CHAPTER – II

PROBLEMS ON HYDROMAGNETIC FLOW OF A DUSTY VISCO-ELASTIC MAXWELL FLUID
CHAPTER II

Introduction:

This Chapter contains some problems on unsteady MHD flows of an incompressible visco-elastic (Maxwell type) electrically conducting fluid with embedded small spherical particles. The first paper considers the motion through a porous rectangular channel with non-conducting impermeable walls. In presence of an external transverse magnetic field, the flow is generated by a time dependent pressure gradient. The second paper deals with the flow in an open rectangular channel with non-conducting boundary. In presence of a transverse uniform magnetic field, the motion starts from rest under the influence of a time varying pressure gradient.

Basic Equations:

The constitutive equation for Maxwell fluid is

\[ \tau_{ij} = -p \delta_{ij} + \tau'_{ij} \]  

(1)

\[ \left(1 + \lambda \frac{\partial}{\partial t}\right) \tau'_{ij} = 2 \mu e_{ij} \]  

(II)

\[ e_{ij} = (u_{i,j} + u_{j,i})/2 \]  

(III)

The equation of continuity for fluid is

\[ u_{i,i} = 0 \]  

(IV)

We assume that the number of density \( N \) of the dust particles is constant, i.e. \( N = N_0 \) (constant). Then the equation of continuity of the dusty particles is

\[ \nu_{i,i} = 0 \]  

(V)

The equations of motion of a dusty fluid are (Saffman [72]):

\[ \rho \left( \frac{\partial u_i}{\partial t} + u_{i,j} u_j \right) = -p_i + \tau'_{ij,j} + K_0 N_0 (v_i - u_i) + \left( \mathbf{j} \times \mathbf{B} \right)_i - \frac{\mu}{K} u_i \]  

(VI)

and

\[ m \left( \frac{\partial v_i}{\partial t} + v_{i,j} v_j \right) = K_0 (u_i - v_i) \]  

(VII)

where \( \mathbf{j} = \sigma [\mathbf{E} + (\mathbf{u} \times \mathbf{B})] \) is the density of electric current, \( (\mathbf{j} \times \mathbf{B}) \) is the Lorentz force and \( \mathbf{B} = \mu_e \mathbf{H}_e \) is the magnetic induction vector.
For most of the conducting fluids the magnetic Reynolds number $R_m$ is much less than unity so that the induced magnetic field can be neglected in comparison with the applied magnetic field (Shercliff [81] and Gupta [39]). Since no external electric field is applied, the effect of polarisation of fluid is neglected so that we can take $\mathbf{E} = (0,0,0)$. Then the Lorentz force in absence of electric field will be $\sigma(\mathbf{u} \times \mathbf{B}) \times \mathbf{B}$.

**NOMENCLATURE**

$\tau_{ij}$ = total stress tensor
$\tau_{ij}^{\prime}$ = deviatoric stress tensor
$\epsilon_{ii}$ = rate of strain tensor
$p$ = pressure
$\mu$ = co-efficient of viscosity
$\lambda$ = stress relaxation time
$\mu_c$ = magnetic permeability
$u_i$ = velocity component of the fluid
$N_0$ = Number of density of the dust particles
$\sigma$ = electrical conductivity
$v_i$ = velocity component of the dust particles
$m$ = mass of the dust particle
$\rho$ = fluid density
$\mathbf{u}$ = velocity of the conducting medium
$K$ = permeability of the porous medium
$\mathbf{H}_c$ = magnetic field intensity
$\mathbf{B}$ = magnetic induction vector
$\mathbf{E}$ = electric field intensity
$\mathbf{J}$ = density of electric current
$K_0$ = Stoke's resistance co-efficient which for a spherical particles of radius $r$ is $6\pi \mu r$. 
2.1: ON THE HYDROMAGNETIC FLOW OF A DUSTY VISCO-ELASTIC MAXWELL FLUID THROUGH A RECTANGULAR CHANNEL*

Introduction:

Interest in problems of flow of an incompressible visco-elastic electrically conducting fluid with embedded small spherical particles have increased enormously in recent years because of its practical applications in a wide range of technical problems. These areas include flow through packed beds, sedimentation, environmental pollution and blood rheology etc. Model equations describing the motion of dusty fluid has been given by Saffman [72]. Ghosh (Dutta), S. and Sanyal, D. C. [30] have studied unsteady incompressible dusty viscous fluid flow through rectangular duct with time varying pressure gradient. The problem of unsteady MHD flow of an electrically conducting dusty visco-elastic liquid between two parallel plates has been studied by Ghosh, N. C. et-al [33]. The problem of unsteady flow of a dusty visco-elastic fluid through an inclined rectangular channel has been considered by Kumar, G. and Singh K. K. [48]. Panja, S. and Sengupta, P. R. [67] have studied the unsteady flow of a dusty visco-elastic Rivlin-Ericksen fluid through a tube with sector of a circle as cross-section under axial pressure gradient. Baruah, B. and Hazarika [10] have considered the problem of Non-Newtonian fluid flow along a continuous stretching surface which is rotating. Samad, Sk. A. [73] has considered the steady flow of viscous, incompressible, electrically conducting fluids through circular pipes in presence of an applied uniform transverse magnetic field. In his analysis, the finite conductivity and wall thickness of the pipe have been taken into account. An exact solution and its numerical calculation have been presented by the author [73]. Ersoy [27] studied the problem of MHD flow of an Oldroyd–B fluid between eccentric rotating disk. Sharma and Kumar [78] have investigated unsteady flow of Non-Newtonian fluid down an open inclined channel under the influence of a time dependent pressure gradient and a time dependent tangential stress at the free surface.

In the present paper, the author has investigated the unsteady MHD flow of

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an electrically conducting visco-elastic Maxwell type fluid with suspended small spherical particles through a rectangular porous channel with non-conducting walls. In presence of an external transverse magnetic field, the flow is generated by a time dependent pressure gradient. Considering the flow field under the influence of an arbitrary time varying pressure gradient, the exact velocities of the fluid and the dust particles are obtained by Laplace transform method. Three particular cases are considered : (i) harmonically oscillating pressure gradient, (ii) the pressure gradient is of the form $A(1 - e^{\beta t})$, where $A > 0$, $\beta > 0$ and (iii) constant pressure gradient acting for a finite time ‘$T$’. The expressions for the velocity of the fluid and the dust particles due to impulsive pressure gradient are derived from 3rd case. The results of each case are discussed in detail with the aid of graphs and tables.

**Formulation of the Problem:**

Let us consider an unsteady flow of an incompressible visco-elastic dusty conducting fluid of Maxwell type along a rectangular channel under the influence of transversely imposed magnetic field and time varying pressure gradient. The flow takes place in presence of porous medium. It is considered that the depth of the channel (i.e. the distance between the upper plate and the lower plate) is ‘$a$’ and the width of it (i.e. distance between the boundary walls) is ‘$b$’.

Let $(x, y, z)$ be the rectangular co-ordinate system. It is assumed that $z$-axis is parallel to the axis of the tube (which is shown in fig. 1), $x$-axis is measured in upward direction and is perpendicular to $z$-axis, $y$-axis is measured along the width of the channel and is perpendicular to $x$-axis and $z$-axis. We take the non-conducting walls of the channel as the planes $x = 0$, $x = a$, $y = 0$ and $y = b$.

Let us make the following assumptions:

(i) The dust particles are small in size, spherical in shape and uniformly distributed.
(ii) Chemical reaction, radiation and the interaction between the particles themselves are not considered.
(iii) Induced magnetic field and the Hall effect are assumed to be negligible.
(iv) The bulk concentration of the dust is very small and the Stoke’s resistance law between the particles and the fluid can be applied.
(v) The buoyancy force has been neglected.
(vi) The temperature is uniform within the particles.

Fig. 1: Geometry of the flow configuration

In the present analysis we take the velocity fields of the fluid and the dust particles as

\[ [0,0,u] \text{ and } [0,0,v] \] \hspace{1cm} (1)

Then from equation (IV) and (V), we get respectively

\[ \frac{\partial u}{\partial z} = 0 \Rightarrow u = u(x,y,t) \] \hspace{1cm} (2)

\[ \frac{\partial v}{\partial z} = 0 \Rightarrow v = v(x,y,t) \] \hspace{1cm} (3)

If the magnetic Reynolds number be assumed to be very small, the induced magnetic field produced by the motions of electrically conducting fluid is negligible in comparison with the applied magnetic field so that we can take \( \vec{B} = (B_1, B_2, 0) \), where \( B_1 \) and \( B_2 \) are constants. Since no external electric field is applied, the effect of polarisation of fluid is neglected so that we can assume \( \vec{E} = (0,0,0) \).

The electromagnetic body force i.e. Lorentz force

\[ \vec{j} \times \vec{B} = \sigma(\nabla \times \vec{B}) \times \vec{B} = -\vec{i}_e \sigma B_0^2 u \] \hspace{1cm} (4)
where \( \hat{\imath}_z \) is the unit vector along z-axis and \( B_{0z}^2 = B_1^2 + B_2^2 \)

Now, the equations of motion (VI) and (VII) with the help of (I) and (III) become

\[
0 = -\frac{1}{\rho} \left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial x}
\]

\[
0 = -\frac{1}{\rho} \left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial y}
\]

\[
\left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = -\frac{1}{\rho} \left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial z} + \alpha \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{K_0 N_0}{\rho} \left( 1 + \lambda \frac{\partial}{\partial t} \right) (v - u)
\]

\[
- \left( 1 + \lambda \frac{\partial}{\partial t} \right) \left( \frac{\sigma B_0^2}{\rho} u + \frac{\alpha}{K} u \right)
\]

and

\[
m \frac{\partial v}{\partial t} = K_0 (u - v)
\]

where \( \alpha = \frac{\mu}{\rho} \) is the kinematic coefficient of viscosity and \( \frac{m}{K_0} \) is the time relaxation parameter of the dust particles.

Introducing the non-dimensional quantities

\[
x' = \frac{x}{a}, \quad y' = \frac{y}{a}, \quad z' = \frac{z}{a}, \quad p' = \frac{pa^2}{\rho a^2}, \quad t' = \frac{\alpha t}{a^2}, \quad \lambda' = \frac{\alpha \lambda}{a^2}
\]

\[
u' = \frac{\nu a}{a}, \quad v' = \frac{va}{a}, \quad c = \frac{b}{a}, \quad L = \frac{K_0 a^2}{m \alpha}, \quad K' = \frac{K}{a^2}
\]

into the equation (6), (7), (8) and (9), we get (omitting dashes)

\[
0 = -\left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial x}
\]

\[
0 = -\left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial y}
\]

\[
\left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial u}{\partial t} = -\left( 1 + \lambda \frac{\partial}{\partial t} \right) \frac{\partial p}{\partial z} + \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{1}{\alpha} \left( 1 + \lambda \frac{\partial}{\partial t} \right) (v - u) - \left( 1 + \lambda \frac{\partial}{\partial t} \right) Hu
\]

and

\[
\frac{\partial v}{\partial t} = L (u - v)
\]

where \( M = B_{0a} \sqrt{\frac{\sigma}{\rho \alpha}} \) (Hartmann number), \( f = \frac{m N_0}{\rho} \) (mass concentration) and \( H = M^2 + \frac{1}{K} \).
The boundary conditions of the problem in non-dimensional form are

\[ u = v = 0 \quad \text{at} \quad x = 0, 1 \]  \hspace{1cm} (14)

and

\[ u = v = 0 \quad \text{at} \quad y = 0, c \]  \hspace{1cm} (15)

and the initial conditions of the problem in non-dimensional form are

\[
\begin{aligned}
&u = v = 0 \\
&\frac{\partial u}{\partial t} = 0
\end{aligned}
\at \quad t \leq 0
\]  \hspace{1cm} (16)

From (10) and (11) it is clear that \( \frac{\partial p}{\partial z} \) is a function of \( t \) only. So in (5), we take

\[ -\frac{\partial p}{\partial z} = \phi(t) \]  \hspace{1cm} (17)

**Solution of the Problem:**

To solve the equation (12) satisfying the conditions (14) and (15), we assume

\[ u(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{m,n}(t) \sin q_m x \sin q_n y \]  \hspace{1cm} (18)

where

\[ q_m = (2m + 1)\pi, \quad q_n = (2n + 1)\frac{\pi}{c} \]  \hspace{1cm} (19)

and

\[ A_{m,n}(t) = \frac{\partial A_{m,n}(t)}{\partial t} = 0 \]  \hspace{1cm} (20)

Applying Laplace transform to the equations (12), (13) and (18), we get respectively

\[
\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2} - \left[ \lambda s^3 + (1 + \lambda L + \lambda H + \lambda fL) s^2 + (L + H + fL + \lambda LH) s + LH \bar{u} \right] \frac{1}{s + L} \\
+ (1 + \lambda s) \bar{\phi}(s) - \lambda \phi(0) = 0
\]  \hspace{1cm} (21)

\[ \bar{u}(x, y, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{m,n}(s) \sin q_m x \sin q_n y \]  \hspace{1cm} (22)

and

\[ \bar{v} = \frac{L \bar{u}}{s + L}, \quad \text{i.e.} \quad \bar{v} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{A_{m,n}(s) \sin q_m x \sin q_n y}{s + L} \]  \hspace{1cm} (23)

Now substituting the value of \( \bar{u} \) given by (22) into (21) and then multiplying both sides by \( \sin q_m x \sin q_n y \) and then integrating w. r. t. \( x \) and \( y \) from \( x = 0 \) to \( 1 \) and \( y = 0 \) to \( c \), we get
\[
\tilde{A}_{m,n}(s) = \frac{S_{m,n}(s + L)[(1 + \lambda s)\bar{\phi}(s) - \lambda\phi(0)]}{\lambda s^3 + (1 + \lambda L + \lambda H + \lambda fL)s^2 + (L + H + R_{m,n} + fL + \lambda HL)s + L(R_{m,n} + H)}
\]  

(24)

where 's' is the parameter of Laplace transform and

\[
S_{m,n} = \frac{16}{q_m q_n}, \quad R_{m,n} = q_m^2 + q_n^2
\]  

(25)

From (22), (23) and (24), we have

\[
\tilde{u} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} S_{m,n} \frac{(s + L)[(1 + \lambda s)\bar{\phi}(s) - \lambda\phi(0)] \sin q_m \cos q_n y}{\lambda s^3 + (1 + \lambda L + \lambda H + \lambda fL)s^2 + (L + H + R_{m,n} + fL + \lambda HL)s + L(R_{m,n} + H)}
\]  

(26)

and

\[
\tilde{v} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} S_{m,n} \frac{[(1 + \lambda s)\bar{\phi}(s) - \lambda\phi(0)] \sin q_m \cos q_n y}{\lambda s^3 + (1 + \lambda L + \lambda H + \lambda fL)s^2 + (L + H + R_{m,n} + fL + \lambda HL)s + L(R_{m,n} + H)}
\]  

(27)

Inverting (26) and (27) by convolution theorem, we get

\[
u = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{S_{m,n}}{\psi(s)} \left[ E\left(s^{(i)}_{m,n}\right) \frac{t^{(i)}_{m,n}}{\lambda s^{(i)}_{m,n} + L} \phi(t) \right] \sin q_m \cos q_n y
\]  

(28)

and

\[
\nu = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{S_{m,n}}{\psi(s)} \left[ (1 + \lambda s^{(i)}_{m,n}) \frac{t^{(i)}_{m,n}}{\lambda s^{(i)}_{m,n} + L} \phi(t) \right] \sin q_m \cos q_n y
\]  

(29)

where \( E(s) = \lambda s^2 + (1 + \lambda L)s + L \),

\[
\psi(s) = \lambda(s - s^{(1)}_{m,n})(s - s^{(2)}_{m,n})(s - s^{(3)}_{m,n})
\]  

(30)

and \( s^{(i)}_{m,n} \) are the roots of the equation

\[
\lambda s^3 + (1 + \lambda L + \lambda H + \lambda fL)s^2 + (L + H + R_{m,n} + fL + \lambda HL)s + L(R_{m,n} + H) = 0
\]  

(32)

The equation (32) has no positive roots. This equation must allow at least one real negative root. Indeed, the real parts of all the three roots must be negative. This follows from the theorem that a necessary and sufficient condition for the real parts of all the three roots of the cubic,

\[
\Theta^3 + B\Theta^2 + C\Theta + D = 0
\]
to be negative is that \( BC > D > 0 \). For the cubic (32), \( D > 0 \) and \( BC - D > 0 \).

So the expressions (28) and (29) describe the velocity of the fluid and the dust particles respectively valid for large \( t \).

When \( \lambda \rightarrow 0 \) in (28) and (29), we get the corresponding flows for ordinary dusty viscous fluid and they are given respectively by

\[
\begin{align*}
    u &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{2}{i=1} \frac{S_{m,n}}{\psi'(s_{m,n})} \left[ (s_{m,n} + L) \int_{0}^{1} e^{s_{m,n} T} \phi(t-T) dT \right] \sin m \pi x \sin n \pi y \\
    v &= L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{2}{i=1} \frac{S_{m,n}}{\psi'(s_{m,n})} \left[ \int_{0}^{1} e^{s_{m,n} T} \phi(t-T) dT \right] \sin m \pi x \sin n \pi y
\end{align*}
\]

where \( s_{m,n}^{(1)} \) and \( s_{m,n}^{(2)} \) are the roots of the equation

\[
s^2 + (L + H + R_{m,n} + fL)s + L(R_{m,n} + H) = 0
\]

and

\[
\psi_{1}(s) = (s - s_{m,n}^{(1)})(s - s_{m,n}^{(2)})
\]

**Particular Cases**

**Case (i). Flow under harmonically oscillating pressure gradient:**

In this case, let us take \( \phi(t) = A \sin \omega t \),

where \( A (> 0) \) and \( \omega (> 0) \) are constants.

Now, from (28), (29) and (1.1), we have

\[
\begin{align*}
    \frac{u}{A} &= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{3}{i=1} \frac{S_{m,n}}{\psi'(s_{m,n})} \left[ E(s_{m,n}) \left( \omega e^{s_{m,n}^2} - s_{m,n} \sin \omega t - \omega \cos \omega t \right) \right] \frac{s_{m,n}^2}{s_{m,n}^2 + \omega^2} \times \sin m \pi x \sin n \pi y \\
    \frac{v}{A} &= L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{3}{i=1} \frac{S_{m,n}}{\psi'(s_{m,n})} \left[ (1 + \lambda s_{m,n}) \left( \omega e^{s_{m,n}^2} - s_{m,n} \sin \omega t - \omega \cos \omega t \right) \right] \frac{s_{m,n}^2}{s_{m,n}^2 + \omega^2} \times \sin m \pi x \sin n \pi y
\end{align*}
\]

Expressions (1.2) and (1.3) give the velocity of the fluid and the dust particles respectively.
When $l \to \infty$ and $f \to 0$ in (1.2), we get

$$\frac{u}{A} = \sum_{m=1}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1,2} \frac{S_{m,n} E(s_{m,n}^{(i)})}{\psi'(s_{m,n}^{(i)})} \left[ \frac{(1 + \lambda s_{m,n}^{(i)}) \left( \cos \frac{s_{m,n}^{(i)}}{\lambda} - s_{m,n}^{(i)} \sin \theta \cos \theta \right)}{s_{m,n}^{(i)} + \omega^2} \right] \times \sin q_m \sin q_n \gamma$$

where $s_{m,n}^{(1)}$ and $s_{m,n}^{(2)}$ are the roots of the equation

$$\lambda s^2 + (1 + \lambda H)s + (R_{m,n} + H) = 0$$

and

$$\psi_2 (s) = \lambda(s - s_{m,n}^{(1)})(s - s_{m,n}^{(2)})$$

Equation (1.4) describes the velocity of clean fluid.

**Case (ii). Flow under the pressure gradient of the form $A(1 - e^{-\beta t})$:**

Let us now consider the case in which the pressure gradient increases numerically with the time and ultimately tends to a constant value. In other words we assume

$$\phi(t) = A (1 - e^{-\beta t})$$

where $A$ and $\beta$ are positive constants.

Then from (28), (29) and (2.1), we have

$$\frac{u}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{S_{m,n} E(s_{m,n}^{(i)})}{\psi'(s_{m,n}^{(i)})} \left[ \frac{\beta(e^{s_{m,n}^{(i)}} - 1) + s_{m,n}^{(i)} (e^{-\beta t} - 1)}{s_{m,n}^{(i)}} \right] \times \sin q_m \sin q_n \gamma$$

and

$$\frac{v}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{S_{m,n} (1 + \lambda s_{m,n}^{(i)})}{\psi'(s_{m,n}^{(i)})} \left[ \frac{\beta(e^{s_{m,n}^{(i)}} - 1) + s_{m,n}^{(i)} (e^{-\beta t} - 1)}{s_{m,n}^{(i)} \left( s_{m,n}^{(i)} + \beta \right)} \right] \times \sin q_m \sin q_n \gamma$$

Equations (2.2) and (2.3) describe the velocity of the fluid and the dust particles respectively.

When $\beta \to \alpha$ in (2.3) and (2.4), we have respectively

$$\frac{u}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} \frac{S_{m,n} E(s_{m,n}^{(i)}) (e^{s_{m,n}^{(i)}} - 1)}{\psi'(s_{m,n}^{(i)}) s_{m,n}^{(i)}} \times \sin q_m \sin q_n \gamma$$

and
\[
\frac{v}{A} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{3}{3} S_{m,n} \left( 1 + \lambda S_{m,n}^{(i)} \right) \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right) \psi' \left( S_{m,n}^{(i)} \right) S_{m,n} \times \sin q_m \times \sin q_n (2.5)
\]

Equations (2.4) and (2.5) describe the velocity of the fluid and the dust particles due to constant pressure gradient.

When \( \lambda \to 0 \) in (2.3) and (2.4), we get respectively

\[
\frac{u}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1,2} S_{m,n} \left( \frac{\beta \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right) + S_{m,n}^{(i)} \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right)}{S_{m,n}^{(i)} \left( S_{m,n}^{(i)} + \beta \right)} \right) \times \sin q_m \times \sin q_n (2.6)
\]

and

\[
\frac{v}{A} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1,2} S_{m,n} \left( \frac{\beta \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right) + S_{m,n}^{(i)} \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right)}{S_{m,n}^{(i)} \left( S_{m,n}^{(i)} + \beta \right)} \right) \times \sin q_m \times \sin q_n (2.7)
\]

where \( S_{m,n}^{(1)} \) and \( S_{m,n}^{(2)} \) are the roots of the equation (35) and \( \psi_i(s) \) is given by (36).

Equations (2.6) and (2.7) represent the velocity of the fluid and the dust particles respectively for ordinary dusty viscous fluid.

Case (iii). Flow under constant pressure gradient acting for a finite time ‘T’:

In the present circumstances, let us take

\[
\phi(t) = A [H(t) - H(t - T)] (3.1)
\]

where \( A > 0 \) is a constant and \( H(t) \) is the Heaviside unit function defined by

\[
H(t) = 0 \quad \text{for} \quad t \leq 0
\]

\[
= 1 \quad \text{for} \quad t > 0
\]

From (28), (29) and (3.1), we get

\[
\frac{u}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} E \left( S_{m,n}^{(i)} \right) \left( e^{S_{m,n}^{(i)} \lambda} - 1 \right) \psi' \left( S_{m,n}^{(i)} \right) S_{m,n} \times \sin q_m \times \sin q_n, \quad \text{when} \ 0 < t \leq T (3.2)
\]

\[
= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} \frac{E \left( S_{m,n}^{(i)} \right) e^{S_{m,n}^{(i)} \lambda} \left( 1 - e^{-S_{m,n}^{(i)} \lambda} \right)}{\psi' \left( S_{m,n}^{(i)} \right) S_{m,n}^{(i)}} \times \sin q_m \times \sin q_n, \quad \text{when} \ t > T (3.3)
\]

and
\[ \frac{v}{A} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} \left(1 + \lambda s_{m,n}^{(i)} \right)e^{s_{m,n}^{(i)}t} \left(1 - e^{-s_{m,n}^{(i)}T} \right) \times \sin q_m \times \sin q_n y \quad \text{when } 0 < t \leq T \]  

\[ = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} \left(1 + \lambda s_{m,n}^{(i)} \right)e^{s_{m,n}^{(i)}t} \left(1 - e^{-s_{m,n}^{(i)}T} \right) \times \sin q_m \times \sin q_n y \quad \text{when } t > T \]  

Equations (3.2) and (3.4) describe the velocity of the fluid and the dust particles within the time interval \(0 < t \leq T\) in which duration the pressure gradient is active.

Equation (3.3) and (3.5) describe the velocity of the fluid and the dust particles after ceasing the pressure gradient at time \(t = T\).

When \(L \to \infty \) and \(f \to 0\) in (3.2) and (3.3), we have respectively

\[ \frac{u}{A} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{2} S_{m,n} \left(1 + \lambda s_{m,n}^{(i)} \right)e^{s_{m,n}^{(i)}t} \left(1 - e^{-s_{m,n}^{(i)}T} \right) \times \sin q_m \times \sin q_n y \quad \text{when } 0 < t \leq T \]  

\[ = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{2} S_{m,n} \left(1 + \lambda s_{m,n}^{(i)} \right)e^{s_{m,n}^{(i)}t} \left(1 - e^{-s_{m,n}^{(i)}T} \right) \times \sin q_m \times \sin q_n y \quad \text{when } t > T \]  

Where \(s_{m,n}^{(1)}\) and \(s_{m,n}^{(2)}\) are the roots of the equation (1.5) and \(\psi_2(s)\) is given by (1.6).

Expression (3.6) and (3.7) describe the velocity of clean fluid within the prescribed interval of time. Making \(T \to 0\) and \(A \to \infty\) in (3.3) and (3.4) in such a manner that \(TA = F\), then we respectively get

\[ \frac{u}{F} = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} E(s_{m,n})e^{s_{m,n}^{(i)}t} \times \sin q_m \times \sin q_n y \]  

\[ \frac{v}{F} = L \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{i=1}^{3} S_{m,n} \left(1 + \lambda s_{m,n}^{(i)} \right)e^{s_{m,n}^{(i)}t} \times \sin q_m \times \sin q_n y \]  

Equations (3.8) and (3.9) describe the velocity of the fluid and the dust particles due to the impulsive pressure gradient \(F \delta(t)\).
Table 1.1 (Case I)

Numerical values of u and v for different values of y and f when

\[ M = 1, K = 0.1, \lambda = 0.1, L = 6.67, t = 0.1, x = 0.2 \text{ and } \omega = 6.0. \]

<table>
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<tr>
<th>y</th>
<th>u (f = 0.1)</th>
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<th>u (f = 0.2)</th>
<th>v (f = 0.1)</th>
<th>v(f = 0.15)</th>
<th>v (f = 0.2)</th>
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<td>0.000702</td>
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Table 1.2 (Case I)

Numerical values of u and v for different values of x and \( \lambda \) when

\[ M = 1, K = 0.1, f = 0.1, L = 6.67, t = 0.1, y = 0.2 \text{ and } \omega = 6.0. \]

<table>
<thead>
<tr>
<th>x</th>
<th>u (( \lambda = 0.1 ))</th>
<th>u (( \lambda = 0.2 ))</th>
<th>u(( \lambda = 0.3 ))</th>
<th>v (( \lambda = 0.1 ))</th>
<th>v (( \lambda = 0.2 ))</th>
<th>v (( \lambda = 0.3 ))</th>
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<td>0.000817</td>
</tr>
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<td>0.002138</td>
</tr>
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### Table 1.3 (Case I)
Numerical values of $u$ and $v$ for different values of $x$ and $L$ when $M = 1$, $K = 0.1$, $\lambda = 0.1$, $f = 0.1$, $t = 0.1$, $y = 0.2$ and $C) = 6.0$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$u$ ($L=6.67$)</th>
<th>$u$ ($L=50.0$)</th>
<th>$u$ ($L=100.0$)</th>
<th>$v$ ($L=6.67$)</th>
<th>$v$ ($L=50.0$)</th>
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### Table 2.1 (Case II)
Numerical values of $u$ and $v$ for different values of $y$ and $f$ when $M = 1$, $K = 0.1$, $\lambda = 0.1$, $L = 6.67$, $t = 0.1$, $x = 0.2$ and $\beta = 1.0$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$u$ ($f=0.10$)</th>
<th>$u$ ($f=0.15$)</th>
<th>$u$ ($f=0.20$)</th>
<th>$v$ ($f=0.10$)</th>
<th>$v$ ($f=0.15$)</th>
<th>$v$ ($f=0.20$)</th>
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</tr>
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<td>0.000240</td>
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</table>
Table 2.2 (Case II)

Numerical values of $u$ and $v$ for different values of $x$ and $L$ when $M = 1, K = 0.1, \lambda = 0.1, f = 0.1, t = 0.1, y = 0.2, \beta = 1.0$

<table>
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<tr>
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<th>$u (L=100.0)$</th>
<th>$v (L=6.67)$</th>
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Table 3.1 (Case III for $0 < t \leq T$)

Numerical values of $u$ and $v$ for different values of $x$ and $f$ when $M = 1, K = 0.1, \lambda = 0.1, L = 6.67, t = 0.1, y = 0.2$ and $T = 0.3$

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<th>$u (f=0.20)$</th>
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<th>$v (f=0.15)$</th>
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<tr>
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</table>
Table 3.1 a (Case III for $t > T$)

Numerical values of $u$ and $v$ for different values of $x$ and $f$ when $M = t$, $K = 0.1$, $\lambda = 0.1$, $L = 6.67$, $t = 0.1$, $y = 0.2$ and $T = 0.3$

<table>
<thead>
<tr>
<th>$x$</th>
<th>$u (f = 0.10)$</th>
<th>$u (f = 0.15)$</th>
<th>$u (f = 0.20)$</th>
<th>$v (f = 0.10)$</th>
<th>$v (f = 0.15)$</th>
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<tr>
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</tr>
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</tr>
<tr>
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<td>0.53136E-03</td>
<td>0.52544E-03</td>
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Table 3.2 (Case III for $0 < t < T$)

Numerical values of $u$ and $v$ for different values of $x$ and $L$ when $M = 1$, $K = 0.1$, $\lambda = 0.1$, $f = 0.1$, $t = 0.1$, $y = 0.2$ and $T = 0.3$

<table>
<thead>
<tr>
<th>$x$</th>
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<th>$u$ ($L= 50.0$)</th>
<th>$u$ ($L=100.0$)</th>
<th>$v$ ($L= 6.67$)</th>
<th>$v$ ($L= 50.0$)</th>
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</tr>
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</tr>
<tr>
<td>0.80</td>
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<td>0.0137825</td>
<td>0.0058609</td>
<td>0.0140656</td>
<td>0.0144445</td>
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<tr>
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Table 3.2a (Case III for \( t > T \))

Numerical values of \( u \) and \( v \) for different values of \( x \) and \( L \) when \( M = 1, K = 0.1, \lambda = 0.1, f = 0.1, t = 0.6, y = 0.2 \) and \( T = 0.3 \)

<table>
<thead>
<tr>
<th>( x )</th>
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<th>( u ) (( L = 50.0 ))</th>
<th>( v ) (( L = 6.67 ))</th>
<th>( v ) (( L = 50.0 ))</th>
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</table>

Discussions and Conclusions:

The analysis of incompressible, one dimensional magneto-hydrodynamic flow of a dusty visco-elastic Maxwell fluid through a porous rectangular channel with arbitrary pressure gradient leads to the following conclusions:

If we take \( \lambda \to 0 \) in (23) and (24), then we have the corresponding flow of dusty viscous fluid. Taking \( L \to \infty \) and \( f \to 0 \) in (23) and (24), one may get the unsteady flow of clean visco-elastic fluid through a porous rectangular channel in presence of uniform transverse magnetic field and time depending pressure gradient.

If we make \( M \to 0 \), (23) and (24) represent the corresponding non-MHD flow. When \( K \to \infty \) in (23) and (24), one may get the corresponding flow when no resistance is offered by the porous medium. Three particular cases, considered here, are interesting.

In first case, we have considered harmonically oscillating pressure gradient. Analytical expression of clean fluid for first case have been derived by making \( L \to \infty \) and \( f \to 0 \). In second case the pressure gradient increases numerically with time and ultimately tends to a constant i.e. the pressure gradient is of the type \( A(1 - e^{\beta t}) \). \( A, \beta > 0 \), the results are interesting. In this case analytical expressions of the fluid and
the dust particles for ordinary viscous fluid have been derived by putting $\lambda \to 0$. The results of third case are interesting. Here the pressure gradient is a constant ceased after a finite time $T$. The velocities of the fluid and the dust for constant pressure gradient may be obtained from (3.2) and (3.4) respectively by making $T \to \infty$. The analytic expression for the velocities of the fluid and the dust particles for impulsive pressure gradient have been derived from 3rd case.

For numerical calculations we have taken $A = 1$ and $c = 0.5$

**case (i)**: From fig. 1.1, it is seen that the velocities of the fluid and the dust particles oscillate as time advances. We observe from fig. 1.1 that the velocities of the fluid and that of the dust particles decrease and increase with $M$, the Hartmann number, as time $t$ goes on increasing. In other words, the effect of the magnetic field oscillates with the oscillation of the velocities of the fluid and the dust particles. Fig. 1.1 depicts that the amplitude of oscillation of the velocities of the fluid and the dust particles diminishes with higher values of $M$. This figure also depicts that the amplitude of oscillation of the velocity of the fluid is greater than that of the dust particles. From table 1.1 it is obvious that the velocities of both the fluid and the dust particles go on decreasing with the increasing values of $f$, the mass concentration of dust particles. It is also obvious from table 1.1 that the velocity profiles of both the fluid and the dust particles are symmetric about the plane $y = 0.25$. Fig. 1.2 describes that the velocities of the fluid and the dust particles fall off as $K$, the permeability of the porous medium takes lower values i.e. the resistance offered by the porous medium on the velocities of the fluid and the dust particles increases with the decrease of the permeability of the porous medium. Table 1.2 shows that the velocities of the fluid and the dust particles increase with the increase of $\lambda$, the time relaxation of the fluid. Table 1.3 reveals that the velocities of the fluid and the dust particles fall off and augment respectively with the increase of $L$ (reciprocal of time relaxation of dust). This implies that for coarse or fine particles the velocity of the fluid increases or decreases respectively and for coarse or fine particles the velocity of the dust particles decreases or increases respectively. From tables 1.2, 1.3 and graph 1.2 it is clear that the velocity profiles of the fluid and the dust particles are symmetric about the plane $x = 0.5$. 
case (ii) : From fig. 2.1 we see that as t the time increases the velocities of the fluid and the dust particles increase. It is also obvious from fig. 2.1 that the velocities of the fluid and the dust particles go on decreasing with higher values of M, the Hartmann number. The decrease of velocities with increasing values of M has its physical significance. It is observed practically that as the value of M goes on increasing, the magnetic lines of force act like rubber bands tending to decrease the velocity.

Table 2.1 depicts that as f, the mass concentration of the dust particles, increases the velocities of both the fluid and the dust particles decrease. Table 2.1 also depicts that the velocity profiles of the fluid and the dust particles are symmetric about the plane $y = 0.25$. Fig. 2.2 describes that the velocities of the fluid and the dust particles augment with the increment of K, the permeability of the porous medium. It is seen that the effect of K on the velocity of the fluid is pronounced at $x = 0.5$.

From the fig. 2.2 it is noted that the velocity of the dust particles is slightly changed due to variation of K, the permeability of the porous medium. Fig. 2.3 describes that the velocities of the fluid augment as $\lambda$, the time relaxation of the fluid takes higher values. The effect of $\lambda$ on the velocity of the dust is all most all negligible. Table 2.2 narrates that the velocity of the fluid and the dust particles fall off and increase (like case I) respectively with the augment of L, the reciprocal of time relaxation of dust particles i.e. with the decrease of the time relaxation of the dust particles. From fig. 2.2, 2.3 and table 2.2 it is seen that the velocity profiles of the fluid and the dust particles are symmetrical about the plane $x = 0.5$. From all the figures and tables it is clear that the velocity of the dust particles is slower than that of the fluid.

case (iii) Figs 3.1 and 3.2 describe that for $0 < t \leq T$, the effects of $\lambda$, the time relaxation of the fluid and K, the permeability of the medium on the velocity profiles of the fluid and that of the dust particles are like as in case (i) and as in case (ii). It is seen from the fig. 3.1a that the velocities of the fluid and the dust particles increase and decrease respectively as $\lambda$ takes higher values when $t > T$. From the fig. 3.1a it is clear that for small values of $\lambda$ the velocity of the fluid and the dust particles are in two opposites in the sense of direction. Fig. 3.2a depicts that it is clear that when $t > T$ the effect of K on the velocity profiles of the fluid and the dust particles are like as the
previous cases. It is also clear from the figs. 3.1a and 3.2a that when \( t > T \), for small and large values of \( \lambda \) the velocity profiles of the fluid and the dust particles are respectively of M-shaped, as noted by Hunt [42], Sloan and Smith [86] for MHD flows through rectangular ducts. The velocity profiles of both the fluid and the dust particles are symmetric about the plane \( y = 0.25 \) when \( 0 < t \leq T \) and \( t > T \). From fig. 3.3, we observe that the velocities of both the fluid and the dust particles initially increase rapidly and then the velocity of the fluid decreases sharply and becomes more or less a common constant at \( t = 0.2 \) for different values of \( M \). Then after the velocity of the fluid augments slightly and takes a constant value for \( 0.2 < t \leq T \). Fig. 3.3 reveals that the velocity of the dust particles decreases as \( M \) takes higher values for \( 0 < t \leq T \) and that of the fluid falls off as \( M \) increases for \( 0 < t < 0.2 \) and \( 0.2 < t \leq T \). Also from fig. 3.3 it is seen that immediately after withdrawing the pressure gradient at \( t = T \) the velocities of the fluid and the dust particles decrease abruptly and for \( t > T \) the velocity of the fluid oscillates with damped vibration and ultimately the velocities of both the fluid and the dust particles asymptotically fall to zero. The amplitude of oscillation of the fluid falls off with the increase of \( M \) when \( t > T \). For \( t > T \) the velocity of the dust particles oscillates with very small amplitude and damped vibrations, and the amplitude (small) of oscillation of the dust decreases with the increase of \( M \). From table 3.1, we see that as \( f \) takes higher values for \( 0 < t \leq T \), the velocity of the fluid as well as the dust particles decreases. It is noted from table 3.1a that for \( t > T \) the velocity of the fluid increases but that of the dust particles decreases as \( f \) increases. Also from the table 3.2 it is noted that as \( L \) increases the velocities of the fluid and the dust particles increase for \( 0 < t \leq T \). Table 3.2a describes that for \( t > T \), the velocity of the fluid increases with the increase of \( L \) but the velocity of the dust particles increases upto a certain values of \( L \) whereafter the opposite happens.
Fig. 1.1 (Case I): Velocity profiles of the fluid and the dust particles for various values of $M$.  

Fig. 1.2 (Case I): Velocity profiles of the fluid and the dust particles for various values of $K$.  

Fig. 2.1 (Case II): Velocity profiles of the fluid and the dust particles for various values of $M$.  

Fig. 2.2 (Case II): Velocity profiles of the fluid and the dust particles for various values of $K$.  

\[
\begin{array}{cccccccc}
0.1 & 0.2 & 0.2 & 6.67 & 0.1 & 6 & 0.1 \\
0.1 & 0.2 & 0.2 & 6.67 & 0.1 & 6 & 0.1
\end{array}
\]
Fig. 2.3 (Case II) : Velocity profiles of the fluid and the dust particles for various values of $\lambda$.

Fig. 3.1 (Case III for $0 < t \leq T$) : Velocity profiles of the fluid and the dust particles for various values of $\lambda$.

Fig. 3.1a (Case III for $t > T$) : Velocity profiles of the fluid and the dust particles for various values of $\lambda$. 
Fig. 3.2 (Case III for 0 < t ≤ T) : Velocity profiles of the fluid and the dust particles for various values of K.

Fig. 3.3 (Case III) : Velocity profiles of the fluid and the dust particles for various values of M.

Fig. 3.2a (Case III for t > T) : Velocity profiles of the fluid and the dust particles for various values of K.
2.2: UNSTEADY MHD FLOW OF A VISCO-ELASTIC CONDUCTING DUSTY FLUID THROUGH AN OPEN RECTANGULAR CHANNEL

Introduction:

Due to importance of dusty viscous flows in petroleum industry, in the purification of crude oil, in physiological flows and in other technological fields, various studies have appeared in the literature. The dispersion and fall out of pollutants in air or in water has necessitated the study of flows of dusty fluids. Saffman [72] has formulated the basic equations, for the flow of dusty fluid. Since then many researchers have discussed the problems of dusty fluid. Fan and Chao [28] have studied unsteady laminar incompressible fluid flow through rectangular duct with time varying pressure gradient. Gupta and Gupta [40] have studied the flow of dusty gas in a rectangular channel with arbitrary pressure gradient. Singh and Singh [83] have investigated the unsteady flow of dusty visco-elastic liquid through conformal elliptical duct. Yadav and Singh [98] have considered the unsteady flow of a dusty conducting Rivlin-Ericksen fluid through a uniform pipe with sector of a circle as cross-section. Recently Kumar and Singh [48] have investigated the unsteady flow of a dusty visco-elastic fluid through an inclined rectangular channel. Kumar and Singh [50] have studied a three dimensional laminar flow of an electrically conducting viscous incompressible fluid with non-conducting identical spherical dust particles through a long rectangular channel under the influence of constant magnetic field and time varying pressure gradient.

The present problem considers the motion of an electrically conducting visco-elastic (Maxwell fluid) incompressible dusty fluid along a long rectangular open channel under the influence of transverse magnetic field and time varying pressure gradient. Taking the fluid initially at rest, the expressions for the velocities of the fluid and the dust particles have been calculated for four different cases when the pressure gradient is:

1. an exponential function of time of the type \( A(1 - e^{-\gamma t}) \), \( A > 0, \gamma > 0 \),

* Communicated to International Journal of Heat and Fluid Flow, U. K.
(II) varying periodically with time,  
(III) impulsive type and  
(IV) constant acting for a finite time.

**Formulation of the problem:**

Let us consider an unsteady flow of incompressible visco-elastic (Maxwell fluid) dusty conducting fluid along an open rectangular channel under the influence of transversely imposed magnetic field and time varying pressure gradient. The depth of the channel (i.e. the distance between the free surface and the lower surface) is \( h \) and the width (i.e. the distance between the boundary walls) is \( 2d \). It is assumed that \( x \)-axis is on the central plane along the flow of the fluid, \( y \)-axis along the depth and \( z \)-axis along the width of the channel, the origin is chosen in the free surface of the liquid. The analysis of the present paper is based on the following assumption:

(i) Chemical reaction, mass transfer, radiation and the interaction between the particles themselves have not been considered.

(ii) Induced magnetic field, perturbation of the field and Hall effect are assumed to be negligible.

In the present analysis we take the velocity fields of the fluid and the dust particles as 
\[
[u(y, z, t), 0, 0] \quad \text{and} \quad [v(y, z, t), 0, 0].
\]
And magnetic induction vector \( \mathbf{B} = [0, 0, B_0] \).

Now, in absence of porous medium, the equations (VI) and (VII) with the help of (I) – (III) become

\[
\left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = -\frac{1}{\rho} \left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial \rho}{\partial x} + \alpha \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{K_0 N_0}{\rho} \left(1 + \lambda \frac{\partial}{\partial t}\right) (v - u)
\]

\[
-\left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\sigma B_0^2}{\rho} u
\]

where \( \alpha = \frac{\nu}{\rho} \) is the kinematic coefficient of viscosity and \( \frac{\nu}{K_0} \) is the time relaxation parameter of the dust particles.
The initial and the boundary conditions of the problem are
\begin{equation}
\begin{align*}
  u &= 0 \quad \text{when } t = 0 \\
  \frac{\partial u}{\partial t} &= \frac{\partial v}{\partial t} = 0 \quad \text{when } t = 0
\end{align*}
\end{equation}

and
\begin{equation}
\begin{align*}
  u &= v = 0 \quad \text{when } z = \pm d, \quad t > 0 \\
  u &= v = 0 \quad \text{when } y = h, \quad t > 0 \\
  \frac{\partial u}{\partial y} &= \frac{\partial v}{\partial y} = 0 \quad \text{when } y = 0, \quad t > 0
\end{align*}
\end{equation}

Introducing the non-dimensional quantities
\[
x' = \frac{x}{h}, \quad y' = \frac{y}{h}, \quad z' = \frac{z}{h}, \quad p' = \frac{ph^2}{\rho \alpha^2}, \quad t' = \frac{\alpha t}{h^2}, \quad \lambda' = \frac{\alpha \lambda}{h^2}, \quad u' = \frac{uh}{\alpha},
\]
\[
v' = \frac{vh}{\alpha} \quad \text{and} \quad \tau = \frac{m\alpha}{K_0 h^2}
\]
in (1) and (2), we respectively get (omitting dashes)
\begin{equation}
\left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + f \left(1 + \lambda \frac{\partial}{\partial t}\right) (v - u) - M^2 \left(1 + \lambda \frac{\partial}{\partial t}\right) u
\end{equation}

and
\begin{equation}
\tau \frac{\partial v}{\partial t} = (u - v)
\end{equation}

where \( M = B_0 h \sqrt{\frac{\sigma}{\alpha \rho}} \) (Hartmann number), \( f = \frac{mN_0}{\rho} \) (mass condensation of dust).

The initial and boundary conditions in non-dimensional form are (omitting dashes)
\begin{equation}
\begin{align*}
  u &= v = 0 \quad \text{at } t = 0 \\
  \frac{\partial u}{\partial t} &= \frac{\partial v}{\partial t} = 0
\end{align*}
\end{equation}

and
\begin{equation}
\begin{align*}
  u &= v = 0 \quad \text{when } z = \pm \frac{d}{h} = \pm b (\text{say}), \quad t > 0 \\
  u &= v = 0 \quad \text{when } y = h, \quad t > 0 \\
  \frac{\partial u}{\partial y} &= \frac{\partial v}{\partial y} = 0 \quad \text{when } y = 0, \quad t > 0
\end{align*}
\end{equation}
on putting $z = \frac{2b\xi}{\pi} - b$ in (5), we get

\[
\left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial p}{\partial x} + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\pi^2}{4b^2} \frac{\partial^2 u}{\partial \xi^2}\right) + \frac{f}{\tau} \left(1 + \lambda \frac{\partial}{\partial t}\right)(v - u) - M^2 \left(1 + \lambda \frac{\partial}{\partial t}\right) u
\]

(9)

Now the boundary conditions of (8) reduces to

\[
\begin{align*}
\text{when } \xi &= 0 \text{ and } \xi = \pi, \quad t > 0, \\
\frac{\partial u}{\partial \xi} &= \frac{\partial v}{\partial \xi} = 0 \quad \text{when } \xi = 0, \quad t > 0
\end{align*}
\]

(10)

**Solution of the problem:**

Let us take $-\frac{\partial p}{\partial x} = f_1(t)$

(11)

From (11) and (9), we get

\[
\left(1 + \lambda \frac{\partial}{\partial t}\right) \frac{\partial u}{\partial t} = \left(1 + \lambda \frac{\partial}{\partial t}\right) f_1(t) + \left(\frac{\partial^2 u}{\partial y^2} + \frac{\pi^2}{4b^2} \frac{\partial^2 u}{\partial \xi^2}\right) + \frac{f}{\tau} \left(1 + \lambda \frac{\partial}{\partial t}\right)(v - u) - M^2 \left(1 + \lambda \frac{\partial}{\partial t}\right) u
\]

(12)

Applying Laplace transform

\[
\bar{u} = \int_0^\infty u e^{-st} dt, \quad \bar{v} = \int_0^\infty v e^{-st} dt, \quad \text{Re}(s) > 0
\]

to the equations (6) and (12) under the initial conditions of (7), we get respectively

\[
\bar{v} = \frac{\bar{u}}{st + 1}
\]

(13)

and

\[
\left(\frac{\partial^2 \bar{u}}{\partial y^2} + \frac{\pi^2}{4b^2} \frac{\partial^2 \bar{u}}{\partial \xi^2}\right) - \frac{(1 + \lambda s)}{(1 + st)} \left[t \bar{s}^2 + (1 + f + M^2 \tau) \bar{s} + M^2 \bar{u} + (1 + \lambda s)f_1(s) - P \bar{f}_1(0) = 0
\]

(14)

The boundary conditions of (10) now transform to

\[
\begin{align*}
\bar{u} = 0 &= \bar{v} \quad \text{when } \xi = 0 \text{ and } \xi = \pi, \\
\frac{\partial \bar{u}}{\partial \xi} &= \frac{\partial \bar{v}}{\partial \xi} = 0 \quad \text{when } y = 0
\end{align*}
\]

(15)
Applying finite Fourier Sine transform

$$\tilde{u}_F = \int_0^\pi \tilde{u} \sin n\xi d\xi, \quad \tilde{v}_F = \int_0^\pi \tilde{v} \sin n\xi d\xi,$$

where $n$ is a positive integer, to the equation (14), we get

$$\frac{d^2 \tilde{u}_F}{dy^2} - \rho^2 (s) \tilde{u}_F + \left[\frac{(1 + \lambda s)\tilde{f}_1(s) - \lambda f_1(0)}{n}\right] (1 - \cos n\pi) = 0$$

(16)

where $P^2(s) = \frac{(1 + \lambda s)}{(1 + s\tau)}\left[\tau^2 s^2 + (1 + M^2\tau + M^2) + \frac{n^2\pi^2}{4b^2}\right]$ (17)

Under the above finite Fourier Sine transform the last two boundary conditions of (16) transform to

$$\begin{align*}
\tilde{u}_F &= 0 = \tilde{v}_F \quad \text{when } y = 1 \\
\frac{d\tilde{u}_F}{dy} &= 0 = \frac{d\tilde{v}_F}{dy} \quad \text{when } y = 0
\end{align*}$$

(18)

The solution of (16), under the boundary conditions of (18) is

$$\tilde{u}_F = \frac{\left[(1 + \lambda s)\tilde{f}_1(s) - \lambda f_1(0)\right]}{nP^2} \left[1 - \frac{\cosh Py}{\cosh P}\right] (1 - \cos n\pi)$$

(19)

Applying inverse finite Fourier Sine transform to the equation (19), we have

$$\tilde{u} = 2 \sum_{n \in \mathbb{N}} \frac{\sin n\xi (1 - \cos n\pi)}{nP^2} \left[1 - \frac{\cosh Py}{\cosh P}\right] (1 - \cos n\pi)$$

(20)

From (13) and (20), we get

$$\tilde{v} = 2 \sum_{n \in \mathbb{N}} \frac{\sin n\xi (1 - \cos n\pi)}{(1 + s\tau)nP^2} \left[1 - \frac{\cosh Py}{\cosh P}\right] (1 - \cos n\pi)$$

(21)

Applying convolution theorem to (20) and (21), we get respectively

$$u = \frac{8}{\pi^2} \sum_{n = 1}^{\infty} \sum_{r = 0}^{\infty} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r + 1)}{n(2r + 1)} \frac{\pi y}{2} \Delta^{(i)}_{n} \times$$

$$\left[\left(1 + \lambda s_{n}^{(i)}\right) f_{1}(1 - \eta) e^{s_{n}^{(i)} \eta} d\eta - \lambda f_{1}(0) e^{s_{n}^{(i)} \eta}\right]$$

(22)

and

$$v = \frac{8}{\pi^2} \sum_{n = 1}^{\infty} \sum_{r = 0}^{\infty} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r + 1)}{n(2r + 1)} \frac{\pi y}{2} \Delta^{(i)}_{n} \times$$

$$\left[\left(1 + \lambda s_{n}^{(i)}\right) f_{1}(1 - \eta) e^{s_{n}^{(i)} \eta} d\eta - \lambda f_{1}(0) e^{s_{n}^{(i)} \eta}\right]$$

(23)
Where \( s_{nr}^{(i)} (i = 1, 2, 3) \) are the roots of the equation

\[
\lambda \tau s^3 + [\lambda (1 + f + M^2 \tau) + \tau] s^2 + \left[ 1 + f + M^2 \tau + \lambda M^2 + \frac{n^2 \pi^2}{4b^2} - \tau + (2r + 1)^2 \frac{\pi^2}{4} \right] s
\]

\[
+ \left[ M^2 + \frac{n^2 \pi^2}{4b^2} + (2r + 1)^2 \frac{\pi^2}{4} \right] = 0 \quad (24)
\]

and

\[
\Lambda_{nr}^{(i)} = \frac{(1 + s_{nr}^{(i)} \tau)^2}{2 \lambda \tau^3 s_{nr}^{(i)} + [\lambda (4 + f + M^2 \tau) + \tau] s_{nr}^{(i)} + 2 [\lambda (1 + f + M^2 \tau) + \tau] s_{nr}^{(i)} + (1 + f + \lambda M^2)]} \quad (25)
\]

The equation (24) has no positive roots. This equation must allow at least one real negative root. Indeed, the real parts of all the three roots must be negative. This follows from the theorem that a necessary and sufficient condition for the real parts of all the three roots of the cubic,

\[
\theta^3 + B \theta^2 + C \theta + D = 0
\]

to be negative is that \( BC > D > 0 \). For the cubic (24), \( D > 0 \) and \( (BC - D) > 0 \). So the results (22) and (23) describe respectively the fluid and the particle velocities exist for large \( t \).

We now consider particular cases of pressure gradient in which the problem is highly interesting and applicable.

**Case (1). Flow under the pressure gradient of the form \( A(1-e^{-\gamma}) \):**

In the present circumstances let us take

\[
f_1(t) = \Lambda (1-e^{-\gamma}), \text{ where } \Lambda \text{ and } \gamma \text{ are positive constants.}
\]

\[
\therefore \frac{u}{\Lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{\infty} \frac{(-1)^r \sin n \xi (1 - \cos n \pi) \cos (2r + 1) \frac{\pi y}{2}}{n (2r + 1) . \Lambda_{nr}^{(i)}} \times \frac{(1 + s_{nr}^{(i)} \tau)}{s_{nr}^{(i)} (\gamma + s_{nr}^{(i)})} \left[ \gamma (e^{s_{nr}^{(i)} t} - 1) + s_{nr}^{(i)} (e^{-\gamma t} - 1) \right] \quad (1.1)
\]

and

\[
\frac{v}{\Lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{\infty} \frac{(-1)^r \sin n \xi (1 - \cos n \pi) \cos (2r + 1) \frac{\pi y}{2}}{n (2r + 1) . \Lambda_{nr}^{(i)}} \times \frac{(1 + s_{nr}^{(i)} \tau)}{s_{nr}^{(i)} (\gamma + s_{nr}^{(i)})} \left[ \gamma (e^{s_{nr}^{(i)} t} - 1) + s_{nr}^{(i)} (e^{-\gamma t} - 1) \right] \quad (1.2)
\]
When $\gamma \to \infty$ in (1.1) and (1.2), we have respectively

$$u/A = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r+1)}{n(2r+1)} \frac{\pi y}{2} \Lambda_{nr}^{(i)} \times \left( 1 + \frac{\lambda s_{nr}^{(i)}}{s_{nr}^{(i)}} (e^{s_{nr}^{(i)}t} - 1) \right)$$

(1.3)

and

$$v/A = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r+1)}{n(2r+1)} \frac{\pi y}{2} \Lambda_{nr}^{(i)} \times \frac{(1 + \lambda s_{nr}^{(i)})(e^{s_{nr}^{(i)}t} - 1)}{s_{nr}^{(i)} (1 + s_{nr}^{(i)} t)}$$

(1.4)

Equations (1.3) and (1.4) give the velocity of the fluid and the dust particles due to constant pressure gradient $A$.

**Case (II). Flow under periodic pressure gradient:**

Let us consider the case in which

$$f(t) = A\sin(\omega t), \text{ where } A \text{ and } \omega \text{ are positive constants.}$$

$$u/A = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r+1)}{n(2r+1)} \frac{\pi y}{2} \Lambda_{nr}^{(i)} \times$$

$$\frac{(1 + \lambda s_{nr}^{(i)})(\omega e^{s_{nr}^{(i)}t} - s_{nr}^{(i)} \sin \omega t - \omega \cos \omega t)}{(s_{nr}^{(i)} + \omega^2)}$$

(2.1)

and

$$v/A = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r+1)}{n(2r+1)} \frac{\pi y}{2} \Lambda_{nr}^{(i)} \times$$

$$\frac{(1 + \lambda s_{nr}^{(i)})(\omega e^{s_{nr}^{(i)}t} - s_{nr}^{(i)} \sin \omega t - \omega \cos \omega t)}{(s_{nr}^{(i)} + \omega^2)(1 + \tau s_{nr}^{(i)})}$$

(2.2)

When $\tau \to 0$ and $t \to 0$ in (2.1), we have

$$u/A = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{2} \frac{(-1)^r \sin n\xi (1 - \cos n\pi) \cos(2r+1)}{n(2r+1)} \frac{\pi y}{2} \Lambda_{nr}^{(i)} \times$$

$$\frac{(1 + \lambda s_{nr}^{(i)})(\omega e^{s_{nr}^{(i)}t} - s_{nr}^{(i)} \sin \omega t - \omega \cos \omega t)}{(s_{nr}^{(i)} + \omega^2)}$$

(2.3)

where $s_{nr}^{(i)}$ and $s_{nr}^{(j)}$ are the roots of the equation.
\[ \lambda s^2 + (\lambda M^2 + 1)s + \left( M^2 + \frac{n^2 \pi^2}{4b^2} + (2r + 1)^2 \frac{\pi^2}{4} \right) = 0 \]

and \[ D_{nr}^{(i)} = 2\lambda s_{nr}^{(i)} + \lambda M^2 + 1 \]

Expression (2.3) represents the velocity of dust free fluid in the present case.

**Case (III). Flow under impulsive pressure gradient:**

Let us consider the case in which

\[ f_i(t) = A \delta(t), \quad (3.1) \]

where \( A \) is a constant and \( \delta(t) \) is Dirac delta function defined as

\[ \delta(t) = \begin{cases} 0, & t \leq 0 \\ \frac{1}{\varepsilon}, & 0 < t < \varepsilon \\ 0, & t \geq \varepsilon \end{cases} \]

where \( \varepsilon > 0 \) and no matter how small \( \varepsilon \) is.

From (22), (23) and (3.1), we get

\[ \frac{u}{\lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^{r} \sin n \xi (1 - \cos \pi r) \cos (2r + 1)}{n(2r + 1)} \frac{\pi y}{2} \Delta_{nr}^{(i)} \times \]

\[ \times (1 + \lambda s_{nr}^{(i)}) e^{s_{nr}^{(i) t}}, \quad t > 0 \]

and

\[ \frac{v}{\lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^{r} \sin n \xi (1 - \cos \pi r) \cos (2r + 1)}{n(2r + 1)} \frac{\pi y}{2} \Delta_{nr}^{(i)} \times \]

\[ \times \frac{(1 + \lambda s_{nr}^{(i)}) e^{s_{nr}^{(i) t}}}{(1 + s_{nr}^{(i) t})}, \quad t > 0 \]

When \( \lambda \to 0 \) in (3.2) and (3.3), we have respectively

\[ \frac{u}{\lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1,2} \frac{(-1)^{r} \sin n \xi (1 - \cos \pi r) \cos (2r + 1)}{n(2r + 1)} \frac{\pi y}{2} E_{nr}^{(i)} e^{s_{nr}^{(i) t}}, \quad (3.4) \]

and

\[ \frac{v}{\lambda} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1,2} \frac{(-1)^{r} \sin n \xi (1 - \cos \pi r) \cos (2r + 1)}{n(2r + 1)} \frac{\pi y}{2} \times \frac{E_{nr}^{(i)} e^{s_{nr}^{(i) t}}}{(1 + s_{nr}^{(i) t})}, \quad (3.5) \]
where \( s_n^{(i)} (i = 1, 2) \) are the roots of the equation

\[
\tau s^2 + \left[ 1 + \frac{M^2}{4b^2} + \frac{n^2 \pi^2}{(2r + 1)^2} \frac{\pi^2}{4} \right] s + \left[ M^2 + \frac{n^2 \pi^2}{4b^2} + \frac{(2r + 1)^2 \pi^2}{4} \right] = 0 \quad (3.6)
\]

and

\[
E_{nr}^{(i)} = \frac{(1 + s_{nr}^{(i)} \tau)}{\tau^2 s_{nr}^{(i)} + 2 \tau s_{nr}^{(i)} + (1 + \tau)^2} \quad (3.7)
\]

Expressions (3.4) and (3.5) give the velocity of the fluid and the dust particles for ordinary viscous fluid in the present case.

**Case (IV). Flow due to constant pressure gradient acting for a finite time \( T \):**

Under the present circumstances we take the help of Heaviside unit function \( H(t) \) defined as

\[
H(t) = \begin{cases} 
0 & \text{if } t \leq 0 \\
1 & \text{if } t > 0.
\end{cases}
\]

Let us take

\[
f_1(t) = A[H(t) - H(t - T)] \quad (4.1)
\]

From (22), (23) and (4.1), we have

\[
\frac{u}{A} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} (-1)^r \sin n \xi (1 - \cos n \pi) \cos (2r + 1) \frac{\pi^2}{2} \Delta_{nr}^{(i)} \times \frac{(1 + \lambda s_{nr}^{(i)})(e^{s_{nr}^{(i)}t} - 1)}{s_{nr}^{(i)}} \quad \text{for } 0 < t \leq T \quad (4.2)
\]

\[
= \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} (-1)^r \sin n \xi (1 - \cos n \pi) \cos (2r + 1) \frac{\pi^2}{2} \Delta_{nr}^{(i)} \times \frac{(1 + \lambda s_{nr}^{(i)})(1 - e^{s_{nr}^{(i)}T})e^{s_{nr}^{(i)}t}}{s_{nr}^{(i)}} \quad \text{for } t > T \quad (4.3)
\]

and

\[
\frac{v}{A} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} (-1)^r \sin n \xi (1 - \cos n \pi) \cos (2r + 1) \frac{\pi^2}{2} \Delta_{nr}^{(i)} \times \frac{(1 + \lambda s_{nr}^{(i)})(e^{s_{nr}^{(i)}t} - 1)}{s_{nr}^{(i)}(1 + s_{nr}^{(i)} \tau)} \quad \text{for } 0 < t \leq T \quad (4.4)
\]
\[
\frac{u}{A_0} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n \xi (1 - \cos n \pi \cos(2r + 1)) \pi y}{n(2r + 1)} \left(1 + \lambda s_{nr}^{(i)} \right) \Delta^{(i)}_{nr} e^{s_{nr}^{(i)}} \frac{1}{1 + s_{nr}^{(i)} t} \quad \text{for } t > T
\]

If we make \( T \to 0 \) and \( A \to \infty \) in (4.3) and (4.5) in such manner that \( AT = A_0 \), we respectively get

\[
\frac{u}{A_0} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n \xi (1 - \cos n \pi \cos(2r + 1)) \pi y}{n(2r + 1)} 2 \Delta^{(i)}_{nr} e^{s_{nr}^{(i)}} \frac{1}{1 + s_{nr}^{(i)} t} \quad \text{for } t > 0 \tag{4.6}
\]

and

\[
\frac{v}{A_0} = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \sum_{r=0}^{\infty} \sum_{i=1}^{3} \frac{(-1)^r \sin n \xi (1 - \cos n \pi \cos(2r + 1)) \pi y}{n(2r + 1)(1 + s_{nr}^{(i)} t)} (1 + \lambda s_{nr}^{(i)}) \Delta^{(i)}_{nr} e^{s_{nr}^{(i)}} \quad \text{for } t > 0 \tag{4.7}
\]

Expressions (4.6) and (4.7) represent the velocity of the fluid and the dust particles due to the impulsive pressure gradient \( A_0 \delta(t) \). The results (4.6) and (4.7) agree with the results (3.2) and (3.3) respectively.

**Discussions:**

The analysis of incompressible, unidirectional magneto-fluid-dynamic equation of dusty visco-elastic fluid of Maxwell type through a rectangular channel with arbitrary pressure gradient forming a system of ‘translates’ leads to the following conclusions:

Making \( \lambda \to 0 \) in (22) and (23), one may get the corresponding flow of dusty viscous fluid. Setting \( M \to 0 \) in (22) and (23), we may get the corresponding non-MHD flow. If we take \( \tau \to 0 \) and \( f \to 0 \) in (22) and (23), we have the corresponding flow of dust free fluid.

Four particular cases, considered here, are interesting. In first case the pressure gradient increases numerically with time and ultimately tends to a constant i.e. the pressure gradient is of the type \( A(1 - e^{-\gamma t}) \), \( A, \gamma > 0 \), the results are interesting. From this case analytical expression of the velocity of the fluid and the dust particles for
constant pressure gradient have been derived by putting $\gamma \to 0$. In second case, we have considered harmonically oscillating pressure gradient. Analytical expression of the velocity of clean fluid for second case has been derived by making $\tau \to 0$ and $f \to 0$ in the equation (2.1). In third case, we have taken the pressure gradient is of the impulsive type. In this case, the analytical expression of the velocity of the fluid and the dust particles for ordinary dusty viscous fluid have been derived by putting $\lambda \to 0$ in the equations (3.2) and (3.3) respectively. The results of fourth case are interesting. Here the pressure gradient is a constant, which persists for a finite time $T$. By putting $T \to \infty$ in (4.2) and (4.4), one may get the velocity profiles of the fluid and the dust particles for constant pressure gradient. The analytical expressions of the velocity of the fluid and the dust particles for impulsive pressure gradient have been derived from fourth case. These results agree with the results of third case.

In order to discuss results, numerical computations have been made by taking typical values of the various parameters and using $b = 1$ and $\tau = 0.01, A = 1$.

From figs. 1.1, 2.1, 3.1, 4.1 and 4.1a it is seen that as $M$, the Hartmann number, increases the velocity of the fluid and the dust particles decrease i.e. the magnetic field impedes the velocity of the fluid and the dust particles in all the four cases. Thus, the implementation of magnetic field is a device to control the flow. The above mentioned figs. show that the velocity of the fluid and the dust particles decrease with the increase of $y$.

The tables 1.1, 2.1 and 3.1 depict that as $\lambda$, the time relaxation, increases the velocity of both the fluid and the dust particles increase in first three cases. In fourth case, tables 4.1 and 4.1a reveal that when $0 < t \leq T$ the velocity of both the fluid and the dust particles go on increasing with $\lambda$, the time relaxation, but the opposite happens when $t > T$ i.e. the velocity of both the fluid and the dust particles go on decreasing with the increase of $\lambda$, the time relaxation. It is also observed from tables 1.1, 2.1, 3.1, 4.1 and 4.1a that the velocity profiles of the fluid and the dust particles are symmetric about the plane $z = 0$.

Fig. 1.3 depicts that the velocity of both the fluid and the dust particles go on decreasing with the increase of $f$ the mass concentration of dust particles. Fig. 1.3 also
depicts that the velocity of the fluid and the dust particles go on increasing with time and the effect of $f$ is being pronounced as time goes on increasing.

Fig. 2.3 shows that the velocity profiles of the fluid and the dust particles are periodic w. r. t. time $t$. Fig. 2.3 also shows that as $f$, the mass concentration of the dust, increases the amplitude of oscillation of the velocity of the fluid and the dust particles diminishes. Thus, the velocity of the fluid and the dust particles are retarded by the presence of dust particles.

Fig. 3.3 narrates that the flow fields of the fluid and the dust particles decrease with the increasing value of $f$, the mass concentration of the dust. This figure also narrates that at the beginning of the motion the velocity of the fluid is greater than that of the dust particles but after a short while the opposite happens i.e. the velocity of the fluid is slower than that of the dust particles. From fig. 3.3 it is observed that the velocity of the fluid and the dust particles decrease with the increase of time $t$ and ultimately vanish.

From the fig. 4.3 it is obvious that the velocity of the fluid remains greater than that of the dust particles as long as $(0 < t \leq T)$ the pressure gradient is active. Immediately after ceasing the pressure gradient at time $t = T$, the dust particles move along with the fluid for a short while. Afterwards, the velocity of the fluid is slower than that of the dust particles and the velocity of both the fluid and the dust particles asymptotically fall to zero. From fig. 4.3 it is seen that as $f$, the mass concentration of dust, increases the velocity of the fluid and the dust particles decrease. This happens upto a certain value $t_c$ (say) of time and $t_c > T$. When $t > t_c$, the opposite happens i.e. the presence of dust particles accelerates the motion of the fluid and the dust particles. Fig. 4.3 depicts that the velocity of the fluid and the dust particles initially increase rapidly. The fig. 4.3 also depicts that immediately after ceasing the pressure gradient at time $t = T$, the velocity of both the fluid and the dust particles decrease abruptly.
### Table 1.1 (Case I)

Numerical values of $u$ and $v$ for different values of $z$ and $\lambda$ when $M = 3.317$, $y = 0.7$, $f = 0.1$, $\tau = 0.01$, $t = 0.1$, $\gamma = 0.06$.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$u (\lambda = 0.15)$</th>
<th>$u (\lambda = 0.2)$</th>
<th>$u (\lambda = 0.25)$</th>
<th>$v (\lambda = 0.15)$</th>
<th>$v (\lambda = 0.2)$</th>
<th>$v (\lambda = 0.25)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>-0.8</td>
<td>0.00252</td>
<td>0.00305</td>
<td>0.00392</td>
<td>0.0023</td>
<td>0.00283</td>
<td>0.00375</td>
</tr>
<tr>
<td>-0.6</td>
<td>0.0026</td>
<td>0.00325</td>
<td>0.00474</td>
<td>0.0024</td>
<td>0.003</td>
<td>0.00461</td>
</tr>
<tr>
<td>-0.4</td>
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<td>0.00385</td>
<td>0.00585</td>
<td>0.00252</td>
<td>0.00367</td>
<td>0.00581</td>
</tr>
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### Table 3.1 (Case III)

Numerical values of $u$ and $v$ for different values of $z$ and $\lambda$ when $M = 3.317$, $y = 0.7$, $f = 0.1$, $\tau = 0.01$ and $t = 0.1$.

<table>
<thead>
<tr>
<th>$z$</th>
<th>$u (\lambda = 0.2)$</th>
<th>$u (\lambda = 0.25)$</th>
<th>$u (\lambda = 0.3)$</th>
<th>$v (\lambda = 0.2)$</th>
<th>$v (\lambda = 0.25)$</th>
<th>$v (\lambda = 0.3)$</th>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>-0.8</td>
<td>0.48355</td>
<td>0.66853</td>
<td>1.16123</td>
<td>0.53809</td>
<td>0.74378</td>
<td>1.27035</td>
</tr>
<tr>
<td>-0.6</td>
<td>0.67505</td>
<td>0.96847</td>
<td>1.84747</td>
<td>0.71845</td>
<td>1.04299</td>
<td>1.98532</td>
</tr>
<tr>
<td>-0.4</td>
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<td>1.11575</td>
<td>2.32585</td>
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<td>1.21625</td>
<td>2.51130</td>
</tr>
<tr>
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<td>1.34146</td>
<td>2.83731</td>
</tr>
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</tr>
<tr>
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<td>2.63941</td>
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<tr>
<td>0.6</td>
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<td>1.84747</td>
<td>0.71845</td>
<td>1.04299</td>
<td>1.98532</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.66853</td>
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<td>0.74378</td>
<td>1.27035</td>
</tr>
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</table>
Table 4.1 (Case IV, \( t < T \))

Numerical values of \( u \) and \( v \) for different values of \( z \) and \( \lambda \) when 

\[ M = 3.317, \ y = 0.7, \ f = 0.1, \ \tau = 0.01, \ t = 0.1 \ and \ T = 0.4. \]

<table>
<thead>
<tr>
<th>( z )</th>
<th>( u (\lambda = 0.1) )</th>
<th>( u (\lambda = 0.15) )</th>
<th>( u (\lambda = 0.2) )</th>
<th>( v (\lambda = 0.1) )</th>
<th>( v (\lambda = 0.15) )</th>
<th>( v (\lambda = 0.2) )</th>
</tr>
</thead>
<tbody>
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<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>-0.8</td>
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<td>0.07144</td>
<td>0.08771</td>
<td>0.05759</td>
<td>0.07056</td>
<td>0.08738</td>
</tr>
<tr>
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<td>0.07491</td>
<td>0.09810</td>
<td>0.05858</td>
<td>0.07051</td>
<td>0.09629</td>
</tr>
<tr>
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<td>0.11332</td>
<td>0.05923</td>
<td>0.07931</td>
<td>0.11210</td>
</tr>
<tr>
<td>-0.2</td>
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<td>0.08111</td>
<td>0.11636</td>
<td>0.06102</td>
<td>0.07798</td>
<td>0.11530</td>
</tr>
<tr>
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<td>0.12155</td>
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<td>0.08229</td>
<td>0.11850</td>
</tr>
<tr>
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<td>0.08111</td>
<td>0.11636</td>
<td>0.06102</td>
<td>0.07798</td>
<td>0.11530</td>
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<tr>
<td>0.4</td>
<td>0.06085</td>
<td>0.08106</td>
<td>0.11332</td>
<td>0.05923</td>
<td>0.07931</td>
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</tr>
<tr>
<td>0.6</td>
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<td>0.07491</td>
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<td>0.09629</td>
</tr>
<tr>
<td>0.8</td>
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<td>0.08738</td>
</tr>
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</tbody>
</table>

Table 4.1a (Case IV, \( t > T \))

Numerical values of \( u \) and \( v \) for different values of \( z \) and \( \lambda \) when 

\[ M = 3.317, \ y = 0.7, \ f = 0.1, \ \tau = 0.01, \ t = 0.6 \ and \ T = 0.4. \]

<table>
<thead>
<tr>
<th>( z )</th>
<th>( u (\lambda = 0.1) )</th>
<th>( u (\lambda = 0.15) )</th>
<th>( u (\lambda = 0.2) )</th>
<th>( v (\lambda = 0.1) )</th>
<th>( v (\lambda = 0.15) )</th>
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<td>0.00332</td>
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<td>0.00120</td>
<td>0.00443</td>
</tr>
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<td>0.00668</td>
<td>0.01652</td>
<td>0.00387</td>
<td>0.00828</td>
<td>0.01944</td>
</tr>
<tr>
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<td>0.00757</td>
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<td>0.000000</td>
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</tr>
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</table>
Fig. 1.1 (Case I): Velocity profiles of the fluid and the dust particles for various values of $M$.

Fig. 1.3 (Case I): Velocity profiles of the fluid and the dust particles for various values of $f$. 
Fig. 3.1 (Case III) : Velocity profiles of the fluid and the dust particles for various values of M.

Fig. 2.1 (Case II) : Velocity profiles of the fluid and the dust particles for various values of M.
Fig. 2.3 (Case II) : Velocity profiles of the fluid and the dust particles for various values of $f$.

Fig. 3.3 (Case III) : Velocity profiles of the fluid and the dust particles for various values of $f$. 
Fig. 4.1a (Case IV, t > T) : Velocity profiles of the fluid and the dust particles for various values of M.

Fig. 4.3 (Case IV) : Velocity profiles of the fluid and the dust particles for various values of f.
Fig. 4.1 (Case IV, t < T): Velocity profiles of the fluid and the dust particles for various values of M.

Table 2.1 (Case II)

Numerical values of u and v for different values of z and λ when $M = 3.317$, $y = 0.7$, $f = 0.1$, $\tau = 0.01$, $t = 0.1$ and $\omega = 8.0$.

<table>
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<th>$u (\lambda = 0.2)$</th>
<th>$v (\lambda = 0.1)$</th>
<th>$v (\lambda = 0.15)$</th>
<th>$v (\lambda = 0.2)$</th>
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<td>0.03100</td>
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<td>0.04170</td>
<td>0.02105</td>
<td>0.02964</td>
<td>0.03871</td>
</tr>
<tr>
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<tr>
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<td>0.03662</td>
<td>0.05271</td>
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<td>0.04932</td>
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