INTRODUCTION

The flow past an infinite horizontal plate started impulsively in its own plane was first studied by Stokes (1) which is also the first exact solution of the Navier-Stokes equation. It is referred in all the books on viscous flow theory. The flow past an impulsively started semi-infinite horizontal plate was studied analytically by Stewartson (2) and by finite-difference method by Hall (3). The flow past an impulsively started infinite vertical isothermal plate of an incompressible viscous fluid was first studied by Soundalgekar (4) who considered the effects of free-convection currents due to the existence of a temperature-difference between the plate temperature and the temperature of the fluid far away from the plate. An exact solution to this problem was derived by the Laplace-transform technique on the assumption that the viscous dissipative heat is negligible. However, when the plate is moving rather fast, the viscous dissipative heat cannot be neglected and in this case, the flow is governed by non-linear coupled partial differential equations. Such a non-linear system does not have an exact analytic solution. So Soundalgekar, et. al. (5) solved this system of non-linear partial differential equations by finite-difference method. The plate was assumed to be isothermal.
However, in many technological applications, the assumption of isothermal plate is rather very restrictive. In many cases, the heat is supplied to the moving plate and hence the boundary condition should be changed to variable or constant heat flux. So for a flow past an impulsively started infinite vertical plate under constant heat flux and without viscous dissipative heat, an exact solution was first derived by Soundalgekar and Patil (6) and with viscous dissipative heat, it was solved by finite-difference method by Soundalgekar, et. al (7).

However, in all these papers, it was assumed that the flow of air or water in pure form, which is rather a very restrictive assumption. In nature, air or water are not found in pure form, but mixed up with other foreign masses like water vapour, carbon dioxide etc. and the presence of these foreign masses does affect the flow past bodies. In literature, this phenomenon is studied as mass transfer effects. Gebhart and Fera (8), studied the effects of mass transfer on steady free-convection flow past a semi-infinite vertical plate without viscous dissipative heat, whereas recently, Soundalgekar and Takhar (9), studied this phenomenon by considering viscous dissipative heat. Mass transfer effects on flow past an impulsively started infinite vertical plate were also studied by Soundalgekar J A M Mech (10). It is now proposed to study the effects of mass transfer on flow past an infinite vertical plate started impulsively in its own plane in a dissipative fluid and with constant heat flux at the plate. The problem is now governed by coupled non-linear equations which
are solved by finite-difference technique. In section 2, the mathematical analysis is presented and in section 3, the conclusions are set out.

**MATHEMATICAL ANALYSIS**

We consider the flow of an incompressible viscous fluid past an infinite vertical plate, started impulsively in its own plane. The $X'$-axis is taken along the plate in the vertically upward direction and the $Y'$-axis is taken normal to the plate. It is assumed that the foreign mass is present in the fluid at a very low level. This assumption leads us to neglect the Soret and Dufour effects. Initially at $t' < 0$, both the plate and the fluid are stationary and at the same temperature $T_{\infty}'$ and the level of foreign mass is maintained at the same level throughout at $C_{\infty}'$. At time $t' > 0$, the plate starts moving impulsively in its own plane with a velocity $U_0$ and the heat is supplied to the plate at a constant rate. Also, the concentration level near the plate is raised to $C_\alpha'$ such that $C_\alpha' - C_{\infty}'$ is maintained still at a low level. Then under usual Boussinesq's approximations, the flow can be shown to be governed by the following system of coupled differential equations:

\[
\begin{align*}
\frac{\partial \theta}{\partial t'} &= \rho C_\ell (\tau' - T_{\infty}') + \rho C_\ell \left( C' - C_{\infty}' \right) + 2 \frac{\partial \theta}{\partial y} \ \cdots (2.1) \\
\rho C_p \frac{\partial \tau}{\partial t'} &= k \frac{\partial^2 \tau}{\partial y^2} + \mathcal{M} \left( \frac{\partial \theta}{\partial y} \right)^2 \ \cdots (2.2) \\
\frac{\partial \epsilon'}{\partial t'} &= D \frac{\partial^2 \epsilon'}{\partial y^2} \ \cdots (3.3)
\end{align*}
\]
with following initial and boundary conditions:

\[
\begin{align*}
\eta' &= 0, \quad T' \rightarrow T_{ad}' , \quad C' = C_x' \text{ for all } y', \quad t' \leq 0 \\
\eta' &= U_0, \quad \frac{\partial T'}{\partial y'} = -\frac{g}{k} , \quad C' = C_x' \text{ at } y' = 0 \\
\eta' &= 0, \quad T' \rightarrow T_{ad}' , \quad C' \rightarrow C_x' \quad \text{ as } y' \rightarrow \infty
\end{align*}
\]  

\[\ldots(3, 6)\]

Here \( u' \) is the velocity component of the fluid in the \( x' \) direction, \( t' \) the time, \( g \) the acceleration due to gravity, \( \beta \) the coefficient of volume expansion, \( \beta^* \) the coefficient of species expansion, \( T' \) the temperature of the fluid near the plate, \( C' \) the species concentration near the plate, \( \rho \) the density, \( C_p \) the specific heat at constant pressure, \( k \) the thermal conductivity, \( D \) the chemical molecular diffusivity, \( q \) is the constant heat supply to the plate. The derivation of these equations is given by Gebhart (11).

We now introduce the following non-dimensional quantities:

\[
\begin{align*}
&u = u'/U_0 , \quad \Theta = (T' - T_{ad}')/(2 \gamma^3/k U_0) , \\
&C = (C' - C_{\infty}')/(C_{\infty}' - C_x') , \quad \gamma = y' U_0/2 , \quad \tau = \eta' U_0^2/2 , \\
&\gamma^3 = \nu \beta \theta^3/k U_0 , \quad \gamma^* = \nu \beta^* (C_{\infty}' - C_x')/U_0^3 , \\
&S_c = \nu/D , \quad Pr = \lambda C_p/k , \quad \chi = \rho \nu , \quad E = U_0^2/C_p (2 \gamma^3/k U_0)
\end{align*}
\]  

\[\ldots(2, 5)\]
Then equations (3.1) - (3.4), in view of (3.5), reduces to following:

\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \alpha \Theta + \nu C \quad \ldots \quad (3.6)
\]

\[
\frac{\partial \Theta}{\partial t} = \frac{\partial^2 \Theta}{\partial y^2} + \rho E \left( \frac{\partial u}{\partial y} \right) \quad \ldots \quad (3.7)
\]

\[
Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \quad \ldots \quad (3.8)
\]

and the initial and boundary conditions are

\[
\begin{align*}
    \Theta &= 0, \quad C = 0 \quad \text{for all } y, \quad t \leq 0, \\
    \Theta &= 1, \quad \frac{\partial \Theta}{\partial y} = -1, \quad \Theta = 1 \quad \text{at } y = 0, \quad t > 0, \\
    \Theta &= 0, \quad C = 0 \quad \text{at } y \rightarrow \infty.
\end{align*}
\]

Here \( \Theta \) is the Grashof number, \( Gm \), the modified Grashof number, \( Pr \) the Prandtl number and \( Sc \) is the Schmidt number.

As exact solutions to equations (3.6) - (3.9) are not possible, we now solve these equations by finite-difference technique.

An explicit finite-difference form for equations (3.6) - (3.9) is as follows:

\[
\frac{u(i,j+1)}{\Delta t} - u(i,j) = \alpha \Theta(i,j) + \nu C(i,j) + \frac{u(i,j+1) - 2u(i,j) + u(i,j-1)}{(\Delta y)^2} \quad \ldots \quad (3.10)
\]

\[
\frac{\Theta(i,j+1) - \Theta(i,j)}{\Delta t} = \frac{\Theta(i,j+1) - 2\Theta(i,j) + \Theta(i,j-1)}{(\Delta y)^2} + \rho E \left( \frac{u(i,j+1) - u(i,j)}{2\Delta y} \right) \quad \ldots \quad (3.11)
\]

\[
Sc \frac{C(i,j+1) - C(i,j)}{\Delta t} = \frac{C(i,j+1) - 2C(i,j) + C(i,j-1)}{(\Delta y)^2} \quad \ldots \quad (3.12)
\]

Here index \( i \) refers to \( y \) and \( j \) to \( t \). We have chosen \( \Delta y = 0.1 \). Then initial conditions at \( y = 0 \) take the form

\[
u(0,0) = 1, \quad \Theta(0,0) = 0, \quad C(0,0) = 0 \quad \ldots \quad (3.13)
\]
This implies that due to the sudden velocity given to the plate, the velocity at \( y = 0 \) changes discontinuously to 1 from its value 0 at \( t < 0 \). But as the effects of the constant heat flux at \( y = 0 \) can change the temperature at the wall gradually and therefore \( \Theta(0,0) \) is taken as zero.

The initial condition for \( y > 0 \) are

\[
\begin{align*}
&u(i,0) = 0, \quad \Theta(i,0) = 0, \quad C(i,0) = 0 \quad \text{for all } i (>0) \quad \ldots \quad (3.14)
\end{align*}
\]

From the first of equation (3.9b), the boundary condition at \( y = 0 \) for \( u \) takes the form

\[
\begin{align*}
u(0,j) &= 1 \quad \text{for all } j \quad \ldots \quad (3.15)
\end{align*}
\]

Now, we have to compute from (3.11) and \( C \) from (3.12) at \( i = 0 \) i.e. on the plate itself and will involve on the right hand side of (3.11) the values of \( v \) at \( i = -1 \) i.e. at a hypothetical grid-point to the left of the time-axis.

The second equation in (3.9b) converted to the finite-difference scheme can be written in either of the forms

\[
\begin{align*}
\frac{\Theta(1,j) - \Theta(-1,j)}{2A_j} &= -1 \quad \ldots \quad (3.16a, b)
\end{align*}
\]

Each of these equations gives us an expression for \( \Theta \) at the hypothetical grid-point \( (i, j) \) and the average of these is taken to be used on the right hand side of (3.11) in evaluating \( \Theta(0,j+1) \). However, \( \Theta(-1,0) \) is calculated purely from (3.16b) for the initial time-step i.e. for \( j = 0 \).
Although the boundary condition (3.9c) applies at \( y = d' \), we shall regard \( y = 4.1 \) as corresponding to \( y = d' \) and therefore set \( u(41,j) = 0 \), \( \theta(41,j) = 0 \), and \( \rho(41,j) = 0 \) for all \( j \). The velocity at the end of a time-step is computed from (3.10) in terms of velocities, temperatures and species concentration at points on the earlier time-step. Similarly, \( u(i,i+1) \) and \( \rho(i,i+1) \) are computed from (3.11) and (3.12).

The procedure is repeated till \( t = 1 \) (i.e., \( j = 2000 \)) with \( \Delta t = 0.0005 \).

These computations were carried out on HCL Magnum system at Gauhati University for air (Pr = 0.71), Gr = 1, Gm = 0.0, 0.1, 0.4, E = 0.1, 0.3, 0.5 and the values of the Schmidt number were chosen as follows:

```
<table>
<thead>
<tr>
<th>Pr</th>
<th>Species</th>
<th>Sc</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>Hydrogen</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>He</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>H2O</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>NH3</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>CO2-</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>Ethyl benzene</td>
<td>2.00</td>
</tr>
</tbody>
</table>
```

To test the convergence, the program was run with smaller values of \( \Delta t = 0.0001 \) and no significant change in the values of \( u, \theta, \rho \) were observed.
The velocity profiles are shown on Fig. 1. We observe from this figure that an increase in the Schmidt number leads to a decrease in the velocity, but an increase in the modified Grashof number or greater viscous dissipative heat leads to an increase in the velocity.

On Fig. 2, the temperature profiles are shown. We observe from this figure that an increase in the Schmidt number or the modified Grashof number leads to a decrease in the temperature of the air, but greater viscous dissipative heat causes a rise in the temperature. The temperature also increases with time.

We now study the skin-friction which is given in non-dimensional form by

\[ \tau = -\frac{\partial u}{\partial y} \bigg|_{y=0} \]

where

\[ \tau = \frac{\tau'}{\rho u_0^2} \]

We have computed the numerical values of \( \tau \) by using Newton's interpolation formula by taking 5 points and these are plotted on Fig. 3. We observe from this figure that as the Schmidt number increases, the skin-friction also increases. But the frictional effects are reduced at large values of the modified Grashof number or owing to greater viscous dissipative heat.

It is now proposed to study the rate of heat transfer at the plate. The Nusselt number is now given by

\[ Nu = \frac{1}{\theta} \frac{\partial \theta}{\partial y} \bigg|_{y=0} = -\frac{1}{\theta(0)} \frac{\partial \theta}{\partial y} \bigg|_{y=0} = -1 \]

The numerical values of \( Nu \) are shown on Fig. 4 which indicates that the
Nusselt number decreases with increasing the Schmidt number or owing to greater viscous dissipative heat.

As there are no experimental results available in such problems in the literature, our results cannot be compared with any experimental data. However, our results will help an experimentalist to undertake an experimental job on such problems.

CONCLUSIONS

1. The velocity, the temperature of the air decreases with increasing the Schmidt number.
2. The velocity of air increases and the temperature decreases owing to an increase in the modified Grashof number.
3. Both the velocity and the temperature of air increase owing to greater viscous dissipative heat.
4. An increase in the Schmidt number leads to an increase in the skin-friction and a decrease in the Nusselt number.
5. Owing to greater viscous dissipative heat, both the frictional forces and the Nusselt number are observed to reduce in magnitude.

REFERENCES

1. G. G. Stokes, 1851, Camb. Phil. Trans. IX, B.
9. V. M. Soundaigekar and H. S. Takhar, (To be published).
Fig. 1. THE VELOCITY PROFILE

Fig. 2. THE TEMPERATURE PROFILE

Pr = 0.71, Gr = 1.0
Fig. 3: The Skin-Friction

Fig. 4: Rate of heat transfer