3.1 Introduction

The study of sheath related phenomena is getting more importance for understanding the boundary layer between the plasma and the solid surface, because of its practical applications in plasma processing, fusion devices using probes, antenna, divertor, limiter etc. The sheath is a non-neutral region between the physical boundary and plasma and is a natural process in any practical plasma device. The energy distribution of the particles striking the wall is of significant importance for many applications in low pressure plasma processing (Chapman¹ 1980). The shape and magnitude of the sheath determine the energy and flux of the particles at the surface of the boundary wall. The strong electric field within the sheath is followed by a weaker presheath region, which is nothing but the well known Bohm
The existence of sheath induced instability in the presheath region is due to the occurrence of some perturbations in plasmas.

Though there have been several studies of sheath related phenomena, the study of plasmas at the sheath edge i.e. the study of plasma presheath was first done by Measick et al. (1985). They determined the plasma potential in a multidipole plasma system with the help of emissive probe by using inflection point method in the limit of zero emission. They described the particles in the bulk plasma by a specified version of plasma sheath equation and observed the temporal evolution of ion acoustic waves propagating through the plasmas.

There are also few reports which include the properties of sheath induced instability [Stenzel et al. (1988, 1989), Chutia et al. (1991), Popa et al. (1976), Barrett et al. (1989), Ohno et al. (1981), Amstrong et al. (1991), Piel at al. (1996), etc.]. However the growth of instability due to the combined interaction of sheath, plasma and ion beam in a double plasma device have not been studied in details yet. Stenzel (1988, 1989) observed the instability frequency in the range of electron plasma frequency due to the resonance of sheath and plasma elaborately. He concluded that the excitation of instability is due to the negative radio frequency sheath resistance associated with the electron inertia.

The first low frequency instability induced due to a negatively biased grid was reported by Popa et al. (1976) in a Q-machine. They explained the cause of instability as due to the space charge formed between negatively biased grid and the cold end plate, which is situated at a long distance from the grid. Same kind of instability characteristics was reported by some other authors also, but in a larger size of the machine such as Double Plasma Device, where the separation grid is used.
For instance, Barrett et al. (1989), reported an instability near the ion plasma frequency in a double plasma device, where plasma is produced only in the source chamber. The target chamber plasma is nothing but the plasma leaking through the grid from the source side of the device. They determined the oscillation frequency in such a case by the transit time model through the sheath. They speculated the mechanism might be the relaxation effect involving the surplus and deficit of electrons from the source plasma. However the analogy between the transit time effect for diodes and plasma sheath have been pointed out in a case of ion rich sheath [Rosa et al. (1971)]. Oliver et al. (1973) performed an experiment on ac sheath impedance due to ion inertia effect at frequencies above the ion plasma frequency, but no instabilities were observed considering the lack of resonance in the system. Further studies were done by Chutia et al. (1991) on Berratt and Greaves’s work by including plasma in the target chamber also. They observed that in such cases the oscillations are considered to be excited by Klystron bunching effect of the incident ions toward the ion rich sheath. Recently, excitation mechanism of the instability has been identified as the negative RF (radio frequency) resistance associated with the ion inertia by an ion rich sheath explained by Komori et al. (1992). This current carrying in the sheath, thus, ultimately leads to the cascading bifurcation and then chaos. The low frequency instability observed by Popa et al. (1994) was explained by resonance phenomena between the ion bounce around the negatively biased grid and the periodic relaxation of the plasma potential in the target chamber. They determined the density and the grid bias dependence frequency as

\[ f^2 = 1.2 \times 10^2 \left( \frac{n}{V_g^{1/3}} \right) \quad --- \quad --- \quad (3.1) \]
Where 'n' and $V_s$ are the density and the grid biasing voltage respectively.

Armstrong\textsuperscript{10} \textit{et al.} (1991) observed the ion beam driven instability at a probe in a double plasma device. They performed the experiment in a stainless steel cylindrical vacuum vessel of 120 cm. in length and 60 cm. in diameter. In this 120 cm. long chamber, they considered approximately 25 cm. source chamber and rest as the target chamber. In this operating condition, they have observed both the instability and intermittency on a plane probe under the influence of ion beams. Piel\textsuperscript{11} \textit{et al.} (1996) observed the ion sheath oscillations around a negatively biased grid experimentally and compared the results with PIC simulation. They concluded that due to the asymmetry densities of the source and the target chamber, the ion sheath in the low density side becomes unstable to virtual anode oscillations and acts as the origin of ion bounce oscillation in the sheath region. They found that the observed frequency tallies well with the ion plasma frequency at the sheath edge. They however concludes that there are three resonating elements in their experiment which gives rise to the unstable condition.

Though there have been numbers of report on sheath induced instability, but studies on the low frequency instability due to combined effect of sheath, plasma and ion beam are very few. The asymmetry of the ion rich sheath as well as the potential difference of the source and the target chamber due to the introduction of the ion beam into the source section are the main contributors to the excitation of such type of sheath driven low frequency instability. In this chapter the basic theory of the sheath formation along with the minimum requirements criteria i.e. Bohm criteria regarding the sheath formation is described. But this chapter deals mainly with the beam enhanced sheath instability in a double plasma device,
under the equilibrium density condition in both chambers. Also the dependence of instability frequency on different parameters has been observed and discussed.

3.2 Theory

3.2.1 Sheath Formation

The sheath formation is a natural consequence of a boundary plasma system. It can also be defined as a non-neutral space charge located in the vicinity of the surface of the boundary wall. It provides a self-consistent electrostatic potential barrier for the confinement of the highly mobile plasma species like electrons. So the boundary wall floats at some negative steady state potential with respect to the bulk plasma. Hence, the repulsion of electrons will take place and as a result of this there will be formation of positive space charge region called “sheath” (Riemann16 1991). In steady state case, the distribution of the charger species (within the space time scale ~ a few Debye length) is so adjusted that to leave the bulk plasma quasineutral.

So, sheath can shield the neutral plasma from the negative wall. The potential energy \((e\Phi)\) is normalised with the electron thermal energy \((kT_e)\) in order to get

\[
\Phi = -\left(\frac{e\Phi}{kT_e}\right)
\]

(3.2)

If \(x\) is the space co-ordinate, with \(\lambda_{De}\) as the Debye length which is given by

\[
\lambda_{De} = \left(\frac{e_o k T_e}{n_o e^2}\right)^{1/2}
\]

(3.3)

where \(e_o\) and \(n_o\) are the permittivity constant and charge particle density of plasma respectively.
Let us consider a basic physical model for the formation of stable ion sheath in a two component plasma system (Dwivedi17 1995) with the Maxwellian distribution of particles. The electrons are assumed to follow Boltzmann distribution within the steady state time scale of sheath and is given by

\[ n_e = n_0 \exp \left( \frac{e\phi}{T_e} \right) \]  

where \( n_0, n_e \) and \( T_e \) are the electron density, neutral particle density and electron temperature of plasma respectively. The heavier ions are described by full inertial dynamics with the ion density distribution as

\[ n_i = n_0 \left[ 1 - \frac{2e\phi}{m_i v_n^2} \right] \]  

where \( v_n \) and \( m_i \) are velocity of ions and ion mass respectively. These two equations satisfy the Poission’s equation, which reduces the energy integral equation as

\[ \frac{1}{2} \left( \frac{e^2}{\varepsilon_0} \right) + V(\varphi) = 0 \]  

where \( V(\varphi) = 1 - \exp(\varphi) \) and \( M^2 \left[ 1 - \left( \frac{2\varphi}{M^2} \right)^{1/2} \right] \).
edge restrict the ion flow velocity at the entrance of the sheath by the condition $M > 1$, which is called the well known Bohm sheath criterion. So the plasma arranges itself in such a way that the electric field is localised near the wall of the system. As a result, in the sheath region there exists a strong charge separation and a decreasing potential and increasing drift velocity (Bohm velocity) of ions toward the sheath region take place.

Bohm$^2$ (1949) himself led the idea of presheath of the order of potential drop ($T_e/2$), where ions are accelerated to a sufficient velocity so as the ion density exceeds the electron density in the sheath region. Its size is given by half of the device dimension in case of a collisionless plasma, while it is equal to ion mean free path in case of collisional plasma. So, an electric field is required in the presheath region in order to get the velocity by the ion beam, which was first introduced by Hu$^{18}$ et al. (1966). The electric field does not vanish at the sheath edge, but it goes on extending into the background plasma and becoming smaller and smaller. There are several theoretical reports on the properties of the sheath and presheath at different plasma conditions [Emmert$^{19}$ et al. (1989), Scheuer$^{20}$ et al. (1980), Godyak$^{21}$ et al. (1990) and Sheridan$^{22}$ et al. (1989)]. However, very few experimental measurements on presheath have been reported so far (Measick$^3$ et al. 1985, Meyer$^{23}$ et al. 1992). In the limiting case $\frac{\lambda_{pe}}{l} \to 0$, where $l$ is the complete scale length of the chamber, the presheath is treated on a scale $\sqrt{l}$, while the sheath is treated on a scale $\xi = \frac{x}{\lambda_{pe}}$. The sheath edge is the region of drastic fall of plasma potential and it is defined in the approach on a presheath scale by the location where $\frac{d\phi}{dn} \to -\infty$, while on the Debye scale length by
\[
d\frac{d\phi}{dz} > 0 \text{ at } \xi \to \infty. \text{ The sheath edge represents the boundary at which the quasineutrality breaks down. The Bohm criteria is mathematically given by Stangeby}^{24} \text{ et al. (1994) as}
\]

\[
\left[ \frac{1}{v_n^2} - \frac{k_B(T_e + T_i)}{m_i} \left[ \frac{dn_e}{dr} \bigg|_{r=r-} - \frac{dn_i}{dr} \bigg|_{r=r-} \right] \frac{dv}{dr} \right] \bigg|_{dr=0} = 0 \quad --- \quad --- \quad (3.7)
\]

where \( v' \) and \( r \) are the flow velocity and distance respectively. So, at the entrance

\[
\frac{dn_e}{dr} = \frac{dn_i}{dr} \text{ and } \frac{dv}{dr} \neq 0 \text{ and hence } v' = C_s. \text{ For the marginal case and } \frac{dn_e}{dr} > \frac{dn_i}{dr} \text{ at } r = \infty,
\]

within which \( v' > C_s. \)

Thus the generalised form of Bohm criterion can be written as (Allen\(^{25} \) et al. 1976)

\[
\left[ \frac{f(v)}{v^2} \right]^{1/7} dv < \frac{m_i}{kT_i} \quad --- \quad --- \quad (3.8)
\]

where \( f(v) \) is the normalised ion distribution function at the plasma sheath interface, \( k \) is the Boltzmann constant. This result is outcome of the plasma sheath boundary condition

\[
\frac{dn_e}{d\phi} \leq \frac{dn_i}{d\phi}, \text{ with the } \phi \text{ as the sheath potential at the plasma sheath boundary. This derivation is given by Bissel}^{26} \text{ et al. (1987) in a simple plasma with Boltzmann distribution of electrons and full ion inertial dynamics without source or sink of the plasma. The above inequality represents the condition that the electron density should fall rapidly than the ions on the potential within the sheath.}
3.2.2. Theory of the beam plasma instability

The excitation mechanism due to the sheath driven low frequency instability by a positive ion rich sheath is proposed by a simple model (Dwivedi 1995). This model is based on the natural property of sheath which requires the ions to enter the sheath region with a streaming flow velocity greater than the ion acoustic velocity i.e. Bohm velocity. Thus an ion rich sheath may be technically treated as the source of ion beam propagating across the potential well from the sheath - boundary plasmas. So, the sheath driven instability can be explained as the streaming instability excited by these ion beams which are propagating across the potential well from the bulk plasma region.

For the illustration of this model in relation with the experiment, let us consider the source ion beams are those entering from the source plasma side and target ion beams are those entering from the target plasma side of a double plasma device. In symmetric sheath these beams after passing through the potential well formed at the grid, interact with the counter stream of ions from the bulk plasma of the source and the target chamber. Since it is well observed that the symmetric potential cannot excite any instability even for any grid biasing voltage the asymmetry of the sheath becomes unstable. This asymmetry can be produced by biasing the source plasma at a higher positive potential with respect to the target plasma. In this condition the ion beam will propagate to the target side with a free fall velocity. At the same time the ion beams which are propagating from the target side toward the grid will be reflected back to the target side due to the asymmetry of the potential structure. As a result, there forms a combination of three sets of ion beams (two sets are comoving and one set is countermoving) at the entrance point of the target sheath. Assuming the Boltzmann distribution of electrons and full inertial dynamics of the ion beams, normal
mode analysis can be carried out to derive the following dispersion relation of the sheath driven mode:

\[ 1 + \frac{1}{k^2 \lambda_{DC}^2} \left( \omega_s^2 \right) - \frac{\omega_t^2}{(\omega - k \cdot v_s)^2} - \omega_t^2 \left[ \frac{1}{(\omega - k \cdot v_t)^2} + \frac{1}{(\omega + k \cdot v_t)^2} \right] = 0 \quad \cdots \quad (3.9) \]

Where \( \omega_s \) and \( \omega_t \) are the source and the target ion plasma frequencies, \( v_s \) and \( v_t \) are source and target ion beam velocities respectively. In the source ion beam frame, the above dispersion relation can be read as

\[ \frac{\Omega^2 - \Omega_s^2}{\Omega_t^2} = 2 \Omega_t^2 \frac{[\Omega^2 + 2 \Omega k \cdot v_s + (k \cdot v_s)^2 + (k \cdot v_t)^2]}{[\Omega + k \cdot (v_s + v_t)] [\Omega + k \cdot (v_s - v_t)]^2} \quad \cdots \quad (3.10) \]

Where \( \Omega_s^2 = \frac{k^2 \lambda_{DC}^2 \omega_s^2}{(1 + k^2 \lambda_{DC}^2 \omega_s^2)} \), \( \Omega = \omega - k \cdot v \) With \( \alpha = s, t \)

Under the mode-mode resonant coupling model with eigen frequencies; \( \Omega \equiv \Omega_s, \Omega \equiv [k \{v_s - v_t\}] \) and \( \Omega - \Omega_s = \Delta \Omega \), Eq. (3.10) can be reduce to cubic dispersion relation as

\[ (\Delta \Omega)^3 = \Omega_s \Omega_t^2 F(k) \quad \cdots \quad \cdots \quad (3.11) \]

Where

\[ F(k) = \frac{\Omega_s^2 + 2 \Omega k \cdot v_s + (k \cdot v_s)^2 + (k \cdot v_t)^2}{[\Omega + k \cdot (v_s + v_t)]^2} \]

In the resonant condition, \( k \cdot (v_s - v_t) < 0 \), restrict the fluctuation to propagate toward the higher potential side of the asymmetry sheath for \( v_s > v_t \). It is obvious that for \( \Delta \Omega < 0 \), an unstable solution exist with the growth rate

\[ \gamma = [\Omega_s \Omega_t^2 F(k)]^{1/3} \quad \cdots \quad \cdots \quad (3.12) \]
Therefore it is clear that the asymmetry causes the resonance to occur with the resonance wave vector value \( k_r \approx \frac{\Omega_s}{(v_r - v_i)} \). Some of the result of Eq. (3.9) are shown in Fig. 3.2 and Fig. 3.3, which is the existence of one unstable root. The spatial localization of the fluctuation near the sheath may not be explained within the model of uniform ion beams. For full analysis of the realistic solution of eigen value problem would have to be solved with due consideration of the presheath potential profile. In symmetric sheath the dispersion relation is given as

\[
1 + \frac{1}{k^2 \lambda_{pe}^2} - \omega_p^2 \left[ \frac{1}{(\omega - k.v_o)^2} + \frac{1}{(\omega + k.v_o)^2} \right] = 0 \quad (3.13)
\]

Where \( \omega_p \) is the ion plasma frequency of the counter streaming ion beams with velocity \( v_o \).

From the root characteristics of this equation, the instability condition \( k.v_o < \frac{kC_s}{(1 + k^2 \lambda_{pe}^2)^{1/2}} \), which is not satisfied for sonic beams, it rules out the possibility of any instability in a symmetric sheath.

3.3 Experimental Setup.

The experiment is carried out in a double plasma device equipped with multidipole magnets for surface plasma confinement. The schematic diagram of the experimental setup is shown in Fig. 3.1. The mesh grid across which the sheath is produced is of 20 lines per cm. The chamber is evacuated down to \( 1 \times 10^{-6} \) Torr with the help of a rotary and a diffusion pump. The Ar plasma is produced by injecting Ar gas in to the chamber at a working
FIG. 3.2 & 3.3 Existence of unstable roots by numerical calculation of Eq. 3.9.
FIG. 3 Schematic diagram of the experimental setup.
pressure $4 \times 10^{-4}$ Torr under continuous pumping. The discharge voltage and the current is fixed at 70V and 60 mA respectively in both the chambers. The ion sheath is produced by biasing the grid negatively with respect to the ground. An extra positive voltage ($V_s$) is applied to the source anode with respect to the target in order to get an ion beam in the target section. The different plasma parameters are measured with the help of plane Langmuir probes ($L_1$, $L_2$). Retarding ion energy analyzer (R) is used determine the ion beam temperature and the ion beam energy. The energy analyzer consist of two fine nickel grids and a collector plate placed inside a stainless still housing of 2.3 cm diameter and 0.5 cm in length. The first grid is left floating at plasma floating potential to repel electrons and allow most of the ions to pass through. The second grid, called discriminator grid, is biased positively. The collector is biased negatively at $-90V$ to collect all the ions and repel residual high energy electrons, which penetrates through the first grid.

The plasma potential profile has been measured on both sides of the chambers, with the help of emissive probes ($E_1$, $E_2$). The emissive probe is 1% thoraited tungsten wire of 0.005 cm in diameter and 0.5 cm in length. The probe is heated by a half wave rectified DC voltage. In order to get accuracy of measurement, utmost care is taken and the experiments are repeated for several times. Under the maintained experimental conditions the plasma parameters are: density $\sim 10^8 - 10^9$ cm$^{-3}$, electron temperature ($T_e$) $\sim 1.5$ eV to 2 eV and the ion temperature ($T_i$) $\sim 0.1$ eV.

The unstable oscillatory signals are picked by the Langmuir probe in the target chamber, which is fed to the spectrum analyzer. The Langmuir probe, which picks up the signal, is positively biased with respect to the plasma potential in order to detect any plasma
fluctuation in the electron saturation current. To know the dispersion relation of the different modes excited in plasma, usual interferometer method is used by applying continuous sinusoidal signals in to the anode of the source chamber.

3.4 Experimental results

With the different conditions mentioned above, a low frequency instability is found to be excited in the ion beam plasma system. This instability depends on different plasma parameters like plasma potential, sheath thickness, source anode biasing voltage ($V_A$), grid biasing voltage ($V_g$) etc. The behaviour of the sheath instability on different parameters has been studied in detail and described as follows:

3.4.1 Measurement of Potential Profile

The plasma potential profile on both sides of the chamber has been measured with the help of an emissive probe. The floating point of an strongly emitting probe technique is used to determine the plasma potential as described by Kemp et al. (1966). The probe tip is in the form of a small wire loop so that it can be heated to emission by the passage of current. Whenever the probe is sufficiently positive, the emitted electrons will be drawn back to the probe and collected electron current will be essentially unaffected by the emission. To get the proper condition of the emissive probe for plasma potential measurement, firstly I-V characteristics is drawn with respect to the probe emitting voltage ($V_{EIP}$) as shown in Fig. 3.4 and the $V_{EIP}$ is fixed in such a way that the floating potential is almost equal to the plasma
FIG. 3.4 I-V characteristics drawn by an emissive probe with respect to probe emitting voltage ($V_{EH}$).
space potential. This technique is used to determine the plasma potential spatially along the axial direction starting from the grid.

The axial distribution of the time averaged plasma potential on both sides of the grid are obtained by using emissive probe is shown in Fig. 3.5. In Fig. 3.5 (a) and (b), the plasma potential profile for source anode biasing voltage $V_s$'s are shown for constant grid biasing voltage $V_g = -100$V and $-80$V respectively. Obviously the plasma potential in the source chamber becomes higher when the source anode biasing voltage is increased with respect to the grounded target chamber. However without the application of source biasing voltage (i.e. $V_s = 0$V), the potential is found to be almost equal in both the chamber.

3.4.2. Measurement of Sheath Thickness

An ion rich sheath is produced across the separation grid by biasing the grid with a negative voltage ($-V_g$) with respect to ground. The sheath thickness is measured from the region of sharp fall of plasma potential near the grid as shown by the arrow mark of Fig. 3.5 (a) and (b). It is seen that, when there is no difference of plasma potential between the source and the target plasma i.e. $V_s = 0$V, the sheath thickness is smaller at floating grid voltage. The thickness, however, increases with the slight increase of $V_s$, but it remains constant beyond $V_s = 5$V.

The sheath thickness on the other hand changes considerably with the negative grid biasing voltage ($V_g$). The plasma potential profile for different $V_g$'s are shown in Fig. 3.6, when $V_s$ is kept at 10V. It is seen that sheath thickness increases more prominently in the target plasma with respect to $V_g$. However on the source side the sheath thickness is much
FIG 3.5 Axial plasma potential profile on both sides of the grid measured by an emissive probe for different value of $V_e$.

(a) When $V_e = -80V$

(b) When $V_e = -100V$.
FIG. 3.6 Axial plasma potential profile on both sides of the grid for different $V_r$, when $V_r = 10V$
smaller and does not change considerably with the change of $V_g$. The current–voltage characteristic of the grid ion current ($I_g$) drawn across a resistance (150 kΩ) connected to the grid, with respect to $V_g$ are recorded for different values of $V_s$. The plot of log $I_g$ verses log $V_g$ at $V_s = 10V$ is shown in Fig. 3.7. The variation of $I_g$ with respect to $V_g$ follows the well known Child Langmuir Law as

$$I_g = \left( \frac{4\varepsilon_0}{9} \right) \left( \frac{2e}{M_i} \right)^{1/2} \left( \frac{V_s}{d^2} \right)$$  \hspace{1cm} (3.14)

Where ‘$\varepsilon_0$’ is the permittivity of the free space, ‘$e$’ is the electron charge, ‘$M_i$’ is the ion mass and ‘$d$’ is the sheath thickness. However, this law is found to be valid completely for lower $V_s$, but for $V_s > 25V$, this law is slightly modified.

Substituting the values of different $V_g$ and $I_g$ in Eq. (3.14), the sheath thickness is calculated and shown in Table-I. The experimentally measured sheath thickness $d(= d_s + d_T)$, where $d_s$ and $d_T$ are sheath thickness in the source and the target chamber respectively from the potential profile curve is also shown in the same table. It is found that the calculated values from Eq. (3.14) are approximately equal to the measured values from the potential profile curve. So, the dependence of $I_g$ on $V_g$ and $d$ confirms the Child-Langmuir law.

3.4.3 Measurement of Ion-Beam Energy

The ion beam is found to exist in the target section as soon as a positive potential is applied to the source anode with respect to the grounded target. The application of $V_s$ in the source chamber indicates the presence of extra beam ions in the target chamber. These ion
FIG. 3.7 Variation of log $I_g$ for different log $V_g$ at $V_z = 10V$. 
TABLE I. The theoretically calculated and experimentally measured sheath thickness for different $V_g$ and corresponding instability frequency $f_o$ when $V_g = 10V$.

<table>
<thead>
<tr>
<th>Grid bias voltage $V_g$ (Volts)</th>
<th>$d$ (cm) Theoretical</th>
<th>$d$ (cm) Experimental</th>
<th>$f_o$ kHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.70</td>
<td>0.50</td>
<td>440</td>
</tr>
<tr>
<td>50</td>
<td>0.72</td>
<td>0.53</td>
<td>500</td>
</tr>
<tr>
<td>60</td>
<td>0.76</td>
<td>0.60</td>
<td>513</td>
</tr>
<tr>
<td>70</td>
<td>0.80</td>
<td>0.65</td>
<td>504</td>
</tr>
<tr>
<td>80</td>
<td>0.83</td>
<td>0.70</td>
<td>483</td>
</tr>
<tr>
<td>90</td>
<td>0.85</td>
<td>0.80</td>
<td>474</td>
</tr>
<tr>
<td>100</td>
<td>0.88</td>
<td>0.80</td>
<td>448</td>
</tr>
<tr>
<td>120</td>
<td>1.05</td>
<td>0.90</td>
<td>432</td>
</tr>
</tbody>
</table>
beams are generally detected with the help of an energy analyser (R). The current-voltage characteristics of the energy analyser for different $V_s$ (10V, 20V and 30V) is shown in Fig. 3.8. The energy distribution function $F(V)$ of the given I-V characteristics are obtained from the first differentiation of the same characteristics and also shown in the same figure by dotted curve. The ion beam appears as a bump on tail in the distribution function curve. 
When $V_s = 10V$ is applied to the source anode, the value of the beam energy from the energy analyser data is found to be 9.6 V i.e. slightly less than the actual value of $V_s$. Similar situation is also obtained, when $V_s = 20 V$ and 30 V. This slightly lower value of the energy analyser data from the actual value is due to the low resolution of the ion energy analyser.

3.4.4. Measurement of Dispersion Relation

The dispersion relation is measured experimentally by interferometer method. A continuous sinusoidal signal of frequencies from 50 KHz to 1000 KHz (50 KHz in steps) is applied to the source anode in order to excite waves in plasmas. A block diagram of the experimental arrangement is shown in Fig. 3.9. Here also Langmuir probe is used as a diagnostics to detect the perturbation, which is propagating through the plasmas. Signals from the probe are amplified and attenuated and then mixed with the input signals. A motor driving system is used to move the probe slowly in the axial direction. The interferometer output results from the mixer are recorded in X-Y recorder as function of distance. In this case to get the dispersion relation in presence of the ion beam, a continuous sinusoidal wave of peak to peak amplitude ~ 500 mV is applied to the source anode. The interferometer patterns are drawn with the help of usual interferometer method. The interferometer results
FIG. 3.8 Current-Voltage characteristics of an energy analyser for $V_s = 10V, 20V$ and $30V$. The corresponding energy distribution functions are also shown.
FIG. 3.9 Block diagram of the experimental arrangement for dispersion relation.
detecting both ion acoustic wave mode and the beam mode for different frequencies are shown in Fig. 3.10 [(a) 350 KHz, (b) 400 KHz, (c) 450 KHz and (d) 500 KHz]. In this case values of $V_s$ are 5, 10, 15 and 20 V and $V_g$ are $-40$V and $-20$V. It is observed that when $V_s < 10$V i.e. $v_b < 3C_s$, where $v_b$ and $C_s$ are velocity of ion beam and ion acoustic speed respectively, the ion acoustic wave does not appear. But with the increase of grid voltage ($V_g$) [Pattern (c)], the ion acoustic wave disappear and only the beam mode propagate through the plasma. The dispersion relation for different $V_s$'s are shown in Fig. 3.11 (a) to (d). Fig. 3.11 (a) - (c) shows the beam mode, when $V_s = 15$V, 10V and 5V respectively. But at $V_s = 15$V, both the beam mode and ion acoustics mode appears simultaneously [Fig. 3.11 (a) - (d)].

Considering the distribution function of the ion beam in this case as drifted Maxwellian, the dispersion relation can be written as [Kawai et al. (1979)]

$$
\varepsilon(\omega, k) = 1 - \left(\frac{k_D^2}{2k^2}\right)[Z'(X_e) + (1 - \alpha)\theta Z'(X_i) + \alpha\theta_b Z'(X_b)] = 0 \quad (3.15)
$$

Where $X_j = \left(\frac{\omega}{k_j^2} - v_j\right)\left(\frac{2T_j}{M_j}\right)^{1/2}$; $j = e$, $i$, $b$ i.e. for electron, ion and beam respectively. $\omega$, $k$ and $v$ represents frequency, wave vector and velocity respectively. Also $k_D = l/\lambda_D$, where $\lambda_D$ is the Debye length and $\theta = T_e/T_b$ (ratio of electron to beam temperature), $\alpha = n_b/n_e$ (ratio of beam to electron density), $M_i = M_b$ is the ion mass and $Z'$ is the derivative of the dispersive function. The numerical results using Eq. (3.15) for $\theta = 10$ and $\theta_b = 25$ agree well with the experimental results when $v_b \geq 3C_s$. 
FIG. 3.10 Interferometer wave pattern for different applied frequencies.

350 kHz, (b) 400 kHz, (c) 450 kHz and (d) 500 kHz at $V_s = 5V, 10V, 15V$ and $20V$. 
FIG. 3.11 Dispersion relation showing the wave and ion beam modes obtained from interferometer patterns, (a) and (d) for \( V_i = 15V \) shows the coexistence of beam and ion acoustic wave respectively, while (b) and (c) for \( V_i = 10V \) and 5V shows only the beam mode.
The presence of ion acoustic wave mode along with the beam mode in the interference pattern is accounted for by calculating the excitation coefficient of the wave together with the dispersion relation as given by Gloud et al. (1964). The excitation coefficient [which is considered to be nearly proportional to \((\frac{ds}{dk})^{-1}\)] of the excited ion acoustic wave for \(V_s = 15V\) (lower \(V_g\) is found to be much higher than that of the beam mode. Therefore it appears along with the beam mode overcoming Landau Damping.

3.4.5 On Beam Enhanced Sheath Instability

It has been observed that the instability frequency \(f_o\), which is much smaller than the ion plasma frequency (MHz) is excited in the target plasma for a threshold value of \(V_s\) and \(V_g\). It is clearly observed that for constant \(V_g\), a minimum value of \(V_s\) is required to excite the instability. A set of spectrum analyser traces observed for \(V_g = -20V, -40V, -60V, -80V, -100V\) and \(-120V\) are shown in Fig. 3.12 (a), (b), (c), (d), (e) and (f) respectively. In all cases it is seen that for any value of \(V_g\) a threshold value of \(V_s\) is required to trigger the instability. It is also noted that for lower value of \(V_g\), the value of \(V_s\) is also required less to trigger the instability. However the value of \(V_s\) increases with the increase of \(V_g\) though the rate of increase of \(V_s\) is very low as observed. The threshold values for excitation of instability and the dependence of \(f_o\) on \(V_s\) for several constant \(V_g\)'s are shown in Fig. 3.13. It is noted that for lower negative \(V_g\), the instability appears at lower \(V_s\) with higher initial \(f_o\).
FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_s$.

(a) When $V_g = -20V$.

(b) When $V_g = -40V$. 
FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_s$.

(c) When $V_g = -60V$. 

---

FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_s$.

(c) When $V_g = -60V$. 
FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_s$.

(d) When $V_R = -80V$. 

$V_s =$

$26V$

$24V$

$20V$

$18V$

$14V$

$10V$

$80V$

$60V$

$34V$
FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_S$.

(e) When $V_R = -100V$. 
FIG. 3.12 A set of spectrum analyser traces of the excited instability for different $V_s$.

(f) When $V_g = -120V$. 
FIG. 3.13 Dependence of the excitation instability frequency on source anode biasing voltages with $V_x$ as parameter.
Since $f_0$ is clearly dependent on $V_g$ and $V_s$, the ion beam and the sheath thickness is playing an important role on the excitation of instability. In case of $V_s = 0V$, i.e. when there is no ion beam, no instability is found to be excited. But for a particular $V_s$, a threshold value of $V_g$ is required to trigger the instability.

3.5 Discussions

The above mentioned instability frequency has some physical mechanism, which is described as follows:

An ion rich sheath is formed around a negatively biased grid. The ions from the presheath region on both sides of the chamber, where the potential difference is of the order of $(T_e/2)$ are moving with a Bohm speed toward the grid in accordance with the Bohm sheath criterion. The ions entering the sheath region with a drift velocity ($>C_s$), are then accelerated toward the grid and due to inertia go to the other side. The ions after crossing the grid from the target side meet the higher potential barrier at the source side, from where the ions are reflected back again to the target side. These reflected ions after crossing the grid again (i.e. sheath around the grid) appear as a reflected beam on the target side. At the same time there is also an ion beam coming from the source plasma to the target plasma due to the free fall of the ions caused by the difference of plasma potential. The energy of these free fall ion beam is calculated by the relation

$$\frac{1}{2} M_i v_h^2 = e\phi$$

(3.16)
Where \( v_h \) is the velocity of the ion beam and \( \phi \) is the plasma potential difference between the source and the target chamber.

Therefore, in the target plasma there exist three sets of ion beams, two of them are moving away from the grid toward the target side, while the other one is counter streaming (i.e. toward the grid from the target plasma) at a Bohm speed. The two experimental controlling parameters of the instability existence are \( V_g \) and \( V_s \), \( V_s \) determines the free fall ion beam velocity as well as reflected ion beam velocity due to difference of plasma potential between the two chambers. While \( V_g \) determines the thickness of the sheath, which increases with the increase of \( V_g \). So, from the over all experimental results it is found that (i) \( f_o \) is inversely proportional to sheath thickness, (ii) the ions enter the sheath region with a velocity greater than the ion acoustic velocity \( C_s \) and (iii) The sheath structure follows the Child-Langmuir law of the space charge limited current. In such a case the transit time model given by Barrett et al. (1989) may be considered to find the average ion drift velocity \( u_o \) of the reflected ions at the sheath edge, using the relation

\[
u_o = \frac{2d}{t_o}
\]

Where \( t_o \) (\( = 1/f_o \)) is the ion bounce time. The velocity \( v_h \) of the ion beams taking part in exciting the instability for different \( V_s \) and \( V_g \) is calculated from Eq. (3.16). Under the same condition the velocity of the reflected ion beam \( u_o \) is calculated from Eq. (3.17) by using different experimental parameters. The values of \( v_h \) and \( u_o \) along with the values of \( f_o \) for some values of \( V_g \) and \( V_s \) are shown in Table II. It is found that for small sheath thickness, the instability starts at higher ratio of \( v_h/u_o \) and for wider range of \( v_h/u_o \) which
### TABLE II. The instability frequency $f_\text{inst}$ calculated $v_b$ and $v_o$ along with their ratio ($v_b/v_o$) versus $V_g$ at different grid biasing voltage $V_g$.

<table>
<thead>
<tr>
<th>$V_g$ (Volts)</th>
<th>$V_e$ (volts)</th>
<th>$f_\text{inst}$ (kHz)</th>
<th>$v_b$ (cm/s)</th>
<th>$v_o$ (cm/s)</th>
<th>$v_b/v_o$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.4</td>
<td>340.9</td>
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<td>3.41x10^5</td>
<td>0.85</td>
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</tr>
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<td>4.12x10^5</td>
<td>0.90</td>
<td></td>
</tr>
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<td>5.0</td>
<td>469.9</td>
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<td>4.70x10^5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>495.3</td>
<td>4.60x10^5</td>
<td>4.95x10^5</td>
<td>0.92</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>502.8</td>
<td>4.97x10^5</td>
<td>5.02x10^5</td>
<td>0.99</td>
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</tr>
<tr>
<td>8.0</td>
<td>448.2</td>
<td>5.30x10^5</td>
<td>4.48x10^5</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
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<td>447.3</td>
<td>5.60x10^5</td>
<td>4.47x10^5</td>
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</tr>
<tr>
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<td>5.94x10^5</td>
<td>4.41x10^5</td>
<td>1.35</td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>324.9</td>
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<td>3.25x10^5</td>
<td>1.95</td>
<td></td>
</tr>
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<td>4.60x10^5</td>
<td>0.70</td>
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<td>6.31x10^5</td>
<td>0.84</td>
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</tr>
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<tr>
<td>12.0</td>
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<td>6.51x10^5</td>
<td>6.84x10^5</td>
<td>0.95</td>
<td></td>
</tr>
<tr>
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<td>6.84x10^5</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
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<td>6.97x10^5</td>
<td>1.07</td>
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</tr>
<tr>
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<td>6.90x10^5</td>
<td>1.15</td>
<td></td>
</tr>
<tr>
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<td>6.71x10^5</td>
<td>1.37</td>
<td></td>
</tr>
<tr>
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<td>6.84x10^5</td>
<td>1.41</td>
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<td>3.7</td>
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<td>370.1</td>
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<td>6.66x10^5</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>432.2</td>
<td>5.94x10^5</td>
<td>7.77x10^5</td>
<td>0.77</td>
<td></td>
</tr>
<tr>
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<td>8.15x10^5</td>
<td>0.90</td>
<td></td>
</tr>
<tr>
<td>20.0</td>
<td>478.3</td>
<td>8.40x10^5</td>
<td>8.60x10^5</td>
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<tr>
<td>25.0</td>
<td>481.2</td>
<td>9.40x10^5</td>
<td>8.60x10^5</td>
<td>1.09</td>
<td></td>
</tr>
<tr>
<td>30.0</td>
<td>489.0</td>
<td>10.20x10^5</td>
<td>8.81x10^5</td>
<td>1.16</td>
<td></td>
</tr>
</tbody>
</table>
corresponds to shorter range of $V_s$. Here, in this experiment, $f_o$ initially increases with $v_b/u_o$ and becomes maximum at $v_b/u_o = 1$. It is then decreases with the increase of $v_b/u_o$. Hence, it is clear that there exists an energy exchange mechanism between two beams with velocities $v_b$ and $u_o$ moving away from the grid toward the target plasma. The resonance condition occurs at $v_b = u_o$. The value of sheath thickness '$d'$ increases with the increase of $V_g$ as confirmed from Table II. Also it is clear from the same table that the threshold values of $v_b/u_o$ for instability excitation decreases and the instability occurs for wider range of $V_s$. However, it is interesting to note from the experimental results that the excitation of instability under the influence beam occurs within the range $0.5 < v_b/u_o < 2.0$.

In order to understand the instability mechanism clearly, the grid current is maintained across a resistance (100 kΩ) with respect to $V_s$ as shown in Fig. 3.14. When $V_s$ is increased from 0V to 5V, the grid ion current increases linearly with $V_s$. In other way the net difference of ion flux of the ions coming toward the grid and going away from the grid increases. Near the saturation region, when $V_s > 5V$, a small amplitude of current hump exist (marked by arrow). In this region the grid collects only small amount of extra ions as $V_s$ increases and the other part are reflected back to the target plasma. These reflected ions with certain average velocity $u0$ couples with direct ion beams coming from the source side with a velocity $v_b$. When $V_s$ is further increased (i.e. $V_s > 15V$ and $V_g = -60V$ and $-80V$), $e\phi$ is so high that in this region $v_b/u_o \geq 2.0$ and the coupling does not occur in this case. At higher $V_s$, the hump flattens for wide range of $V_s$. This observation correlates with the results shown in Fig. 3.13.
FIG. 3.14 Grid current drawn across a 100 kΩ resistance with source anode biasing voltage for different constant $V_g$. 
The growth of instability is also examined by applying a test wave (continuous sinusoidal wave) of different applied frequency \( f_m \) into the source anode using interferometer technique. A typical data of interferometer pattern are shown in Fig. 3.15 (a) and (b) and for \( V_s = 5\text{V}, 10\text{V} \) and \( 15\text{V} \), when the applied frequency is \( 450 \text{kHz} \) and \( V = -80\text{V} \) and \(-100\text{V} \) respectively. Fig. 3.16 (a) and (b) shows the curve for same \( V_s (= 5, 10 \text{ and } 15\text{V}) \) but for different applied frequency i.e. \( 500 \text{kHz} \) at \( V = -80\text{V} \) and \(-100\text{V} \).

For \( f_m (= 450 \text{kHz}) \) at \( V = -80\text{V} \) and for \( V_s = 5, 10 \text{ and } 15\text{V} \), The ratio of \( f_g \) to \( f_m \) are 0.8, 1.04 and 1.12, while at \( V = -100\text{V} \) and for same \( V_s \)'s it is 0.8, 1.0 and 1.11 respectively. Similarly for \( f_m = 500 \text{kHz} \) [ Fig. 3.13(a)] at \( V = -80\text{V} \) and for \( V_s = 5, 10 \text{ and } 15\text{V} \) the ratios are 0.8, .95 and .96 respectively, while at \( V = -100\text{V} \) \( f_g/f_m \)'s are 0.74, 0.90 and 0.96 respectively. So, it is seen that the maximum growth occurs when both the signals are of equal frequency and at a distance of 1.0 to 4.0 cm, i.e. within the presheath region.

Fig. 3.17 shows the amplitude of interferometer pattern versus \( V_g \) at a distance 2.0 cm from the grid for different applied frequencies when \( V_s = 5\text{V} \). Also the instability growth is found to be maximum, when \( f_g = f_m \). It is therefore noted that for particular \( V_s \) and \( V_g \) an instability with fixed oscillation can grow in the presheath region.

### 3.6 Conclusions

Throughout this chapter, it is clearly emphasised on the study of sheath instability in presence of an ion beam in Ar plasma. The different conclusions of this chapter are presented below:
FIG. 3.15 Interferometer wave patterns detecting the instability growth at $V_s = 5V$, 10V and 15V, for 450 kHz applied frequency.

(a) When $V_r = 80$ V.
Fig. 3.15 Interferometer wave patterns detecting the instability growth at $V_s = 5\, \text{V}$, $10\, \text{V}$ $15\, \text{V}$, for 450 kHz applied frequency.

(b) When $V_R = -100\, \text{V}$
FIG 3.16 Interferometer wave pattern detecting the instability growth at $V_s = 5V$, 10V and 15V for 500 kHz applied frequency.

(a) When $V_s = 80 V$
FIG 3.16 Interferometer wave pattern detecting the instability growth at $V_s = 5V$, 10V and 15V for 500 kHz applied frequency.

(b) When $V_g = -100$ V.
FIG. 3.17 Combined amplitude of wave patterns (instability growth), by fixing the probe at 2 cm apart from the grid, versus $V_g$ with applied frequency of external signal as a parameter at $V_s = 5V$. 
(i) To excite the instability the source and target potential must be asymmetric in case of an extra voltage applied to the source anode. In no source anode biasing condition, the potential on both sides of the chamber is found to be equal.

(ii) The ion rich sheath is formed around a negatively biased grid, where the ions are streaming toward the grid with a velocity greater than the ion acoustic velocity (i.e. Bohm velocity). This sheath follows the well known Child-Langmuir law. It increases with the increase of negative grid biasing voltage. The theoretical and experimental values of sheath thickness is found to be approximately equal and it is experimentally found that for $V > 25V$, the Child- Langmuir law is slightly modified.

(iii) The present study reveals that, in an ion beam plasma system, the coupling of three ion beams is necessary for exciting the instability. However, the value of these frequencies are found to be less than the ion plasma frequency. The ions which are coming from the target side toward the grid crosses the sheath edge with a Bohm velocity. The average velocity of these reflected beams from the higher potential side is controlled by the sheath thickness and the plasma potential difference between the two chambers. The free fall ion beam velocity ($v_b$) and the reflected ion beam velocity ($u_r$) is calculated experimentally by using Eq. (3.14) and (3.15) respectively and different values of $v_b$ and $u_r$ are shown in Table II. After calculating all these values of $v_b$ and $u_r$ it is found that the instability occurs only in the presheath region, when the ratio of beam and free fall velocity falls within the range $0.5 < v_b/u_r < 2.0$. 
(iv) The grid current measured across a resistance (100 kΩ) also shows the nonlinearity condition, which is also necessary for the excitation of instability for the same parameters.
References: