CHAPTER-1
NATURAL CONVECTION IN A VERTICAL ANNULAR CYLINDER EMBEDDED WITH POROUS MEDIUM
Fig (A)

Schematic diagram of the Vertical Annular Cylinder Embedded with Porous Medium Annular Cylinder
Case-1: When heat is supplied at the Lower portion of the inner wall of the vertical annular cylinder
Case- II: When heat is supplied at the Center portion of the inner wall of the vertical annular cylinder.
Case- III: When heat is supplied at the Upper portion of the inner wall of the vertical annular cylinder
Case- IV: When heat is supplied at the three different locations of the inner wall of the vertical annular cylinder
Fig (F)

Mesh pattern of Vertical Annular Cylinder Embedded with Porous Medium
Introduction

The study of free convective heat transfer in cylindrical annuli filled with fluid saturated porous media attracted the attention of engineers in the recent past owing to the relevance in high performance insulations for Cryogenic containers and large shipborne LNG tanks, sensible heat storage beds and the risk assessment of radio nuclide migration from depositories of nuclear waste. Among the studies on vertical annular porous media, the analytical solution of Philip [1] is applicable for low modified Rayleigh numbers. Solutions reported by Havstad and Burns [2] using perturbation techniques as well as numerical methods also are valid for low Rayleigh numbers. The numerical studies for low Rayleigh numbers reported by Hickox and Gartling [3] considered anisotropic permeability. Numerical and experimental studies reported by Prasad and Kulacki [4], [5] and Prasad et al [6] cover a wide range of Rayleigh numbers, aspect ratios and radius ratios. Yucel [7] investigated the influence of injection or withdrawal of a fluid on a free convection about a vertical cylinder in a porous medium whereas Kumari et al [8] have discussed about finite difference and improved perturbation solutions for free convection on a vertical cylinder embedded in a saturated porous medium. Rajamani et al [9] have
investigated the convective heat transfer in a vertical annular cylinder embedded with porous medium by making use of finite element method and discussed the effect of Aspect ratio and Radius ratio of the annulus on the heat transfer rate.

Yih [10] analysed the radiation effect on natural convection over a vertical cylinder embedded with porous medium. The radiation term was treated with Rosseland approximation and the solution of equations was obtained by implicit finite difference method together with Keller box scheme. Results were presented in terms of Nusselt number and temperature profiles. It was found that thermal boundary layer increases with the increased radiation effect. Ali [11] has studied numerically the natural convection in a rectangular porous medium. In this case the vertical walls of the cavity were maintained at different temperatures and the horizontal walls were adiabatic. He considered two important parameters for the study, which are the Rayleigh number and the Aspect ratio of cavity. He found that the flow is parallel to the cavity and the isotherms are stratified in the core of the cavity when the Aspect ratio is large. Rajamani et al [12] have studied the natural convective heat transfer in an annular cylinder.

Yao [13] has studied theoretically the natural convection along a vertical wavy surface. He found that the local heat transfer rate is smaller than that of the flat plate case and decreases with increase of the wave amplitude. The average Nusselt number also shows the same trend. Adjlout et al. [14] reported a numerical study of the effect of a hot wavy wall in an inclined differentially heated square cavity. The trend of the local heat transfer is wavy. The mean Nusselt number
decreases comparing the square cavity. More applications and a good insight into the subject are given by Nield and Bejan [15], Vafai [16], Pop and Ingham[17].

In this chapter, we studied the heat transfer in a saturated porous medium embedded in a vertical annular cylinder for four different cases of supplying heat to the vertical annulus i.e. the vertical annulus is supplied with heat at i) Lower half of the annulus ii) Center half of the annulus iii) Upper half of the annulus and iv) Inner wall heated at three different positions as shown in the figures (B, C, D and E). Finite Element Method has been used to convert the partial differential equations into a matrix form of equations, which can be solved iteratively with the help of a computer code. The Galerkin Finite Element Method of three nodded triangular elements is used to divide the physical domain into smaller segments, which is a pre-requisite for finite element method. Flow and heat transfer characteristics (streamlines, isothermals and the average Nusselt number) are investigated for Aspect ratio (Ar), Radius ratio (Rr) and Rayleigh number (Ra).
Mathematical Formulation:

A vertical annular cylinder of inner radius $r_i$ and outer radius $r_o$ as depicted by schematic diagram as shown in figure (A), is considered to investigate the heat transfer behavior. The co-ordinate system is chosen such that the $r$-axis points towards the width and $z$-axis towards the height of the cylinder respectively. Because of the annular nature, two important parameters emerge which are Aspect ratio ($A_r$) and Radius ratio $R_r$ of the annulus. They are defined as

$$A_r = \frac{H}{r_o - r_i}, \quad R_r = \frac{r_o - r_i}{r_i}$$

where $H$, is the height of the cylinder.

The inner surface of the cylinder is maintained at isothermal temperature $T_s$ and outer surface is at ambient temperature $T_\infty$. It may be noted that, due to axisymmetry only half of the annulus is sufficient for analysis purpose, since other half is mirror image of the first half. The top and bottom horizontal surfaces of the vertical annular cylinder are adiabatic.

Following assumptions are made

- The flow inside the porous medium obeys Darcy law and there is no phase change of fluid.
- Porous medium is saturated with fluid.
- The fluid and medium are in local thermal equilibrium in the domain.
- The porous medium is isotropic and homogeneous.
- Fluid properties are constant except the variation of density.
With the above assumptions, the governing equations are given by

Continuity Equation:

\[
\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0
\]  

(1.1.1)

The velocity in \( r \) and \( z \) directions can be described by Darcy law as

velocity in horizontal direction

\[
u = -\frac{K \frac{\partial p}{\partial r}}{\mu}
\]

velocity in vertical direction

\[
w = -\frac{K}{\mu} \left( \frac{\partial p}{\partial z} + \rho g \right)
\]

The permeability \( K \) of porous medium can be expressed as Bejan [16]

\[
K = \frac{D_p \phi^3}{180(1 - \phi)^2}
\]

Momentum Equation:

\[
\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} = \frac{gK \beta}{\nu} \frac{\partial T}{\partial r}
\]

(1.1.2)

Energy Equation:

\[
u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial z^2} \right)
\]

(1.1.3)
The continuity equation (1.1.1) can be satisfied by introducing the stream function $\psi$ as

$$u = -\frac{1}{r} \frac{\partial \psi}{\partial z}$$  \hspace{1cm} (1.1.4)

$$w = \frac{1}{r} \frac{\partial \psi}{\partial r}$$  \hspace{1cm} (1.1.5)

The variation of density with respect to temperature can be described by Boussinesq approximation as

$$\rho = \rho_0 [1 - \beta \gamma (T - T_0)]$$  \hspace{1cm} (1.1.6)

The corresponding boundary conditions are

**Case-I:** Lower heated wall at $r = r_i$ and $0 \leq z \leq \frac{H}{2}$, $T = T_w$, $\psi = 0$

at $r = r_o$ \hspace{2cm} $T = T_w$, $\psi = 0$

**Case-II:** Center heated wall at $r = r_i$ and $\frac{H}{4} \leq z \leq \frac{3H}{4}$, $T = T_w$, $\psi = 0$

at $r = r_o$ \hspace{2cm} $T = T_w$, $\psi = 0$ \hspace{1cm} (1.1.7)

**Case-III:** Upper heated wall at $r = r_i$ and $\frac{H}{2} \leq z \leq H$, $T = T_w$, $\psi = 0$

at $r = r_o$ \hspace{2cm} $T = T_w$, $\psi = 0$

**Case-IV:** When heat is supplied at three different locations at the inner wall of the Vertical Annular Cylinder

at $r = r_i$ and $0 \leq z \leq \frac{H}{6}$, $\frac{5H}{12} \leq z \leq \frac{2H}{3}$, $\frac{5H}{6} \leq z \leq H$, $T = T_w$, $\psi = 0$

at $r = r_o$ \hspace{2cm} $T = T_w$, $\psi = 0$
The new parameters arising due to cylindrical co-ordinates system are

Non-dimensional Radius
\[ \tilde{r} = \frac{r}{L} \]

Non-dimensional Height
\[ \tilde{z} = \frac{z}{L} \]

Non-dimensional Stream function
\[ \tilde{\psi} = \frac{\psi}{\alpha L} \]  \hspace{1cm} (1.1.8)

Non-dimensional Temperature
\[ \tilde{T} = \frac{(T - T_w)}{(T_w - T_o)} \]

Rayleigh Number
\[ Ra = \frac{g\beta_r \Delta TKL}{\nu \alpha} \]

The non-dimensional equations for the heat transfer in vertical cylinder are

Momentum equation:
\[
\frac{\partial^2 \tilde{\psi}}{\partial \tilde{z}^2} + \tilde{r} \left( \frac{1}{\tilde{r}} \frac{\partial \tilde{\psi}}{\partial \tilde{r}} \right) = \tilde{r} Ra \frac{\partial \tilde{T}}{\partial \tilde{r}} \]  \hspace{1cm} (1.1.9)

Energy Equation:
\[
\frac{1}{\tilde{r}} \left[ \frac{\partial \tilde{\psi}}{\partial \tilde{r}} \frac{\partial \tilde{T}}{\partial \tilde{z}} - \frac{\partial \tilde{\psi}}{\partial \tilde{z}} \frac{\partial \tilde{T}}{\partial \tilde{r}} \right] = \frac{1}{\tilde{r}} \left( \frac{\partial}{\partial \tilde{r}} \left( \tilde{r} \frac{\partial \tilde{T}}{\partial \tilde{r}} \right) + \frac{\partial^2 \tilde{T}}{\partial \tilde{z}^2} \right) \]  \hspace{1cm} (1.1.10)
The corresponding non-dimensional boundary conditions are

Case-I: Lower heated wall at \( r = r_i \) and \( 0 \leq z \leq \frac{H}{2} \), \( \bar{T} = 1 \), \( \bar{\psi} = 0 \)

\[
\text{at } r = r_o \quad \bar{T} = 0, \quad \bar{\psi} = 0
\]

Case-II: Center heated wall at \( r = r_i \) and \( \frac{H}{4} \leq z \leq \frac{3H}{4} \), \( \bar{T} = 1 \), \( \bar{\psi} = 0 \)

\[
\text{at } r = r_o \quad \bar{T} = 0, \quad \bar{\psi} = 0
\]  
(1.1.11)

Case-III: Upper heated wall at \( r = r_i \) and \( \frac{H}{2} \leq z \leq H \), \( \bar{T} = 1 \), \( \bar{\psi} = 0 \)

\[
\text{at } r = r_o \quad \bar{T} = 0, \quad \bar{\psi} = 0
\]

Case-IV: When heat is supplied at three different locations at the inner wall of the Vertical Annular Cylinder

\[
\text{at } r = r_i \text{ and } 0 \leq z \leq \frac{H}{6}, \quad \frac{5H}{12} \leq z \leq \frac{7H}{12}, \quad \frac{5H}{6} \leq z \leq H, \quad \bar{T} = 1, \quad \bar{\psi} = 0
\]

\[
\text{at } r = r_o \quad \bar{T} = 0, \quad \bar{\psi} = 0
\]
Solution of the governing equations:

Thus far we derived the partial differential equations, which describe the heat and fluid flow behavior in the vicinity of porous medium is given. There are various numerical methods available to achieve the solution of these equations, but the most popular numerical methods are Finite Difference Method, Finite Volume Method and the Finite Element Method. The selection of these numerical methods is an important decision, which is influenced by variety of factors amongst which the geometry of domain plays a vital role. Other factors include the ease with which these partial differential equations can be transformed into simpler forms, the computational time required and the flexibility in development of computer code to solve these equations.

In the present study, we have used Finite Element Method (FEM). The following sections enlighten the Finite Element Method and present its application to solve the above mentioned equations.

The Finite Element Method is a popular method amongst scientific community. This method was originally developed to study the mechanical stresses in a complex airframe structure Clough [30] and popularized by Zienkiewicz and Cheung [31] by applying it to Continuum mechanics. Since then the application of Finite Element Method has been exploited to solve the numerous problems in various Engineering disciplines.
The great thing about finite element method is its ease with which it can be generalized to myriad Engineering problems comprised of different materials. Another admirable feature of the Finite Element Method (FEM) is that it can be applied to a wide range of geometries having irregular boundaries, which is difficult to achieve with other contemporary methods. FEM Can be said to have comprised of roughly 5 steps to solve any particular problem. The steps can be summarized as

- Discritizing the domain: This step involves the division of whole physical domain into smaller segments known as elements and then identifying the nodes, co-ordinates of each node and ensuring proper connectivity between the nodes.
- Specifying the equation: In this step, the governing equations are specified in terms of nodal values.
- Development of global matrix: The equations are arranged in a global matrix, which takes into account the whole domain.
- Solution: The equations are solved to get the desired variables at each node in the domain.
- Evaluate the quantities of interest: After solving the equations a set of values are obtained for each node, which can be further processed to get the quantities of interest.
There are varieties of elements available in FEM, which are distinguished by the presence of number of nodes. The present study is carried out by using a simple 3-noded triangular element as shown in the figure.

Let us consider that the variable to be determined in the triangular area is \( T \). The polynomial function for \( T \) can be expressed as

\[
T = \alpha_1 + \alpha_2 r + \alpha_3 z
\]  

(1.1.12)

The Variable \( T \) has the value \( T_i, T_j, T_k \) at the nodal position \( i, j \) and \( k \) of the element. The \( r \) and \( z \) co-ordinates at these points are \( r_i, r_j, r_k \) and \( z_i, z_j, z_k \) respectively.

Fig: 1 Typical Triangular element
Substitution of these nodal values in the equation (1.1.12) helps in determining the constants $\alpha_1, \alpha_2, \alpha_3$, which are

$$\alpha_1 = \frac{1}{2A} \left[ (r_{j}z_{k} - r_{k}z_{j})P_{i} + (r_{k}z_{l} - r_{l}z_{k})P_{j} + (r_{l}z_{j} - r_{j}z_{l})P_{k} \right]$$  \hspace{1cm} (1.1.13)

$$\alpha_2 = \frac{1}{2A} \left[ (z_{j} - z_{k})P_{i} + (z_{k} - z_{l})P_{j} + (z_{l} - z_{j})P_{k} \right]$$  \hspace{1cm} (1.1.14)

$$\alpha_3 = \frac{1}{2A} \left[ (r_{k} - r_{j})P_{i} + (r_{j} - r_{l})P_{j} + (r_{l} - r_{k})P_{k} \right]$$  \hspace{1cm} (1.1.15)

$$2A = \begin{vmatrix} 1 & r_{i} & z_{i} \\ 1 & r_{j} & z_{j} \\ 1 & r_{k} & z_{k} \end{vmatrix}$$  \hspace{1cm} (1.1.16)

Substitution of $\alpha_1, \alpha_2, \alpha_3$ in the equation (1.1.12) and mathematical arrangement of the terms results into

$$T = N_{i}T_{i} + N_{j}T_{j} + N_{k}T_{k}$$  \hspace{1cm} (1.1.17)

In equation (1.1.17) $N_{i}, N_{j} \& N_{k}$ are the shape functions given by

$$N_{m} = \frac{a_{m} + b_{m}r + c_{m}z}{2A}, \hspace{1cm} m = i, j \& k$$  \hspace{1cm} (1.1.18)

The constants can be expressed in terms of co-ordinates as

$$a_{i} = r_{j}z_{k} - r_{k}z_{j}, \hspace{1cm} b_{i} = z_{j} - z_{k}, \hspace{1cm} c_{i} = r_{k} - r_{j}$$  \hspace{1cm} (1.1.19)

$$a_{j} = r_{k}z_{l} - r_{l}z_{k}, \hspace{1cm} b_{j} = z_{k} - z_{l}, \hspace{1cm} c_{j} = r_{l} - r_{k}$$

$$a_{k} = r_{l}z_{j} - r_{j}z_{l}, \hspace{1cm} b_{k} = z_{l} - z_{j}, \hspace{1cm} c_{k} = r_{j} - r_{l}$$
Good insight into the FEM is given in Segerlind [27]; ElShayeb and Beng [28] Lewis et al. [29]. Galerkin method is employed to convert the partial differential equations into matrix form of equation for an element. The steps involved are as given below.

Please note that the nodal terms i,j, and k are replaced by 1,2 and 3 respectively in subsequent discussions for simplicity.

Applying Galerkin method to Momentum equation (1.1.9) yields

\[
\{R^e\} = - \int_\Omega \left[ N^T \left( \frac{\partial^2 \psi}{\partial z^2} + \frac{\bar{r}}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \frac{1}{\bar{r}} \frac{\partial \psi}{\partial \bar{r}} \right) - \bar{r} R \frac{\partial \bar{T}}{\partial \bar{r}} \right) \right] dv \tag{1.1.20}
\]

\[
\{R^e\} = - \int_\Omega \left[ N^T \left( \frac{\partial^2 \psi}{\partial z^2} + \frac{\bar{r}}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \frac{1}{\bar{r}} \frac{\partial \psi}{\partial \bar{r}} \right) - \bar{r} R \frac{\partial \bar{T}}{\partial \bar{r}} \right) \right] 2\pi \bar{r} dA \tag{1.1.21}
\]

where \( R^e \) is the residue. Considering the individual terms of equation (1.1.21)

The differentiation of following term results into

\[
\frac{\partial}{\partial \bar{r}} \left( \left[ N^T \right] \frac{\partial \psi}{\partial \bar{r}} \right) = \left[ N^T \right] \frac{\partial^2 \psi}{\partial \bar{r}^2} + \frac{\partial \left[ N^T \right]}{\partial \bar{r}} \frac{\partial \psi}{\partial \bar{r}} \tag{1.1.22}
\]

Thus

\[
\int_\Omega N^T \frac{\partial^2 \psi}{\partial \bar{r}^2} dA = \int_\Omega \frac{\partial}{\partial \bar{r}} \left( \left[ N^T \right] \frac{\partial^2 \psi}{\partial \bar{r}^2} \right) 2\pi \bar{r} dA - \int_\Omega \frac{\partial \left[ N^T \right]}{\partial \bar{r}} \frac{\partial \psi}{\partial \bar{r}} \tag{1.1.23}
\]
The first term on right hand side of equation (1.1.23) can be transformed into surface integral by the application of Greens theorem and leads to inter-element requirement at boundaries of an element. The boundary conditions are incorporated in the force vector.

Making use of (1.1.17) produces

\[
\int_{A} \frac{\partial^{2}\psi}{\partial r^{2}} 2\pi r dA = -\int_{A} \frac{\partial N^{T}}{\partial r} \begin{bmatrix} \psi_{1} \\ \psi_{2} \\ \psi_{3} \end{bmatrix} dA \quad (1.1.24)
\]

Substitution of (1.1.18) into (1.1.24) gives

\[
= -\frac{1}{(2A)^{2}} \int_{A} \begin{bmatrix} b_{1} \\ b_{2} \\ b_{3} \end{bmatrix} \begin{bmatrix} b_{1} & b_{2} & b_{3} \\ b_{2} & b_{2} & b_{3} \\ b_{3} & b_{2} & b_{3} \end{bmatrix} \begin{bmatrix} \psi_{1} \\ \psi_{2} \\ \psi_{3} \end{bmatrix} 2\pi r dA
\]

\[
= -\frac{2\pi R}{4A} \begin{bmatrix} b_{1}^{2} & b_{1}b_{2} & b_{1}b_{3} \\ b_{2}^{2} & b_{2}b_{2} & b_{2}b_{3} \\ b_{3}^{2} & b_{3}b_{2} & b_{3}b_{3} \end{bmatrix} \begin{bmatrix} \psi_{1} \\ \psi_{2} \\ \psi_{3} \end{bmatrix} \quad (1.1.25)
\]

Similarly

\[
\int_{A} N^{T} \frac{\partial^{2}\psi}{\partial z^{2}} 2\pi r dA = -\frac{2\pi R}{4A} \begin{bmatrix} c_{1}^{2} & c_{1}c_{2} & c_{1}c_{3} \\ c_{2}^{2} & c_{2}c_{2} & c_{2}c_{3} \\ c_{3}^{2} & c_{3}c_{3} & c_{3}c_{3} \end{bmatrix} \begin{bmatrix} \psi_{1} \\ \psi_{2} \\ \psi_{3} \end{bmatrix} \quad (1.1.26)
\]

The third term of equation (1.1.21) is

\[
\int_{A} N^{T} \tau R a \frac{\partial T}{\partial r} 2\pi r dA = Ra \int_{A} N^{T} r \frac{\partial T}{\partial r} 2\pi r dA \quad (1.1.27)
\]
In order to get the matrix equation of (1.1.27) the following method can be applied. The triangular element can be subdivided into three triangles with a point in the center of original triangle as shown in the below figure.

![Fig: 2 Showing the sub triangular areas](image)

Defining the new area ratios as

\[ M_1 = \frac{\text{areapij}}{\text{areaijk}}, \quad M_2 = \frac{\text{areapij}}{\text{areaijk}}, \quad M_3 = \frac{\text{areapij}}{\text{areaijk}} \]

It can be shown ElShayeb and Beng [28] that

\[ M_1 = N_1, \quad M_2 = N_2, \quad M_3 = N_3 \]

Replacing shape functions in equation (1.1.27) by (1.1.28) yields

\[
\int_A \mathbf{N}^T \mathbf{\bar{r}} \mathbf{R} \mathbf{a} \frac{\partial \mathbf{\bar{r}}}{\partial \mathbf{r}} 2\pi \mathbf{\bar{r}} dA = \mathbf{\bar{r}} \mathbf{R} \mathbf{a} \begin{bmatrix} \frac{\partial \mathbf{N}}{\partial \mathbf{r}} \end{bmatrix} \begin{bmatrix} \mathbf{T}_1 \\ \mathbf{T}_2 \\ \mathbf{T}_{31} \end{bmatrix} 2\pi dA
\]

(1.1.28)
The area integration can be evaluated by a simple relation Segerlind [27],

\[
\int M_1^d M_2^e M_3^f = \frac{d!e!f!}{(d + e + f + 2)!} 2A
\]  

(1.1.29)

Application of equation (1.1.29) into equation (1.1.28) gives rise to

\[
Ra \frac{2\pi R^2}{3} \begin{bmatrix} 1 \ 1 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} \frac{\bar{T}_1}{2A} \\ \frac{\bar{T}_2}{2A} \\ \frac{\bar{T}_3}{2A} \end{bmatrix}
\]

\[
= \frac{2\pi R^2 Ra}{6} \begin{bmatrix} b_1b_2 + b_2b_3 + b_3b_1 \\ b_1b_2 + b_2b_3 + b_3b_1 \\ b_1b_2 + b_2b_3 + b_3b_1 \end{bmatrix}
\]

Now the Momentum equation leads to

\[
2\pi R \begin{bmatrix} b_1^2 b_1b_2 b_1b_3 \\ b_1b_2 b_2^2 b_2b_3 \\ b_1b_3 b_2b_3 b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 c_1c_2 c_1c_3 \\ c_1c_2^2 c_2c_3 c_2c_3^2 \\ c_1c_2c_3 c_2c_3c_3c_2^3 \end{bmatrix} \begin{bmatrix} \vec{\psi}_1 \\ \vec{\psi}_2 \\ \vec{\psi}_3 \end{bmatrix}
\]

(1.1.30)

In simple form the above equation can be represented as

\[
[K_s][\vec{\psi}] = [f]
\]

Where \(K_s\) is the stiffness matrix and \(f\) is the force vector. For the above Momentum equation they are

\[
[K_s] = \frac{2\pi R}{4A} \begin{bmatrix} b_1^2 b_1b_2 b_1b_3 \\ b_1b_2 b_2^2 b_2b_3 \\ b_1b_3 b_2b_3 b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 c_1c_2 c_1c_3 \\ c_1c_2^2 c_2c_3 c_2c_3^2 \\ c_1c_2c_3 c_2c_3c_3c_2^3 \end{bmatrix} \]

(1.1.31a)
\[
\{\psi\} = \begin{bmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 
\end{bmatrix}
\]  
(1.1.31b)

\[
\{f\} = -\frac{2\pi R^2 Ra}{6} \begin{bmatrix}
\frac{b_1 T_1 + b_2 T_2 + b_3 T_3}{r_1 T_1 + b_2 T_2 + b_3 T_3} \\
\frac{b_1 T_1 + b_2 T_2 + b_3 T_3}{r_2 T_2 + b_3 T_3} \\
\frac{b_1 T_1 + b_2 T_2 + b_3 T_3}{r_3 T_3}
\end{bmatrix}
\]  
(1.1.31c)

The radial distance \( \bar{R} \) to the centroid of an element is given by relation

\[
\bar{R} = \frac{r_1 + r_2 + r_3}{3}
\]

Similarly application of Galerkin method to Energy equation gives

\[
[R^T] = -\int_A \left[ \frac{1}{r} \left( \frac{\partial \psi}{\partial z} \frac{\partial \bar{T}}{\partial z} - \frac{\partial \psi}{\partial \bar{r}} \frac{\partial \bar{T}}{\partial \bar{r}} \right) - \left( \frac{1}{\bar{r}} \frac{\partial}{\partial \bar{r}} \left( \bar{r} \frac{\partial \bar{T}}{\partial \bar{r}} \right) + \frac{\partial^2 \bar{T}}{\partial z^2} \right) \right] 2\pi dA
\]

(1.1.32)

Considering the terms individually of the above equation (1.1.32)

\[
\int_A [N]^T \frac{\partial \psi}{\partial z} \frac{\partial \bar{T}}{\partial \bar{r}} 2\pi dA = \int_A \begin{bmatrix}
M_1 \\
M_2 \\
M_3
\end{bmatrix} \begin{bmatrix}
\frac{\partial [N]}{\partial z} \frac{\partial \psi}{\partial \bar{r}} \\
\frac{\partial \psi}{\partial \bar{r}} \\
\frac{\partial \psi}{\partial \bar{r}}
\end{bmatrix} [T]^T 2\pi dA
\]

(1.1.33)

\[
= \frac{2\pi A}{3} \times \frac{1}{4A^2} \left[ c_1 \psi_1 + c_2 \psi_2 + c_3 \psi_3 \right] \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix} \begin{bmatrix}
\bar{T}_1 \\
\bar{T}_2 \\
\bar{T}_3
\end{bmatrix}
\]

(1.1.34)

\[
= \frac{2\pi}{12A} \left[ c_1 \psi_1 + c_2 \psi_2 + c_3 \psi_3 \right] \begin{bmatrix}
b_1 \\
b_2 \\
b_3
\end{bmatrix} \begin{bmatrix}
\bar{T}_1 \\
\bar{T}_2 \\
\bar{T}_3
\end{bmatrix}
\]

(1.1.35)
Following the same above steps

\[ \int_A \left[ N \right]^T \frac{\partial \bar{\psi}}{\partial r} \frac{\partial \bar{T}}{\partial \bar{z}} 2\pi r dA = \left[ \begin{array}{c} M_1 \\ M_2 \\ M_3 \end{array} \right] \frac{\partial \left[ N \right]}{\partial \bar{r}} \frac{\partial \left[ N \right]}{\partial \bar{z}} \left[ T \right] 2\pi r dA \]

\[ \int_A \left[ N \right]^T \frac{\partial \bar{\psi}}{\partial r} \frac{\partial \bar{T}}{\partial \bar{z}} 2\pi r dA = \frac{2\pi}{12A} \begin{vmatrix} b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \end{vmatrix} \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \]

The remaining two terms of Energy equation can be evaluated in similar fashion of equation (1.1.21)

\[ \int_A \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{T}}{\partial r} \right) 2\pi r dA = -\frac{2\pi R}{4A} \begin{vmatrix} b_1 \bar{b}_1 + b_2 \bar{b}_2 + b_3 \bar{b}_3 \\ b_1 \bar{b}_1 + b_2 \bar{b}_2 + b_3 \bar{b}_3 \\ b_1 \bar{b}_1 + b_2 \bar{b}_2 + b_3 \bar{b}_3 \end{vmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \]

\[ \int_A \frac{\partial^2 \bar{T}}{\partial \bar{z}^2} 2\pi \bar{r} dA = -\frac{2\pi R}{4A} \begin{vmatrix} c_1 c_2 & c_1 c_3 & c_2 c_3 \\ c_1 c_2 & c_1 c_3 & c_2 c_3 \\ c_1 c_2 & c_1 c_3 & c_2 c_3 \end{vmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \]

Thus the Stiffness matrix of Energy equation is given by

\[
\begin{bmatrix}
\frac{2\pi}{12A} \begin{vmatrix} c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \\ c_1 \bar{\psi}_1 + c_2 \bar{\psi}_2 + c_3 \bar{\psi}_3 \end{vmatrix} [b_1 \ b_2 \ b_3] - \frac{2\pi R}{12A} \begin{vmatrix} b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \\ b_1 \bar{\psi}_1 + b_2 \bar{\psi}_2 + b_3 \bar{\psi}_3 \end{vmatrix} [c_1 \ c_2 \ c_3] \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} \\
+ \frac{2\pi R}{4A} \begin{vmatrix} b_1^2 \ b_2 \ b_3 \ b_1 \ b_2^2 \ b_3 \end{vmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} + \begin{vmatrix} c_1^2 \ c_2 \ c_3 \ c_1 \ c_2^2 \ c_3 \end{vmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \end{bmatrix} = 0 
\]

(1.1.36)
Results and Discussion:

Results are obtained in terms of Nusselt number at hot wall for various parameters such as Aspect ratio ($A_r$), Radius ratio ($R_r$) and Rayleigh number ($Ra$), when heat is supplied to the vertical annular cylinder for four different cases i.e., When heat is supplied to the vertical annulus at i) Lower half of the annulus ii) Center half of the annulus iii) Upper half of the annulus and iv) Inner wall heated at three different portions of the inner wall.

The average Nusselt number ($\bar{Nu}$), when heat is supplied at the Lower, Center and Upper half of the vertical annular cylinder is given as

$$\bar{Nu} = -\frac{1}{L} \int_0^L \left( \frac{\partial T}{\partial r} \right)_{r=r_0} \, dr$$

where $L$ is the length of the heated wall.

The average Nusselt number ($\bar{Nu}$), when heat is supplied at the three different locations of the inner wall of the vertical annular cylinder is given as

$$\bar{Nu} = -\frac{1}{L} \int_0^L \left( \frac{\partial T}{\partial r} \right)_{r=r_0} \, dr$$

since $L = L_1 + L_2 + L_3$, where $L_1$, $L_2$ and $L_3$ are the length of the heated wall portions and $L$ is the total length of the heated wall.
Case-I: When heat is supplied at the Lower half of the inner wall of the Vertical Annular Cylinder

a) $\text{Ar} = 0.5$

b) $\text{Ar} = 1$

c) $\text{Ar} = 2$

Fig: 1.2.1: Streamlines (Left) and Isotherms (Right) for $R_r = 3$, $Ra = 100$

a) $A_r = 0.5$ b) $A_r = 1$ c) $A_r = 2$
Fig: 1.2.2: Streamlines (Left) and Isotherms (Right) $A_r=1, Ra=100$
for a) $R_r=3$ b) $R_r=5$ c) $R_r=10$
CASE-I:

Figure 1.2.1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect ratio \((A_r)\) at \(R_r=3\) and \(Ra=100\). The magnitude of the streamlines increases with the increase in Aspect ratio \((A_r)\). The thermal boundary layer thickness increases with the increase in Aspect ratio \((A_r)\). It can be seen from the streamlines and isothermal lines that the fluid movement shifts towards from the lower portion of the hot wall to the upper portion of the cold of the vertical annular cylinder with the increase in Aspect ratio \((A_r)\). The circulation of the fluid covers almost whole domain at both lower and higher values of Aspect ratio \((A_r)\).

Figure 1.2.2 shows the streamlines and isothermal lines inside the porous medium for various values of Radius ratio \((R_r)\) at \(A_r=1\) and \(Ra=100\). It is seen that the streamlines and the isothermal lines tends to move away from the cold wall and reaches nearer to the hot wall of the vertical annular cylinder. This is due to the reason that as the width of the porous medium increases the thermal energy can’t able to travel through the whole width of the porous medium. The thermal boundary layer becomes thinner with the increase in Radius ratio \((R_r)\).
a) $Ra=25$

b) $Ra=50$

c) $Ra=100$

Fig: 1.2.3: Streamlines (Left) and Isotherms (Right) for $A_r=1$, $R_r=3$

a) $Ra=25$ b) $Ra=50$ c) $Ra=100$
Figure 1.2.3 shows the streamlines and isothermal lines inside the porous medium for various values of Rayleigh number (Ra) at $A_r=1$ and $R_r=3$. The magnitude of the streamlines increases with the increase in Rayleigh number. This is due to the reason that the Rayleigh number increases the fluid movement due to high buoyancy force, which in turn allows the high convection heat transfer to take dominant position at the lower portion of the vertical annular cylinder. The streamlines and isothermal lines shifts towards the cold wall of the vertical annular cylinder as Rayleigh number increases. The thermal boundary layer thickness decreases with the increase in Rayleigh number.
Fig: 1.2.4: $\bar{N}u$ Variations with $A_r$ at hot wall for different values of Ra at $R_r=3$

Fig: 1.2.5: $\bar{N}u$ Variations with $R_r$ at hot wall for different values of Ra at $A_r=0.5$
Figure 1.2.4 shows the variation of average Nusselt number ($\overline{Nu}$) at hot wall with respect to Aspect ratio ($A_r$) of the vertical annular cylinder. This figure is obtained for the value of $R_r=3$. The average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder decreases with the increase in Aspect ratio ($A_r$). It is found that the average Nusselt number ($\overline{Nu}$) at $A_r=0.5$ decreased by 17% when Rayleigh number is increased from 25 to 100. The corresponding decrease in average Nusselt number ($\overline{Nu}$) at $A_r=2$ is found to be 49%. We observe that at $A_r=1$, there is a sudden decrease in average Nusselt number as compared to that of lower values of Aspect ratio ($A_r$). This difference becomes more prominent as Aspect ratio ($A_r$) increases.

Figure 1.2.5 illustrates the effect of Radius ratio ($R_r$) on the average Nusselt number ($\overline{Nu}$). This figure corresponds to the value $A_r=0.5$. The average Nusselt number at hot wall of the vertical annular cylinder increases with increase in Radius ratio ($R_r$). It is found that the average Nusselt number at $R_r=1$ increased by 14% when Rayleigh number is increased from 25 to 100. The corresponding increase in average Nusselt number at $R_r=10$ is found to be 33%. This difference between the average Nusselt number at two different values of Rayleigh number increases with increase in Radius ratio ($R_r$). This difference becomes more prominent with the increase in Radius ratio ($R_r$) for high Rayleigh number (Ra).
Case-II: When heat is supplied at the Center half of the inner wall of the Vertical Annular Cylinder

a)

b)

c)

Fig: 1.3.1: Streamlines (Left) and Isotherms (Right) for $R_r=3$, $Ra=100$

a) $A_r=0.5$  b) $A_r=1$  c) $A_r=2$
Fig. 1.3.2: Streamlines (Left) and Isotherms (Right) $A_r=1, Ra=100$
for a) $R_r=3$ b) $R_r=5$ c) $R_r=10$
CASE-II:

Figure 1.3.1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect ratio ($A_r$) at $R_r=3$ and $Ra=100$. The magnitude of the streamlines increases with the increase in Aspect ratio ($A_r$). The thermal boundary layer thickness increases with the increase in Aspect ratio ($A_r$). It can be seen from the streamlines and isothermal lines that the fluid movement shifts towards the right upper portion of the cold wall vertical annular cylinder with the increase in Aspect ratio ($A_r$). The circulation of the fluid covers almost whole domain at lower values of Aspect ratio ($A_r$) as compared to the higher values of Aspect ratio ($A_r$).

Figure 1.3.2 shows the streamlines and isothermal lines inside the porous medium for various values of Radius ratio ($R_r$) at $A_r=1$ and $Ra=100$. It is seen that the streamlines and the isothermal lines tends to move away from the cold wall and reaches nearer to the hot wall of the vertical annular cylinder. This is due to the reason that as the width of the porous medium increases the thermal energy can’t able to travel through the whole width of the porous medium. The thermal boundary layer thickness decreases with the increase in Radius ratio ($R_r$).
Fig. 1.3.3: Streamlines (Left) and Isotherms (Right) for $A_r=1, R_r=3$

a) $Ra=25$  b) $Ra=50$  c) $Ra=100$
Figure 1.3.3 shows the streamlines and isothermal lines inside the porous medium for various values of Rayleigh number (Ra) at \( A_r = 1 \) and \( R_r = 3 \). The magnitude of the streamlines increases with the increase in Rayleigh number. This is due to the reason that the Rayleigh number increases the fluid movement due to high buoyancy force, which in turn allows the high convection heat transfer to take dominant position at the center portion of the vertical annular cylinder. The streamlines and isothermal lines shifts towards the upper portion of the cold wall of the vertical annular cylinder as Rayleigh number increases. The thermal boundary layer thickness decreases with the increase in Rayleigh number.
**Fig: 1.3.4**: $\overline{Nu}$ Variations with $A_r$ at hot wall for different values of $Ra$ at $R_r = 3$

**Fig: 1.3.5**: $\overline{Nu}$ Variations with $R_r$ at hot wall for different values of $Ra$ at $A_r = 0.5$
Figure 1.3.4 illustrates the effect of Aspect ratio ($A_r$) on the average Nusselt number ($\overline{Nu}$). This figure corresponds to the value $R_r=3$. It is seen that the average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder, decreases with the increase in Aspect ratio ($A_r$). It is found that the average Nusselt number ($\overline{Nu}$) at $A_r=0.5$ decreased by 7.5% when Rayleigh number is increased from 25 to 100. The corresponding decrease in Nusselt number ($\overline{Nu}$) at $A_r=2$ is found to be 38.7%. There is a sharp decrease in average Nusselt number ($\overline{Nu}$) at $A_r=1$, as compared to that of lower values of Aspect ratio ($A_r$). This shows that the average Nusselt number ($\overline{Nu}$) decreases with decrease in Rayleigh number (Ra).

Figure 1.3.5 shows the variation of average Nusselt number ($\overline{Nu}$) at hot wall with respect to Radius ratio ($R_r$). This figure is obtained for the value of $A_r=0.5$. The average Nusselt number ($\overline{Nu}$) increases with increase in Radius ratio ($R_r$). When Rayleigh number is increased from 25 to 100, it is found that the average Nusselt number ($\overline{Nu}$) at $R_r=1$ increased by 3.8%. The corresponding increase in average Nusselt number ($\overline{Nu}$) at $R_r=10$ is found to be 17.18%. This difference between the average Nusselt number ($\overline{Nu}$) at two different values of Rayleigh increases with increase in Radius ratio ($R_r$). This is due to the reason of high buoyancy force produced at high Rayleigh number (Ra), which increases the fluid movement at the hot wall and thus increased the average Nusselt number ($\overline{Nu}$).
Case-III: When heat is supplied at the Upper half of the inner wall of the Vertical Annular Cylinder

a) $\theta = 0.5$

b) $\theta = 1$

c) $\theta = 2$

Fig: 1.4.1: Streamlines (Left) and Isotherms (Right) for $R_r=3$, $Ra=100$
a) $A_r=0.5$ b) $A_r=1$ c) $A_r=2$
Fig: 1.4.2: Streamlines (Left) and Isotherms (Right) $A_r = 1$, $Ra=100$
for a) $R_r = 3$ b) $R_r = 5$ c) $R_r = 10$
CASE-III:

Figure 1.4.1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect ratio ($A_r$) at $R_r=3$ and $Ra=100$. The magnitude of the streamlines increases with the increase in Aspect ratio ($A_r$). The thermal boundary layer thickness increases with the increase in Aspect ratio ($A_r$). It can be seen from the streamlines and isothermal lines that the fluid movement shifts towards the right upper portion of the cold wall of the vertical annular cylinder with the increase in Aspect ratio ($A_r$). The circulation of the fluid covers almost whole domain at lower values of Aspect ratio ($A_r$) as compared to the higher values of Aspect ratio ($A_r$).

Figure 1.4.2 shows the streamlines and isothermal lines inside the porous medium for various values of Radius ratio ($R_r$) at $A_r=1$ and $Ra=100$. It is seen that the streamlines and the isothermal lines tend to move away from the cold wall and reaches nearer to the hot wall of the vertical annular cylinder. This is due to the reason that as the width of the porous medium increases the thermal energy can't able to travel through the whole width of the porous medium. The thermal boundary layer thickness decreases with the increase in Radius ratio ($R_r$). It is seen from the figure that the streamlines and the isothermal lines shift from the right upper portion of the cold wall to the left upper portion of the hot wall of the vertical annular cylinder.
Fig 1.4.3: Streamlines (Left) and Isotherms (Right) for $A_r=1$, $R_r=3$

a) $R_a=25$  b) $R_a=50$  c) $R_a=100$
Figure 1.4.3 shows the streamlines and isothermal lines inside the porous medium for various values of Rayleigh number (Ra) at $A_r=1$ and $R_r=3$. The magnitude of the streamlines increases with the increase in Rayleigh number. This is due to the reason that the Rayleigh number increases the fluid movement due to high buoyancy force, which in turn allows the high convection heat transfer to take dominant position at the upper portion of the vertical annular cylinder. The streamlines and isothermal lines shifts towards the upper portion of the cold wall of the vertical annular cylinder as Rayleigh number increases. The thermal boundary layer thickness decreases with the increase in Rayleigh number.
Fig: 1.4.4: $\overline{N_u}$ Variations with $A_r$ at hot wall for different values of $Ra$ at $R_r=3$

Fig: 1.4.5: $\overline{N_u}$ Variations with $R_r$ at hot wall for different values of $Ra$ at $A_r=0.5$
Figure 1.4.4 shows the variation of average Nusselt number ($\overline{Nu}$) at hot wall with respect to Aspect ratio ($A_r$) of the vertical annular cylinder. This figure is obtained for the value of $R_r=3$. It is seen that the average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder, decreases with the increase in Aspect ratio ($A_r$). When Rayleigh number is increased from 25 to 100, it is found that the Nusselt number ($\overline{Nu}$) at $A_r=0.5$ decreased by 11.6%. The corresponding decrease in Nusselt number ($\overline{Nu}$) at $A_r=2$ is found to be 41.15%. The average Nusselt number ($\overline{Nu}$) at low values of Aspect ratio ($A_r$) is very high as compared to the higher values of Aspect ratio ($A_r$). As the Aspect ratio ($A_r$) increases the difference between the average Nusselt number ($\overline{Nu}$) for a particular value of Rayleigh number also increases.

Figure 1.4.5 shows the variation of average Nusselt number ($\overline{Nu}$) at hot wall with respect to Radius ratio ($R_r$). This figure is obtained for the value of $A_r=0.5$. When Rayleigh number is increased from 25 to 100, at the hot wall of the vertical annular cylinder. It is found that the average Nusselt number ($\overline{Nu}$) at $R_r=1$ is increased by 9.69%. The corresponding increase in average Nusselt number ($\overline{Nu}$) at $R_r=10$ is found to be 20%. The difference between the average Nusselt number ($\overline{Nu}$) at two different values of Rayleigh number increases with increase in Rayleigh number. This is due to the reason that high Rayleigh number produces high buoyancy force, which leads to faster fluid movements and thus increased the average Nusselt number ($\overline{Nu}$).
Case-IV: When heat is supplied at the three different locations of the inner wall of the Vertical Annular Cylinder

a)

b)

c)

Fig: 1.5.1: Streamlines (Left) and Isotherms (Right) for $R_r=1$, $Ra=50$

a) $A_r=0.5$  b) $A_r=1$  c) $A_r=2$
Fig: 1.5.2: Streamlines (Left) and Isotherms (Right) \( A_r = 1, R_a = 50 \)
for a) \( R_r = 1 \) b) \( R_r = 5 \) c) \( R_r = 10 \)
CASE-IV:

Figure 1.5.1 shows the evolution of streamlines and isothermal lines inside the porous medium for various values of Aspect ratio ($A_v$) at $R_r=3$ and $Ra=100$. The magnitude of the streamlines increases with the increase in Aspect ratio ($A_v$). The thermal boundary layer thickness increases with the increase in Aspect ratio ($A_v$). It can be seen from the streamlines and isothermal lines that the fluid movement shifts towards the right upper portion of the cold wall of the vertical annular cylinder with the increase in Aspect ratio ($A_v$). The circulation of the fluid covers almost whole domain at lower values of Aspect ratio ($A_v$) as compared to the higher values of Aspect ratio ($A_v$).

Figure 1.5.2 shows the streamlines and isothermal lines inside the porous medium for various values of Radius ratio ($R_r$) at $A_v=1$ and $Ra=100$. It is seen that the streamlines and the isothermal lines tends to move away from the cold wall and reaches nearer to the hot wall of the vertical annular cylinder. This is due to the reason that as the width of the porous medium increases the thermal energy can’t able to travel through the whole width of the porous medium. The thermal boundary layer thickness decreases with the increase in Radius ratio ($R_r$). It is seen from the figure that the streamlines and the isothermal lines shift from the right upper portion of the cold wall to the left center portion of the hot wall of the vertical annular cylinder.
Fig: 1.5.3: Streamlines (Left) and Isotherms (Right) for $A_r=1$, $R_r=1$
 a) $Ra=25$ b) $Ra=50$ c) $Ra=100$
Figure 1.5.3 shows the streamlines and isothermal lines inside the porous medium for various values of Rayleigh number (Ra) at $A_r=1$ and $R_r=3$. The magnitude of the streamlines increases with the increase in Rayleigh number. This is due to the reason that the Rayleigh number increases the fluid movement due to high buoyancy force, which in turn allows the high convection heat transfer to take dominant position at the center portion of the vertical annular cylinder. The streamlines and isothermal lines shift from the center of the hot wall to the upper portion of the cold wall of the vertical annular cylinder as Rayleigh number increases. The thermal boundary layer thickness decreases with the increase in Rayleigh number.
Fig. 1.5.4: $\overline{Nu}$ Variations with $A_r$ at hot wall for different values of $Ra$ at $R_\perp=3$

Fig. 1.5.5: $\overline{Nu}$ Variations with $R_\perp$ at hot surface for different values of $Ra$ at $A_r=0.5$
Figure 1.5.4 illustrates the effect of Aspect ratio ($A_r$) on the average Nusselt number ($\overline{Nu}$). This figure corresponds to the value $R_r=3$. It is seen that the average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder, decreases with the increase in Aspect ratio ($A_r$). It is found that the average Nusselt number ($\overline{Nu}$) at $A_r=0.5$ is decreased by 8% when Rayleigh number is increased from 25 to 100. The corresponding decrease in average Nusselt number ($\overline{Nu}$) at $A_r=2$ is found to be 10%. At $A_r=1$, there is a sharp decrease in average Nusselt number ($\overline{Nu}$) as compared to that of lower values of Aspect ratio ($A_r$). The difference between the average Nusselt number ($\overline{Nu}$) at two different values of Rayleigh number increases with the increase in Aspect ratio ($A_r$).

Figure 1.5.5 shows the variation of average Nusselt number ($\overline{Nu}$) at hot wall with respect to Radius ratio ($R_r$). This figure is obtained for the value of $A_r=0.5$. The average Nusselt number ($\overline{Nu}$) at hot wall of the vertical annular cylinder increases with increase in Radius ratio ($R_r$). It is found that the average Nusselt number ($\overline{Nu}$) at $R_r=1$ increased by 9.8% when $Ra$ is increased from 25 to 100. The corresponding increase in average Nusselt number ($\overline{Nu}$) at $R_r=10$ is found to be 10.8%. This difference between the average Nusselt number ($\overline{Nu}$) at two different values of Rayleigh number increases with increase in Rayleigh number. This difference becomes more prominent with the increase in Radius ratio ($R_r$) for higher values of Rayleigh number ($Ra$).
References:


