CHAPTER 1
PRELIMINARIES

1.1 A SURVEY OF LITERATURE

In 1963, N.Levine introduced semi-open sets[17]. A subset A of a topological space is called semi open if A is a subset of cl(int A).

In 1970, N.Biswas introduced semi-closed sets[6]. The complement of a semi open set is semi closed. The intersection of all semi closed sets containing A is called the semi closure[6] of A and is denoted by scl(A). The union of all semi open sets contained in A is called the semi interior of A and is denoted by sint(A).

In 1982, A.S Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb introduced pre-open sets and pre-closed sets. A subset A of a topological space is called pre-open [23] if A is a subset of int(cl(A)). The complement of a pre-open set is pre-closed [23]. The intersection of all pre-closed sets containing A is called the pre-closure of A and is denoted by pcl(A). The union of all pre-open sets contained in A is called the pre interior of A and is denoted by pint(A).

In 1983, A.S.Mashhour et al. introduced $\alpha$-open sets and $\alpha$-closed sets[24]. A subset A is called $\alpha$ open if $A \subset \text{Int}(\text{cl}(\text{Int}A))$. The complement of an $\alpha$-open set is said to be $\alpha$ closed. The intersection of all $\alpha$ closed sets containing A is called the $\alpha$ closure of A and is denoted
by $\alpha\text{cl}(A)$. The union of all $\alpha$ open sets contained in $A$ is called the $\alpha$ interior of $A$ and is denoted by $\alpha\text{int}(A)$.

In 1983, M.E. Abd El-Monsef introduced $\beta$ open set and $\beta$ closed set. A subset $A$ of a topological space is called $\beta$ open[1] if $A$ is a subset of $\text{cl}(\text{int}(\text{cl}(A)))$. The complement of a $\beta$ open set is $\beta$ closed[1].

In 1986, D. Andrijevic introduced semi-pre open set and also semi-pre closed set. A subset $A$ of a topological space is called semi-pre open [2] if $A$ is a subset of $\text{cl}(\text{int}(\text{cl}(A)))$. The complement of a semi-pre open set is semi-pre closed [2]. $\beta$ sets and semi-pre sets are the same. The intersection of all $\beta$ closed sets containing $A$ is called the $\beta$ closure of $A$ and is denoted by $\beta\text{cl}(A)$. The union of all $\beta$ open sets contained in $A$ is called the $\beta$ interior of $A$ and is denoted by $\beta\text{int}(A)$.
In 1970, N. Levine introduced generalised closed (g-closed) set[18]. A subset A of a topological space is called g-closed if cl(A) ⊆ U whenever A ⊆ U and U is open. The complement of a g-closed set is called a g-open set.

In 1987, P. Bhattacharyya and B. K. Lahiri defined semi generalized closed sets and established that this set has no connection with generalized closed sets defined by Levine. A subset A is called a semi generalized closed set (briefly sg closed[5]) if scl(A) ⊆ U whenever A ⊆ U and U is semi open.

In 1990, S. P. Arya and T. Nour defined generalized semi closed sets. A subset A is a generalized semi closed set (briefly gs closed[3]) if scl(A) ⊆ U whenever A ⊆ U and U is open.

In 1993, H. Maki introduced generalised α closed set. A subset A of a topological space is called a generalised α closed set (briefly ga closed[19]) if αcl(A) ⊆ U whenever A ⊆ U and U is α open.

In 1994, H. Maki introduced α generalised closed sets. A subset A of a topological space is called α generalised closed (briefly, αg closed[20]) if αcl(A) ⊆ U whenever A ⊆ U and U is open.

In 1995, J. Dontchev introduced generalised β closed sets. A subset A of a topological space is called generalised β closed or generalised semi
pre closed (briefly, gβ closed[14] or gsp closed) if βcl(A) ⊆ U whenever A ⊆ U and U is open.

In 1996, H.Maki introduced generalised pre closed set. A subset A of a topological space is called generalised pre closed (briefly, gp closed[21]) if pcl(A) ⊆ U whenever A ⊆ U and U is open.

In 1963, N.Levine defined semi-continuous function. A map f: (X,τ)→(Y,σ) is called semi-continuous[17] if the inverse image f⁻¹(V) of each open set V of (Y,σ) is semi-open in (X,τ).

In 1969, N.Biswas defined semi-homeomorphism. A map f: (X,τ)→(Y,σ) is said to be a semi-homeomorphism[7], if (1) f is continuous (2) f is semi open (i.e. f(U) is semi-open for every open sets U of (X,τ)) and (3) f is bijective.

In 1977, R.Devi, K.Balachandran and H.Maki defined αg-continuous function. A function f: (X,τ)→(Y,σ) is called αg-continuous[12] if f⁻¹(V) is αg-closed in (X,τ) for every closed set V of (Y,σ).

In 1972, S.G.Crossley defined semi-homeomorphism. A map f: (X,τ)→(Y,σ) is said to be a semi-homeomorphism[9], if (1) f is irresolute (that is, f⁻¹(V) is semi-open for every semi-open set V of Y), (2) f is pre-semi open (that is, f(U) is semi-open for every semi-open set U of X) and (3) f is bijective. In 1972 S.G.Crossley defined Irresolute and pre-semiopen maps. A map f: (X,τ)→(Y,σ) is called irresolute[9] if
\( f'(V) \) is semi-open in \( X \) for every semi-open \( V \) of \( Y \) (that is, for every semi-open set \( V \) of \( Y \), \( f'(V) \) is semi-open in \( X \)). A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called pre-semi-open[9] if \( f(U) \) is semi-open in \( (Y, \sigma) \) for every semi-open set \( U \) in \( (X, t) \).

In 1973, T.Noiri defined semi closed map. A mapping \( f: (X, t) \rightarrow (Y, \sigma) \) is said to be semi closed[27] if the image \( f(F) \) of each closed set \( F \) in \( (X, t) \) is semiclosed in \( (Y, \sigma) \).

In 1982, A.S.Mashhour defined pre-continuous function. A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called pre-continuous[23] if \( f'(V) \) is pre-closed in \( (X, t) \) for every closed set \( V \) of \( (Y, \sigma) \). In 1983 A.S.Mashhour defined \( \alpha \)-continuous function. A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called \( \alpha \)-continuous[24] if \( f'(V) \) is \( \alpha \)-closed in \( (X, t) \) for every closed set \( V \) of \( (Y, \sigma) \). In 1983, M.E.Abd El-Monsef defined \( \beta \)-continuous function. A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called \( \beta \)-continuous[1] if \( f'(V) \) is semi-pre-open in \( (X, t) \) for every open set \( V \) of \( (Y, \sigma) \).

In 1985, D.S.Jankovic defined p-open. A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called p-open[16] if \( f(U) \) is pre-open in \( (Y, \sigma) \) for every preopen set \( U \) of \( (X, t) \). In 1991, P.Sundaram defined sg-continuous function. A map \( f: (X, t) \rightarrow (Y, \sigma) \) is called sg-continuous[31] mapping if \( f'(V) \) is sg-closed in \( (X, t) \) for every closed set \( V \) of \( (Y, \sigma) \). A map \( f: (X, t) \rightarrow (Y, \sigma) \) is
gs-continuous [31] if \( f^{-1}(V) \) is gs-closed in \((X,i)\) for every closed set \( V \) of \((Y,\sigma)\). A map \( f: (X,i) \rightarrow (Y,\sigma) \) is called sg-irresolute [31] if \( f^{-1}(V) \) is sg-closed in \((X,i)\) for every sg-closed set \( V \) of \((Y,\sigma)\). A map \( f:(X,i) \rightarrow Y(\sigma) \) is called gs-irresolute [31] if \( f^{-1}(V) \) is gs-closed in \((X,i)\) for every gs-closed set \( V \) of \((Y,\sigma)\).

In 1993, R.Devi defined sg-closed and gs-closed maps. A map \( f: (X,i) \rightarrow (Y,\sigma) \) is called semi-generalised closed [10] if \( f(V) \) is semi-generalised closed in \((Y,\sigma)\) for every closed set \( V \) in \((X,i)\). Every semi-closed map is semi-generalised closed. A map \( f: (X,i) \rightarrow (Y,\sigma) \) is called a generalised semi-closed [10] if for each closed set \( F \) of \((X,i)\), \( f(F) \) is gs-closed in \((Y,\sigma)\). In 1993 H.Maki defined \( g\alpha \)-continuous. A function \( f: (X,i) \rightarrow (Y,\sigma) \) is called \( g\alpha \)-continuous [19] if \( f^{-1}(V) \) is \( g\alpha \)-closed in \((X,i)\) for every closed set \( V \) of \((Y,\sigma)\). In the same year N.Palaniappan defined rg-continuous and regular irresolute [28]. A function \( f: (X,i) \rightarrow (Y,\sigma) \) is called rg-continuous [28] if \( f^{-1}(V) \) is regular generalised closed in \((X,i)\) for every closed set \( V \) of \((Y,\sigma)\).

In 1998, R.Devi, K.Balachandran and H.Maki defined \( \alpha g \) irresolute\([13]\), \( \alpha g \)-closed map\([13]\) and \( g\alpha \) closed map\([13]\).

In 1999, Y.Gnanambal and K.Balachandran defined \( gpr \)-continuous\([15]\) and \( gpr^* \) continuous maps\([15]\).

So far, the generalizations in this context have not been carried out between closed and generalized closed sets. We enter into this venture in this research work.

We have a diagram showing the relationship between the generalised sets mentioned above.

\[
\begin{array}{c}
\text{closed} \quad \longrightarrow \quad \text{\( g \) closed} \\
\downarrow \\
\alpha \text{ closed} \quad \longrightarrow \quad \text{\( g\alpha \) closed} \quad \longrightarrow \quad \text{\( \alpha g \) closed} \\
\downarrow \\
\text{semi closed} \quad \longrightarrow \quad \text{\( sg \) closed} \quad \longrightarrow \quad \text{\( gs \) closed} \\
\downarrow \\
\text{pre closed} \quad \longrightarrow \quad \text{\( gp \) closed} \\
\downarrow \\
\beta \text{ closed} \quad \longrightarrow \quad \text{\( g\beta \) closed}
\end{array}
\]
1.2. Scope of the present work

We define four generalisations and see that all these generalisations are between the class of closed sets and the class of generalised closed sets. We have defined various generalisations of continuous functions. Also we study the impact of these generalisations in the case of fuzzy topological spaces.

In chapter 2 we introduce the concepts - semi generalised star closed sets and semi generalised star open sets briefly written as sg*closed and sg* open sets respectively. A subset A of a topological space X is said to be sg* closed if A \subseteq U, U semi open implies cl(A) \subseteq U. The complement of a sg* closed set is defined to be sg* open. We see that every closed set is sg* closed but not conversely. We see that every sg* closed set is g closed but not conversely. We prove some equivalent characterisations. A set A is sg* closed iff O is open and A \subseteq cl(O) implies A \cup O = cl(A) \cup O. A is sg* open iff for each \ x \in A-\text{int} \ A, scl(x) \cap A^c \text{ is non empty}. We prove the following.

A homeomorphic image of sg* closed set is sg* closed.

A homeomorphic image of sg* open set is sg* open.
We define pre sg* closed map, pre sg* open map, sg* irresolute and sg* homeomorphism. We see that every homeomorphism is sg* homeomorphism. A map $f$ is called pre sg* closed map if $A$ is sg* closed implies $f(A)$ is sg* closed. A map $f: X \rightarrow Y$ is called pre sg* open if $A$ is sg* open in $X$ implies $f(A)$ is sg* open in $Y$.

Every homeomorphism is pre sg* closed. Every homeomorphism is pre sg* open. A map $f: X \rightarrow Y$ is called sg* irresolute if $A$ is sg* closed in $Y$ implies $f^{-1}(A)$ is sg* closed in $X$.

In chapter 3 we introduce $\alpha$ generalized star closed set and $\alpha$ generalized star open set. We see that every closed set is $\alpha g^*$ closed and every $\alpha g^*$ closed set is $g$ closed. We prove some properties and equivalent characterisations of these sets. We prove that a set $A$ is $\alpha g^*$ open iff whenever $A$ contains a $\alpha$ closed set $F$, Int $A$ also contains $F$. We prove that $A$ is $\alpha g^*$ closed iff cl$(A)$-$A$ does not contain any non empty $\alpha$ closed set. We prove that a homeomorphic image of $\alpha g^*$ closed set is $\alpha g^*$ closed and a homeomorphic image of $\alpha g^*$ open set is $\alpha g^*$ open. We define $\alpha g^*$ homeomorphism and derive certain properties.

In chapter 4 we define $\beta g^*$ closed sets, $\beta g^*$ open sets. We prove some properties and equivalent characterisations also. We define $\beta g^*$ homeomorphism and derive certain properties.
In chapter 5 we define $pg^*$ closed sets and $pg^*$ open sets. We prove some properties and equivalent characterizations. We define $pg^*$ homeomorphism and derive certain properties.

In chapter 6 we define $\alpha$ connected set, locally $\alpha$ connected set, $\beta$ connected set, locally $\beta$ connected set, $p$ connected set, and locally $p$ connected set. We define $\alpha$ component, $\beta$ component and $p$ component and we prove in each case that the distinct components form a partition of the topological space.

In chapter 7 we define a crisp topology from a given fuzzy topology. Also we study some generalizations in a fuzzy topological space.

1.3 PRELIMINARY DEFINITIONS

1.3.1 A subset $A$ of a topological space is called semi open[17] if $A$ is a subset of $\text{cl}(\text{int } A)$.

1.3.2 A subset $A$ is said to be $\alpha$-open [24] if $A \subseteq \text{Int}(\text{cl}(\text{Int } A))$. The complement of an $\alpha$-open set is said to be $\alpha$-closed[24]. The intersection of all $\alpha$ closed sets containing $A$ is called the $\alpha$ closure of $A$ and is
denoted by $\alpha \text{cl}(A)$. The union of all $\alpha$ open sets contained in $A$ is called the $\alpha$ interior of $A$ and is denoted by $\alpha \text{int}(A)$.

1.3.3 The complement of a semi open set is said to be semi closed[6]. The intersection of all semi closed sets containing $A$ is called the semi closure[6] of $A$ and is denoted by $\text{scl}(A)$. The union of all semi open sets contained in $A$ is called the semi interior of $A$ and is denoted by $\text{sint}(A)$.

1.3.4 A subset $A$ of a topological space is called g-closed set[18] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open. The complement of a g-closed set is called a g-open set.

1.3.5 A subset $A$ of a topological space is called pre-open [23] if $A$ is a subset of int $\text{cl}(A)$. The complement of a pre-open set is said to be pre-closed [23]. The intersection of all pre-closed sets containing $A$ is called the pre-closure of $A$ and is denoted by $\text{pcl}(A)$. The union of all pre-open sets contained in $A$ is called the pre interior of $A$ and is denoted by $\text{pint}(A)$.

1.3.6 A subset $A$ of a topological space is called $\beta$ open[1] if $A$ is a subset of $\text{cl int cl}(A)$. The complement of a $\beta$ open set is said to be $\beta$ closed[1]. The intersection of all $\beta$ closed sets containing $A$ is called the $\beta$ closure of $A$ and is denoted by $\beta \text{cl}(A)$. The union of all $\beta$ open sets contained in $A$ is called the $\beta$ interior of $A$ and is denoted by $\beta \text{int}(A)$.
1.3.7 A subset $A$ is called a semi generalized closed set (briefly, sg closed[5]) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi open.

1.3.8 A subset $A$ is a generalized semi closed set (briefly, gs closed[3]) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

1.3.9 A subset $A$ of a topological space is called a generalised $\alpha$ closed set (briefly, $g\alpha$ closed[19]) if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$ open.

1.3.10 A subset $A$ of a topological space is called a $\alpha$ generalised closed set (briefly, $\alpha g$ closed[20]) if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

1.3.11 A subset $A$ of a topological space is called generalised $\beta$ closed or generalised semi pre closed (briefly, $g\beta$ closed[14] or gsp closed) if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

1.3.12 A subset $A$ of a topological space is called a generalised pre closed set (briefly, gp closed[21]) if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open.

1.3.13 A map $f:(X,\tau) \rightarrow (Y,\sigma)$ from a space $(X,\tau)$ to a space $(Y,\sigma)$ is called semi-continuous[17] if the inverse image $f^{-1}(V)$ of each open set $V$ of $(Y,\sigma)$ is semi-open in $(X,\tau)$. 


1.3.14 A function \( f: (X, t) \rightarrow (Y, \sigma) \) is called pre-semi-open if \( f(U) \) is semi-open in \((Y, \sigma)\) for every semi-open set \( U \) in \((X, t)\).

1.3.15 A mapping \( f: (X, t) \rightarrow (Y, \sigma) \) is said to be semi closed if the image \( f(F) \) of each closed set \( F \) in \((X, t)\) is semiclosed in \((Y, \sigma)\).

1.3.16 A map \( f: (X, t) \rightarrow (Y, \sigma) \) is called sg-irresolute if \( f^t(V) \) is sg-closed in \((X, t)\) for every sg-closed set \( V \) of \((Y, \sigma)\).

1.3.17 A map \( f: (X, t) \rightarrow Y(\sigma) \) is called gs-irresolute if \( f^1(V) \) is gs-closed in \((X, t)\) for every gs-closed set \( V \) of \((Y, \sigma)\).

1.3.18 A map \( f: (X, t) \rightarrow (Y, \sigma) \) is called irresolute if \( f^1(V) \) is semi-open in \( X \) for every semi-open \( V \) of \( Y \).