fluid at rest or when the body is at rest and the fluid executes a periodic motion.

Very often, problems in non-steady boundary layers involve an essentially steady flow on which there is superimposed a small non-steady perturbation. If it is assumed that the perturbation is small compared with the steady basic flow, it is possible to split the equations into a set of non-linear boundary layer equations for the perturbed quantities.

A well known example is that for which the external stream has the form,

$$U(x,t) = \bar{U}(x) + \epsilon U_1(x,t)$$

where $\epsilon$ is a very small number. The most important special case when the external perturbation is purely harmonic was studied by W.J. Lighthill (1954). The same type of linearization can be employed when the temperature at the wall is represented by the expression

$$\bar{T}_w(x,t) = \bar{T}_w(x) + \epsilon T_{w1}(x,t)$$
1.2 **Review of the relevant literature**

(a) Steady and unsteady hydrodynamic flows along plates:

Uptill now numerous authors have devoted thier study along in finite plates. The forced and free convection heat transfer phenomena in such flows have also received considerable attention. Here we enlist some of the works available in the existing literature mainly related to this thesis.

Lighthill (1954) initiated the study of unsteady two dimensional boundary flow when external flow fluctuates about a steady mean. He devoted his attention in investigating the effects of free stream oscillations on the laminar skinfriction and heat transfer. Soon after the advent of this pioneering work there has been a host of papers in literature using Lighthill's technique. Sama (1964) has generalised Lighthill's work (1954) in sense that the fluctuating unsteady part of the free stream velocity is replaced by an arbitrary function of time and that the on coming free stream is perturbed not only in magnitude but also in direction. Following the work of Watson (1959) for stagnation point flow, he has developed two type of solutions, one for large times and other for small times.

Gill and Casal (1962) have made a theoretical investigation of free convection effects in forced horizontal flows. They have obtained similarity solutions of the boundary layer equations for steady flow over a semi-infinite horizontal plate. Sparrow and Minkowycz (1962) have also considered free convection effects on a horizontal plate by employing a series expansion of the stream
function, which gives the perturbation of a basic forced convection flow due to buoyancy. Sharma and Nanda (1963) have investigated the free convection effects on laminar boundary layers in oscillatory flow. Muhuri and Maiti (1966) have analysed the unsteady free convection flow from a semi-infinite horizontal when the plate temperature varies in time about a constant mean. Gersten and Gross (1974) have studied the effect of transverse sinusoidal suction velocity on the flow and heat transfer over a porous plane wall. Singh, Sharma and Misra have extended this work to consider the free convection effect on skin friction and heat transfer of the flow caused by periodic suction velocity perpendicular to the flow.

Stuart (1955) has found some interesting features for an oscillatory flow over an infinite plate with constant suction at the plate. Reddy (1964) has extended Stuart's work to include a first order velocity slip and temperature jump conditions at the plate. Messiha (1966) has further extended Stuart's problem to consider the effect of variable suction velocity upon the skin friction and heat transfer in the boundary layer flow along an infinite porous plate.

The first exact solution of unsteady flow past an impulsively started infinite plate or oscillating infinite plate in its own horizontal plane was given by Stake (1851). Goldstein and Schert (1960) have studied the steady and transient free convection boundary layers on a uniformly heated vertical plate. Menold and Yang (1962) have obtained asymptotic solutions for unsteady laminar free convection on a vertical plate. Sugawara and Michiyoshi (1951) have
studied the heat transfer by natural convection in the unsteady state on a vertical flat wall. Mizukami (1977) studied the leading effect in unsteady natural convection on a vertical plate with time-dependent surface temperature. The flow past an infinite vertical plate oscillating in its own plane whose temperature $T_w$ differs from the temperature of the fluid far away from the plate was first studied by Soundalgekar (1979). Soundalgekar and Patil (1980) have studied the free convection effects on the flow past a vertical oscillating plate with constant heat flux. Soundalgekar (1987) have also extended his work (1979) in the sense that the plate temperature varies linearly with time.

Stewartson (1951, 1973) has studied analytically the flow past an impulsively started semi-infinite plate whereas Hall (1969) studied this problem by finite difference method. Illingworth (1950) studied the flow of a compressible gas with variable viscosity near an impulsively started infinite vertical plate. The problem was solved by a successive approximation method. By considering the time-dependent motion of an infinite plate, the problem of Illingworth (1950) was studied by Elliott (1969). However in case of the works of Illingworth (1950), Elliott (1969), only mathematical solutions were derived and no attention was given to the physical situation involved in this problem. More important is the situation when the plate temperature is different from that of the fluid far away from the plate. Because of this temperature difference, there exists free convection currents and thus do
affect the flow past the bodies. Such a situation was first considered by Soundalgekar (1977) in the case of an infinite vertical plate started impulsively in its own plane. Without taking into account the dissipative effects, an exact solution was presented by Soundalgekar (1977). When viscous dissipation effects are taken into consideration, the problem is found to be governed by coupled non-linear equations which have been solved with explicit finite difference method by Soundalgekar, Bhat and Mohiuddin (1979).

Schetz and Eichhor(1962) have studied the unsteady natural convection in the vicinity of a doubly infinite vertical plate. Hellums and Churchill (1961) have investigated the natural convection by finite difference methods. The same authors (1962) have also studied numerically the transient and steady state free and natural convection past a isothermal plate. Goldstein and Briggs (1964) have investigated the transient free convection about vertical plates and circular cylinders.

Sparrow and Cess (1961) have considered the steady free convection heat transfer and laminar boundary flow about an isothermal vertical plate. Gebhert (1962) has studied the free convection flow past a vertical plate. Soundalgekar (1973) studied the effects of viscous dissipation on free convection steady motion past an infinite vertical porous plate. Soundalgekar and Gupta (1975) have extended the work of the former author (1973) taking into account the steady motion of the porous plate. The last work (1975) was again reinvestigated by Vajravelu and
Sastri (1978) taking into account the effect of the temperature dependent heat source/sink on the flow and heat transfer characteristics.

Katsuhisa and Ryuchi (1985) have studied a three dimensional analysis of the Navier-Stokes equation for laminar natural convection around a vertical flat plate. Singh, Misra and Narayan (1986) have made a systematic analysis of unsteady two-dimensional free convective flow through a porous medium bounded by an infinite vertical plate when the temperature of the plate is oscillating with time about a constant non-zero mean. Hudson and Dennis (1985) have studied the flow of a viscous incompressible fluid past a normal flat plate at low and intermediate Reynolds numbers.

(b) Steady and unsteady MHD flow past an infinite plate:

Free and forced convective hydromagnetic flows past a infinite plates have been studied widely because of their importance in technical fields.

Gupta and Suryaprakasha Rao (1965) considered hydromagnetic free convection flow past a vertical porous flat plate subjected to suction and injection.

Stokes problem under transversely applied magnetic field was studied by Rossow (1958) whereas corresponding MHD problem for a vertical impulsively started infinite plate was recently studied by Soundalgekar, Gupta and Arnakae (1978). For these last two papers the viscous dissipative heat was assumed to be negligible Soundalgekar (1988) have studied this last problem (1978).
again by finite difference method by considering viscous dissipative heat effect on the flow and heat transfer and induced magnetic field and joule dissipation heat.

Soundalgekar (1974) has studied the free convection effects on steady MHD flow past a vertical porous plate in presence of a uniform transverse magnetic field. Das (1970) has studied the problem of small unsteady perturbation of steady MHD boundary layer flow past a semi infinite plate. Tokis (1985) has investigated the unsteady free convection flow of an electrically conducting fluid near a vertical plate of infinite extent moving in its own plane in the presence of a uniform transverse magnetic field.

The MHD flow past a semi-infinite plate in the presence of a transverse (or crossed) magnetic field has been studied by Clauser (1963), Dix (1963), Sears (1966) and Hector (1967). Mittal (1968) has considered the effect of plate temperature oscillations on the MHD thermal boundary layer on a semi infinite flat plate. Lewis (1968) has considered the boundary layer flow due to the uniform motion of a semi-infinite flat plate through an incompressible conducting fluid at rest, subject to a constant transverse magnetic field.

3. Hydrodynamic Channel Flow:

Berman (1953), Sellars (1955) and Yaun (1956) have studied viscous flow in two-dimensional porous channels. Tyagi (1966) has discussed the laminar forced convection of a dissipative fluid in a channel. Terill (1965) has studied forced convection heat transfer between two porous plates. Blenbons (1942 a, 1942 b) has
studied the natural convection heat transfer between two parallel plates. Ostrach (1952) has presented an analytical study of the laminar fully developed natural convection flow of viscous fluids with and without heat sources between two vertical plates. The same author (1954) has studied the combined natural and forced convection laminar flow and heat transfer of fluids with and without heat sources in channels with linearly varying wall temperatures.

Lekoudis, Nayfeh and Saric (1976) have made a linear analysis of compressible boundary layer flows over a wavy wall. Lessen and Gangwani (1976) have made a very interesting analysis of the effect of small amplitude wall waviness upon the stability of the laminar boundary layer. Vajravelu and Sastri (1978) have studied the free convective heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall. The same authors (1980) have extended this work (1978) to consider the both walls to be wavy. The results of the works of Vajravelu and Sastri (1978, 1980) are meaningful for small values of the Grashof number only and for constant fluid properties. Indeed, Hong and Bergles (1976) and Hwang and Hong (1970) among several others have pointed out that assumption of constances of fluid properties is not reasonable due to physical point of view. Rao and Sastri (1982) have re-investigated the problems of studied by Vajravelu and Sastri (1978, 1980) to consider the viscous dissipation effect on the flow and the fluid properties to be both constant and variable. They have obtained the approximate solutions of the governing equations by Galerkin's method. Gopalswamy, Rao, and Sastri (1982)
have extended this work (1982) to consider the forced convection effect in addition to free convection in the flow and heat transfer. Rao (1982) has further generalised this work (1982) in the sense that the wave lengths of the two wavy walls are not equal and volumetric rate of heat generation is temperature dependent. The governing equations have been solved numerically with the aid of Galerkin's method. Sankar and Sinha (1976) have studied the Rayleigh problem for wavy wall.

(d) MHD Channel Flow:

In recent years, considerable interest has developed in MHD Channel flow because of its application in energy conversion schemes, e.g., power generators, electromagnetic pumps and flow meters. Hartmann (1937) considered the two-dimensional, steady poiseuille flow of mercury between two parallel walls in the presence of an applied crossed magnetic field. Shercliff (1953, 1956) and Tanameuva (1960) have obtained exact solutions for the MHD flow in channels with rectangular and circular cross sections respectively.

The problem of steady flow of an electrically conducting viscous fluid through uniformly porous parallel plates in the presence of transverse magnetic field has been investigated by Suryaprakasrao (1962) and Ternil and Shrestha (1964, 1965 b). Siegel (1958), Pertmuter and Siegel (1961), and Zimin (1961) have studied the magnetic effects on the heat transfer in a fully developed flow between non-conducting parallel plates. Gupta (1960) has investigated the flow of a conducting fluid under pressure gradient between two parallel plates subjected to transverse magnetic field and in particular, solved the energy equation including the effects of viscous
and Joulean dissipations. Gershuni and Zhukhovitski (1958) discussed free convection shear flow between parallel porous walls, combined natural and forced convection flow with transverse magnetic field has been studied by Mori (1961) for parallel plate channel. Singer (1965) has discussed unsteady combined convective flow through vertical channels under transverse magnetic fields. Rao and Sivaprasad (1985) have studied the flow of an incompressible viscous, electrically conducting fluid in a horizontal channel bounded by a wavy wall and a flat wall in the presence of a constant heat source and a uniform transverse magnetic field. Novikov, Lyubin (1985) have investigated the flow of a conducting liquid in a plate channel with permeable (porous) walls under the influence of a transverse magnetic field, assuming the induced electromagnetic field and magnetic Reynolds number to be small.

1.3 Basic Equations:

The basic equations governing the motion of an incompressible viscous and electrically conducting liquid in the presence of a magnetic field and heat sources or sinks are

the equation of continuity:
\[ \nabla \cdot \mathbf{V} = 0 \quad \ldots \quad (1.3.1) \]

the modified Navier-Stokes equation:
\[ \rho \left[ \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} \right] = -\nabla p + \mathbf{F} + \mu \nabla^2 \mathbf{V} + \frac{\mathbf{J} \times \mathbf{B}}{\mu_0} \quad (1.3.2) \]

the energy equation:
\[ \rho C_p \left[ \frac{\partial T}{\partial t} + (\mathbf{V} \cdot \nabla) T \right] = k \nabla^2 T + \phi + \frac{\mathbf{J}^2}{\sigma_0} + Q \quad (1.3.3) \]
Maxwell's equations in rationalized MKS system of units are:

\[
\text{Curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \ldots \quad (1.3.4)
\]

\[
\text{Curl } \vec{B} = \mu_0 \vec{J} \quad \ldots \quad (1.3.5)
\]

\[
\text{Div } \vec{B} = 0 \quad \ldots \quad (1.3.6)
\]

\[
\vec{J} = \sigma \left[ \vec{E} + \vec{v} \times \vec{B} \right] \quad (1.3.7)
\]

Again from the equations 4 to 7, we get the magnetic induction equation:

\[
\frac{\partial \vec{B}}{\partial t} = \eta \nabla^2 \vec{B} + \text{Curl} \left( \vec{v} \times \vec{B} \right) \quad (1.3.8)
\]

The various symbols are:
- \( \vec{v} \) is the velocity vector
- \( \vec{F} \) the non-electric force per unit volume e.g. for free convection flow
- \( \vec{F} = \rho \vec{g} \), \( \vec{g} \) being the force of gravity per unit mass.
- \( \rho \) the pressures
- \( \vec{B} \) the magnetic induction vector
- \( \vec{J} \) the electric current density
- \( \vec{E} \) the electric field
- \( T \) the temperature
- \( t \) the time
- \( \sigma \) the electrical conductivity,
- \( K \) the thermal conductivity
- \( C_p \) the specific heat at constant pressure.
\[
\begin{align*}
\mu & \quad \text{the coefficient viscosity of the fluid} \\
\mu_e & \quad \text{the magnetic permeability,} \\
\rho & \quad \text{the density,} \\
\vec{j} \times \vec{B} & \quad \text{the Lorentz force per unit volume,} \\
Q & \quad \text{the heat source/sink per unit volume} \\
\eta & \quad \text{the magnetic diffusivity} \\
\vec{J} / \sigma_0 & = C_0 \left[ \vec{E} + \vec{v} \times \vec{B} \right]^2 \quad \text{the Joulean heat per unit volume} \\
\vec{\Phi} & \quad \text{the viscous dissipation per unit volume}
\end{align*}
\]

**Boundary conditions:**

The boundary conditions of a flow of an incompressible viscous electrically conducting fluid past a body in presence of a transverse magnetic field are:

1. There is no slip of fluid on the boundary.

2. \( T = 0 \) or \( \partial T / \partial n = 0 \) or \( T = T_w \) on the boundary.

3. \( T = T_\infty \) at a large distance from the boundary.

4. The normal component of the magnetic induction \( \vec{B} \) is continuous across the interface.

5. If none of the regions (fluid, solid or vacuum) adjoining the boundary is perfectly conducting, the tangential component of the magnetic field \( \vec{H} = \frac{\vec{B}}{\mu_e} \) is continuous across the interface. If, however, at least one of two media in contact is perfectly conducting then the magnetic field \( \vec{H} = \frac{\vec{B}}{\mu_e} \) must satisfy the condition.
\[ \mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = \mathbf{J}_s \]

where \( \mathbf{n} \) is the unit vector normal to the surface, \( \mathbf{H}_1, \mathbf{H}_2 \) are the values of the magnetic field on two sides of the interface and \( \mathbf{J}_s \) is the surface current density.

(6) the tangential component of the electric field is continuous across the interface. The equations of continuity, motion and energy can be simplified with the usual boundary layer approximations whenever a problem of boundary layer flow and heat transfer is considered.

As for example for two dimensional convective motion of an incompressible electrically conducting fluid past a vertical plate in presence of a transverse magnetic field, the equation of continuity and the component of the equation of motion along \( x \)-direction, that is in the direction of the free stream take the following forms:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \cdots (1.3.9)
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \rho \beta (T - T_\infty) - \sigma \omega B^2 (u - U_\infty) \quad \cdots (1.3.10)
\]
where the symbols have usual meanings. The equation of motion along Y-direction (normal to the plate) is to be dropped completely. It only implies that the pressure $P$ remains constant along the normal to the plate, that is, free stream pressure is impressed on the boundary layer.

Similarly, the energy equation reduces to the following form

$$PC_p\left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right] = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y}\right)^2$$

$$+ \alpha u^2 B^2 + Q \quad (1.3.11)$$

**Dimensionless quantities**:

The velocity and temperature field of a flow of an incompressible, viscous and electrically conducting fluid past a body in presence of a magnetic field depend on the following dimensionless quantities:

- **Reynolds number** $= R = \frac{U_\infty L}{v}$
- **Prandtl number** $= \Pr = \frac{\mu C_p}{\kappa}$
- **Grashof number** $= Gr = \frac{g \beta (\Delta T)_0 L^3}{\nu^2}$
- **Eckert number** $= E = \frac{U_\infty^2}{C_p (\Delta T)_0}$
- **Hartmann number** $= \text{M} = \frac{\alpha \beta^2 L^2}{\mu}$
Magnetic Prandtl number \( P_m = \frac{\nu}{\eta} \)

In most applications we do not require to know all the details of the temperature and velocity field, but we wish, in the first place, to know the quantity of heat exchanged between the body and the stream. This quantity can be expressed with the aid of a coefficient of heat transfer, \( \alpha \), which is defined either as a local quantity or as a mean quantity over the surface of the body under consideration.

The coefficient of heat transfer is referred to the difference between the temperature of the wall and that of the fluid, the latter being taken at a large distance from the wall. If \( Q(x) \) denotes the quantity of heat exchange per unit area and time (= heat flux) at a point \( x \), then according to Newton’s law of cooling it is assumed that

\[
Q(x) = \alpha(x) x (T_w - T_\infty) = \alpha(x) (\Delta T) \quad \ldots \quad (1.3.12)
\]

The coefficient of heat transfer has the dimension \([\text{J/m}^2\text{sec deg}]\). At the boundary between a solid body and a fluid the transfer of heat is due solely to conduction. In accordance with Fourier’s law the absolute value of the heat flux is given by \( Q(x) = -K \left( \frac{\partial T}{\partial n} \right) \). The nondimensional coefficient of heat transfer known as the Nusselt number \( N \) is given by

\[
N(x) = - \left( \frac{\partial T}{\partial n} \right) |_{n=0} \quad \ldots \quad (1.3.12)
\]

In general the velocity field and temperature field as well as
the local dimensionless coefficient of heat transfer depend on $R_p, C, E, M, \theta_m$. But for special solutions, in most cases, one or more of the dimensionless groups will disappear as the problem will only seldom be of this most general nature. The temperature field and the coefficient of heat transfer depend on Eckert number only when the temperature differences are large and when, simultaneously, the velocities are large and of order of the velocity of sound. With moderate velocities, the temperature and velocity fields depend on $E$ when temperature differences are small. Further, even with moderate velocities, the buoyancy forces caused by temperature differences are small compared with the inertia and friction forces. In such cases the problem ceases to depend on $G$. Such flows are called forced flows. For forced convection flows

$$N = \int (R, P, M, \theta_m).$$

The Grashof number becomes important only at very small velocities of flow, particularly of the motion is caused by buoyancy forces, such as in the stream which rises along a heated vertical plate. Such flows are called natural, and we refer to the problem as one in natural convection. In such cases velocities are very small and therefore effects of $R$ and $\theta_m$ (Induced magnetic field) are negligible

$$N = \int (G, P, M).$$

The Prandtl number constitutes that parameter whose value is decisive for the extent of the thermal boundary layer and, therefore, for the rate at which heat is transferred in forced or free convection. According to its definition $P = \frac{\nu}{\alpha}$, the Prandtl number is equal to the ratio of two quantities. One of them (viscosity)
Characterizes the fluids transport properties with respect to the transport of momentum, the other (thermal diffusivity) doing the same for the transport of heat. If the fluid possesses a particularly large viscosity, it can be stated loosely that its ability to transport momentum is large. Consequently, the destruction of momentum introduced by the presence of a wall (no-slip condition) extends far into the fluid and the velocity boundary layer is comparatively large. Similar statements can be made with respect to the thermal boundary layer. It is, therefore, understandable that the Prandtl number serves as a direct measure for the ratio of the thickness of the two layers in forced flow, the special case when $\text{Pr}=1$ corresponds to flows for which the two boundary layers are approximately equal in extent; they are exactly equal along a flat plate at zero incidence when its temperature is uniform. In addition to this, the two limiting cases when the Prandtl number is either very large or very small are also worthy of attention. It is seen that when $\text{Pr} \to 0$, it is possible to neglect the existence of a velocity boundary layer when thermal boundary layers are calculated; in fact, the whole velocity profile $u(x,y)$ can be replaced by the uniform distribution $U(x)$ which varies only with $x$ and which describes the external non-viscous stream.

1.4 Outline of the thesis

This thesis deals with a few problems of fully developed laminar MHD flow. Excluding one particular problem (Chapter VIII) all are unsteady. In all the chapters, the fluids are considered
to be electrically conducting, viscous and incompressible. In two chapters (VII, VIII) flows between two parallel vertical plates are studied and in one of them (Chapter VIII) one of the plates is wavy. In other chapters flows along infinite plates are investigated. In all problems, excluding the Chapter II, the solutions are supposed to consist of two parts; a mean part and a perturbed part.

In Chapter I, a comprehensive introduction of the subject at hand is prosecuted. In Chapter II, the two dimensional unsteady convective MHD flow of an incompressible, viscous and electrically conducting fluid past an infinite vertical plate moving in its own plane with a plate velocity and temperature which are prescribed functions of time has been discussed.

In Chapter IV, a discussion has been presented in two parts, of the MHD forced convection boundary layer flow past an infinite flat porous plate with time periodic suction at the plate in presence of a transverse magnetic field. In both the parts, it is considered that the external flow velocity as well as the suction velocity normal to the plate varies periodically with time. It is also assumed that there is no transfer of heat between the plate and the fluid. In the first part the applied magnetic field is assumed to be uniform while as in the second part, it is assumed to be periodic with time.

The Chapter IV has been devoted to the study of free and forced convective oscillatory MHD boundary layer flow along an infinite
flat vertical porous plate with variable suction and transverse magnetic field in two different parts. In the first part the induced magnetic field is neglected, but in the second part it has been taken into account.

In Chapter V, we have discussed the combined free and forced convection oscillatory MHD flow past a uniformly moving infinite, vertical porous plate in two different parts. In both the parts, the effect of the presence of heat source/sink on the flow and heat transfer are considered. In the first part, the induced magnetic field is neglected and plate temperature is assumed to be periodic with time. In the other part, the induced magnetic field is taken into account and the plate temperature is supposed to be uniform.

Chapter VI is devoted to an investigation of the problem of three dimensional free convective oscillatory MHD flow and heat transfer past a porous vertical plate in presence of a time periodic transverse magnetic field.

In Chapter VII, we have analysed the free convective oscillatory MHD flow with periodic suction between a porous vertical and a parallel flat wall with variable plate temperature and a transverse magnetic field. In this problem the effects of viscous dissipation and Joule heating are also taken into account.

Finally the Chapter VIII is devoted to the study of the steady free convective MHD flow and heat transfer in a viscous incompressible fluid confined between a long vertical wavy wall and a parallel flat wall in presence of a transverse magnetic
field and heat source/sink. Analytical solutions of the governing equations are obtained.