CHAPTER V

RESULTS AND DISCUSSION

5.1 BARRELLING BEHAVIOUR OF ALUMINIUM SQUARE BILLETS

The results of barrelling behaviour of aluminium square billets during cold upsetting without lubrication are here discussed. Figure 5.1 shows the plot drawn between the axial strain, \( \varepsilon_z = \ln \left( \frac{h_0}{h_t} \right) \) and the new hoop strain, \( \varepsilon_{\theta''} \). This plot is a straight line with almost equal slope irrespective of aspect ratios considered. The new hoop strain \( (\varepsilon_{\theta''}) \) is calculated based on the following expression:

\[
\varepsilon_{\theta''} = \ln \left( \frac{(2l_b^2 + l_c^2)/3a^2}{l_b} \right)
\]

(5.1)

where \( l_b \) is the bulged length \( \left[ \frac{(l_{b1} + l_{b2} + l_{b3} + l_{b4})}{4} \right] \), \( l_c \) is the contact length \( \left[ \frac{(l_{c1} + l_{c2} + l_{c3} + l_{c4})}{4} \right] \) and 'a' is the side of the square.

Using the simple theory of plasticity stresses, namely the hoop stress \( (\sigma_{\theta}) \), the effective stress \( (\sigma) \) and the hydrostatic stress \( (\sigma_m) \) are calculated and plotted against the axial strain \( (\varepsilon_z) \) for different aspect ratios as shown in Figures 5.2(a)–(c). The computational procedure for the aforesaid stresses is discussed in section 3.7. All the stresses namely the axial stress \( (\sigma_z) \), the hoop stress \( (\sigma_{\theta}) \), the effective stress \( (\sigma) \) and the hydrostatic stress \( (\sigma_m) \) increase with the increasing amount of the strain. The hoop stress is tensile in nature because during compressive deformation and the bulged diameter expanded due to the action of secondary tensile stress. However, it is observed that for
Figure 5.1 Variation of the new hoop strain with respect to the axial strain
Figure 5.2(a) Variation of stresses with respect to the axial Strain
Figure 5.2(b) Variation of stresses with respect to the axial Strain.
Figure 5.2(c) Variation of stresses with respect to the axial strain

Axial Strain, $\varepsilon_2$
any given deformation level, the increase in hoop stress due to loading is appreciably lower compared to the axial stress.

Figure 5.3 shows the plot drawn between the measured radius of curvature of the barrel and the calculated radius based on the principle of volume constancy during deformation, on the assumption that the barrel radius follows circular arc. It is seen that the calculated values of the radius of curvature are in close proximity with measured values. This indicates that the calculated value of radius of curvature of the barrel fits a circular arc, which is correct one. Figure 5.4 shows the ln-ln plot between the barrel radius and the new geometrical shape factor for the aspect ratios considered for the study. The new geometrical factor is developed as detailed in section 4.3. The straight-line behaviour obtained (irrespective of the aspect ratios) suggests a power law relationship between the barrel radius and the new geometrical shape factor. The power law relation is expressed as:

\[ R = C_1 S_1^{-m_1} \]  \hspace{1cm} (5.2)

where \( R \) is the barrel radius, \( S_1 \) is the new geometrical shape factor and \( C_1 \) and \( m_1 \) are empirically determined constants.

The effect of hydrostatic stress on the barrel radius is given in Figures 5.5(a) –(b). The radius of curvature of the barrel depends on the level of hydrostatic stress developed during axial compressive deformation of a given material. Figure 5.6(a) is the plot between the radius of curvature of the barrel of square billet and the stress ratio parameter and it suggests that radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter \([\sigma_m / \overline{\sigma}] (h_o-h_d)\) irrespective of the aspect ratios. However it is observed that these curves are not similar for the aspect ratios considered for the experiment. Figure 5.6(b) is drawn to establish a relationship between
Figure 5.3 Variation of Measured Radius with respect to Calculated Radius
MATERIAL: ALUMINIUM SQUARE BILLETS
SYMBOL ASPECT RATIO
- 0.75
- 1.00
- 1.25

SLOPE = 1.894

Lubricant: Dry Condition

Figure 5.4 Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
the barrel radius and the stress ratio parameter on a ln-ln plot and this shows a straight line relationship irrespective of the aspect ratios.

The straight-line behaviour observed irrespective of aspect ratios considered is the manifestation of power law relationship between the barrel radius and the stress ratio parameter of the following form.

\[ R = C_2 \left( \frac{\sigma_m}{\bar{\sigma}} \right) (h_0 - h_f)^{-m_2} \]  

(5.3)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the height before deformation, \( h_f \) is the height after deformation and \( C_2 \) and \( m_2 \) are empirically determined constants.

Since the straight lines are not parallel to each other, the rate of change of barrel radius value with respect to the stress ratio parameter does exhibit a significant difference over the aspect ratios investigated. From Figures 5.5(a) and 5.5(b), it is observed that the behaviour of barrel radius with the hydrostatic stress (\( \sigma_m \)) is same as seen through Figures 5.6(a) and 5.6(b).

5.2 BARRELLING BEHAVIOUR OF SQUARE BILLETS OF ALUMINIUM WITH DIFFERENT LUBRICANTS

Figure 5.7 is the plot drawn between the axial strain (\( \varepsilon_a \)) and the new hoop strain (\( \varepsilon_{ho} \)) for various aspect ratios using different lubricants. These strains were calculated as explained in Chapter 5.1. This plot is a straight line irrespective of the lubricants used and the aspect ratios of the square specimens.

As stated in the Chapter 5.1, using the simple theory of plasticity stresses, namely the hoop stress (\( \sigma_0 \)), the effective stress (\( \bar{\sigma} \)) and the hydrostatic stress (\( \sigma_m \)) are calculated and plotted against the axial strain (\( \varepsilon_a \)) for various lubricants with the same aspect ratio.
Figure 5.5(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.5(b) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
MATERIAL: ALUMINIUM SQUARE BILLET
SYMBOL          ASPECT RATIO
○               0.75
●               1.00
□               1.25

Lubricant: Dry Condition

Figure 5.6(a) Variation of Measured Radius with respect to Stress Ratio Parameter
MATERIAL: ALUMINIUM SQUARE BILLETS
SYMBOL ASPECT RATIO SLOPE

<table>
<thead>
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<th>ASPECT RATIO</th>
<th>SLOPE</th>
</tr>
</thead>
<tbody>
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<td>-1.00</td>
</tr>
<tr>
<td>●</td>
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</tr>
<tr>
<td>□</td>
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<td>-1.00</td>
</tr>
</tbody>
</table>

Lubricant: Dry Condition

Figure 5.6(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
Figure 5.7 Variation of the new hoop strain with respect to the axial strain
as shown in Figures 5.8(a) – (d). From the plots it is observed that the axial stress ($\sigma_a$) and the aforesaid three stresses increase with the increasing amount of the strain for all the lubricants used. It is observed that for any strain level, the increase in hoop stress is appreciably lower as compared to the axial stress ($\sigma_a$). The hoop stress is of tensile in nature and barrelling occurs due to the secondary tensile stress.

Figure 5.9 is the plot drawn between the measured radius of curvature of the barrel of the square billet and the calculated radius of the square billet based on the principle of volume constancy during deformation. The calculated values of the radius of curvature are in close proximity with measured values for all aspect ratios studied irrespective of the type of lubricant used. The use of lubricant does exhibit only circular arc of geometry. Figure 5.10 shows the ln-ln plot between the barrel radius and the new geometrical shape factor with constant slope of 1.6 irrespective of lubricants used. The straight-line relationship may be expressed as a power law relationship between the barrel radius and the new geometrical shape factor as stated below:

\[ R = C_3S_2^{-m_3} \]  

(5.4) where $R$ is the barrel radius, $S_2$ is the new geometrical shape factor and $C_3$ and $m_3$ are empirically determined constants.

Further, the straight line implies that the rate of change of the barrel radius with respect to the new geometrical shape factor exhibits no major difference for different lubricants used. Figure 5.11(a) is the plot between the radius of curvature of the barrel of the square billet and the stress ratio parameter. From the plot it is observed that the radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter $[(\sigma_m/\bar{\sigma})(h_o-h_f)]$ for all lubricants studied. These plots are not similar irrespective of the aspect ratio. Figure 5.11(b) is drawn to establish a relationship
Figure 5.8(a) Variation of stresses with respect to the axial strain
Figure 5.8(b) Variation of stresses with respect to the axial strain.
Figure 5.8(c) Variation of stresses with respect to the axial strain.
Figure 5.8(d) Variation of stresses with respect to the axial strain
Figure 5.9 Variation of Measured Radius with respect to Calculated Radius
Figure 5.10 Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
Figure 5.11(a) Variation of Measured Radius with respect to Stress Ratio Parameter

MATERIAL: ALUMINIUM SQUARE BILLETs
ASPECT RATIO: 1.25

GREASE
○ ZINC STEARATE
□ MOLYBDENUM DISULPHIDE
△ SAE 40 OIL

MATERIAL: ALUMINIUM SQUARE BILLETs
ASPECT RATIO: 1.00

MATERIAL: ALUMINIUM SQUARE BILLETs
ASPECT RATIO: 0.75

Stress Ratio Parameter, \( (\sigma_m/\overline{\sigma}) (h_0-h_f) \)

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between the barrel radius and the stress ratio parameter on a ln-ln plot and it is observed that the straight line relationship is the manifestation of power law relationship between the barrel radius and the stress ratio parameter for all the lubricants used for experiments irrespective of the aspect ratios and it is given below:

\[ R = C_4 \left[ \left( \frac{\sigma_m}{\bar{\sigma}} \right) (h_0-h_f) \right]^{-m_4} \]  

(5.5)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the height before deformation, \( h_f \) is the height after deformation and \( C_4 \) and \( m_4 \) are empirically determined constants.

Since the straight lines are not parallel to each other, the rate of change of barrel radius value with respect to the stress ratio parameter does exhibit a significant difference over the aspect ratios investigated. Figure 5.12(a) is the plot between the measured radius of curvature of the barrel and the hydrostatic stress and it is observed that the measured radius of curvature of the barrel decreases with increasing value of the hydrostatic stress for all the lubricants used. The plots obtained are not similar for all the aspect ratios selected and lubricants used for the experiment. Figure 5.12(b) is the ln-ln plot between the measured radius of curvature of the barrel and the hydrostatic stress and is seen that the straight lines obtained are not parallel for all the aspect ratios selected for the experiment. It is observed that the behaviour of the barrel radius with respect to the hydrostatic stress (\( \sigma_m \)) as given in the Figures 5.12(a) and 5.12(b) is similar to the behaviour of barrel radius with stress ratio parameter given in Figures 5.11(a) - (b).

5.3 EFFECT OF FRICTION ON BARRELLING IN SQUARE BILLETs OF ALUMINIUM

Figure 5.13 shows the plot drawn between the axial strain (\( \varepsilon_a \)) and the hoop strain (\( \varepsilon_0 \)) with different aspect ratios and the study is carried out using different
Figure 5.11(b) Relationship between $\ln$ (Measured Radius) and $\ln$ (Stress Ratio Parameter)
Figure 5.12(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.12(b) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
lubricants namely grease, molybdenum disulphide, zinc stearate, SAE 40 oil and their combinations and under dry lubrication condition. From the above Figure, it is observed that the plot is a straight line with almost same slope values for all aspect ratios considered.

Figure 5.14 is the plot drawn between the friction factor ‘m’ [Ref. Table 100] and the new hoop strain (\(\varepsilon_\theta\)). The plot establishes a straight-line relationship between the two parameters irrespective of the aspect ratios and lubricants used. The lines obtained have different slope for various aspect ratios considered and the lubricants used for the experiment.

Figure 5.15 is drawn between the measured radius of curvature of the barrel of the square billet and the calculated radius of curvature of the square billet to observe the relationship between them. It is observed that the calculated values of the radius of curvature are in close proximity with the measured values. This has proved that the barrel radius follows circular arc.

Figure 5.16 is the plot drawn between the friction factor “m” and the natural logarithmic value of measured values of radius of curvature [\(\ln(R_m)\)] for different load values. Irrespective of aspect ratios and strain levels, the natural logarithmic value of measured radius increases with decreasing value of the friction factor. The graphs establish a straight-line relationship between these two parameters and the lines are not parallel to each other and their slope values are reported in the Figure itself. Figure 5.17 shows the relationship between the friction factor ‘m’ and the new geometrical shape factor (NGF). The new geometrical shape factor developed is explained in the Chapter 4.3. The plot is a straight-line relationship and the friction factor decreases with the increasing value of the new geometrical shape factor (NGF) irrespective of aspect ratios considered. From the plot it is seen that the straight-line relationship suggests a
Figure 5.13 Variation of the hoop strain with respect to the axial strain
Figure 5.14 Variation of the friction factor with respect to the new hoop strain.
Figure 5.15 Variation of Measured Radius with respect to Calculated Radius
MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.00

Slope
1. Load 150 KN = -4.2
2. Load 220 KN = -0.9
3. Load 250 KN = -1.13

Figure 5.16 Relationship between the friction factor and ln (Measured Radius)

Figure 5.16 Relationship between the friction factor and \( \ln (\text{Measured Radius, } R_m) \)
Figure 5.17 Relationship between Friction Factor and $\ln$ (New Geometrical Shape Factor)
power law relationship between the friction factor and the new geometrical shape factor, which is expressed as:

\[ m = C_5 \ln(S_3) + C_6 \]  \hspace{1cm} (5.6)

where \( S_3 \) is the new geometrical shape factor, and \( C_5 \) & \( C_6 \) are empirically determined constants.

Figure 5.18 is a plot drawn between the friction factor 'm' and the natural logarithmic value of stress ratio parameter, which shows a straight-line relationship. As the \( \ln \) (stress ratio parameter) increases under three different loading conditions, the friction factor 'm' value decreases. The rate of change of the friction factor 'm' with respect to the natural logarithmic value of stress ratio parameter is not same for different strain levels. This straight-line relationship is the manifestation of the following relationship.

\[ m = C_7 \ln[(\sigma_m/\bar{\sigma})(h_0-h_f)] + C_8 \]  \hspace{1cm} (5.7)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the initial height of the billet, \( h_f \) is the height of the billet after deformation and \( C_7 \) & \( C_8 \) are empirically determined constants.

5.4 BARRELLING IN RECTANGULAR BILLETS OF ALUMINIUM

The results of barrelling in rectangular billets of aluminium during cold upset forging using different lubricants are discussed. Further, for the study the rectangular billets of three different adjacent sides ratio (a/b) of 0.9, 0.8 and 0.6 with various aspect ratios are considered for this work.

Figures 5.19(a) – (c) show the plot drawn between the axial strain (\( \varepsilon_a \)) and the new hoop strain (\( \varepsilon_h'' \)) for three different adjacent sides ratios. It is observed that for all the
MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.00
Slope
1. Load 150 KN = -5.2
2. Load 220 KN = -6.0
3. Load 250 KN = -10.4

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL
- MOLYBDENUM DISULPHIDE + SAE 40 OIL
- GRAPHITE
- ZINC STEARATE + GREASE
- GRAPHITE + SAE 40 OIL
- DRY FRICTION

Figure 5.18 Relationship between friction factor and ln (Stress Ratio Parameter x 100), ln \(\left(\frac{c_m}{\sigma}\right) (h_o-h_f) \times 10^2\)
three adjacent sides ratios the plot is a straight line irrespective of the geometry, the aspect ratio and the lubricants used. These plots are almost parallel to each other for all aspect ratios considered. The new hoop strain ($\varepsilon_{0''}$) is calculated based on the following expression:

$$\varepsilon_{0''} = \ln \left\{ \frac{2(l_a + l_b)^2 + (l_{ac} + l_{bc})^2}{3(a+b)^2} \right\}$$

(5.8)

where $l_a$ is the bulged side length along the minor axis, $l_b$ is the bulged length along the major axis, $l_{ac}$ is the contact side length along the minor axis, $l_{bc}$ is the contact side length along the major axis, ‘a’ is the side length of rectangle along the minor axis and ‘b’ is the side length of rectangle along the major axis.

Using the simple theory of plasticity, stresses namely the effective stress ($\bar{\sigma}$) and the axial stress ($\sigma_z$) are calculated and plotted against the axial strain ($\varepsilon_z$) as shown in Figures 5.20(a)–5.21(c). Figures 5.22(a)–(c) are plots drawn between the hoop stress or the axial stress and the axial strain. The stresses computed increase with the increase in amount of the strain for the three adjacent side ratios considered irrespective of the aspect ratios and lubricants used. As the applied stress is compressive, the effective stress resulted is of tensile (the bulged diameter expanded due to the action of secondary tensile stress). However, it is observed that for any given deformation level, the increase in effective stress due to loading is appreciably lower compared with the axial stress ($\sigma_z$).

Figures 5.23(a)–(c) are drawn between the measured radius of curvature of the barrel of rectangular billet along the plane parallel to the major axis and the calculated radius along the same plane based on the principle of volume constancy during deformation. For all the three adjacent sides ratios the calculated values of the radius of curvature are in close proximity with measured values irrespective of aspect ratios considered and the lubricants used. Hence, it is true that the radius of curvature fits a circular arc. Figures 5.24(a)–(c) represent the plot between the radius of curvature of
Figure 5.19(a) Variation of the new hoop strain with respect to the axial strain
Figure 5.19(b) Variation of the new hoop strain with respect to the axial strain
Figure 5.19(c) Variation of the new hoop strain with respect to the axial strain
Figure 5.20(a) Variation of stresses with respect to the axial strain
Figure 5.20(b) Variation of stresses with respect to the axial strain

MATERIAL: ALUMINIUM RECTANGULAR BILLET
ASPECT RATIO: 0.9
a/b = 0.9

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL

Axial Strain, $\varepsilon_z$
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.9

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL

Figure 5.20(c) Variation of stresses with respect to the axial strain
Figure 5.21(a) Variation of stresses with respect to the axial strain
Figure 5.21(b) Variation of stresses with respect to the axial strain
Figure 5.21(c) Variation of stresses with respect to the axial strain
Figure 5.22(a) Variation of stresses with respect to the axial strain
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6

Figure 5.22(b) Variation of stresses with respect to the axial strain
Figure 5.22(c) Variation of stresses with respect to the axial strain
RECTANGULAR SHAPE MAJOR WIDTH  
MATERIAL: ALUMINIUM RECTANGULAR BILLETS  
ASPECT RATIO: 1.00  
a/b = 0.9  
Slope = 1.0

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MATERIAL: ALUMINIUM RECTANGULAR BILLETS  
ASPECT RATIO: 0.9  
a/b = 0.9  
Slope = 0.921

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MATERIAL: ALUMINIUM RECTANGULAR BILLETS  
ASPECT RATIO: 0.75  
a/b = 0.9  
Slope = 1.0

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Figure 5.23(a) Variation of Measured Radius with respect to Calculated Radius

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Figure 5.23(b) Variation of Measured Radius with respect to Calculated Radius
Figure 5.23(c) Variation of Measured Radius with respect to Calculated Radius
the barrel of rectangular billets measured along the plane parallel to the minor axis and
the calculated radius along the same plane based on the principle of volume of constancy
during deformation. The measured radius of curvature of the barrel of rectangular billets
is almost equal to the calculated radius irrespective of geometries, aspect ratios and
lubricants used. This confirms that the radius of barrel fits a circular arc. The radius of
curvature measured parallel to the minor axis shows mostly higher radius of curvature
compared with the radius of curvature measured along the plane parallel to major axis for
a given load irrespective of adjacent sides ratios, aspect ratios and lubricants used.

The In-In plot between the measured values of radius of curvature along the plane
parallel to the major axis is plotted against the new geometrical shape parameter (NGF1)
or factor for the three adjacent sides ratios as shown in Figures 5.25(a)–(c). The new
geometrical factor developed is as explained in chapter 4.4. This plot shows a straight-
line relationship between the measured radius of curvature and the new geometrical
shape factor for all the lubricants and aspect ratios considered for the experiment. The
slope of the straight lines in the case of rectangular billet with ‘a/b’ ratio 0.9 is almost
same irrespective of aspect ratios and lubricants used for the experiment. For other ‘a/b’
ratios the slope of the lines is nearly to 2.0. The straight-line relationship may be
expressed by a power law relationship between the barrel radius and the new geometrical
shape factor as expressed in the following equation:

$$R_1 = A_1 S_4^{-m_5}$$

(5.9)

where $R_1$ is the radius of curvature of the barrel of rectangular billet, $S_4$ is the new
geometrical shape factor developed and $A_1$ and $m_5$ are empirically determined constants.

Figures 5.26(a)–(c) show the plot between the measured radius of curvature
along the plane parallel to the minor axis and the new geometrical shape factor (NGF2).
Figure 5.24(a) Variation of Measured Radius with respect to Calculated Radius
Figure 5.24(b) Variation of Measured Radius with respect to Calculated Radius
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = 1.0

• GREASE
○ ZINC STEARATE
□ MOLYBDENUM DISULPHIDE
△ SAE 40 OIL

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = 1.0

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = 1.0

Figure 5.24(c) Variation of Measured Radius with respect to Calculated Radius
Figure 5.25(a) Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
Figure 5.25(b) Relationship between In (Measured Radius) and In (New Geometrical Shape Factor)
Figure 5.25(c) Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
The plot shows a straight-line relationship between these two parameters for all the adjacent side ratios considered. These lines exhibit a similarity having slopes of nearer values. The straight-line behaviour suggests a power law relationship as expressed in equation (5.9).

Figures 5.27(a) – (c) suggest that the radius of curvature of the barrel measured along a plane parallel to the major axis decreases exponentially with increasing values of the stress ratio parameter \( [(\sigma_m/\bar{\sigma}) (h_0-h_f)] \). These plots are not similar for the aspect ratios considered in all three cases irrespective of lubricants applied. Figures 5.28(a) – (c) are drawn to establish a relationship between the barrel radius measured along the plane parallel to minor axis and the stress ratio parameter. It is observed that the barrel radius decreases exponentially with the increasing value of the stress ratio parameter as in the previous plots. And it is seen that these plots are not similar. Figures 5.29(a) – (c) are ln-ln plot between the radius of curvature measured along the plane parallel to the major axis and the stress ratio parameter. It is observed that the plot shows a straight-line relationship. The straight lines obtained are not parallel to each other irrespective of the aspect ratios studied and the lubricants used for all the adjacent sides ratios. The straight-line relationship suggests a power law relationship between the barrel radius and stress ratio parameter as given below:

\[
R_1 = A_2 \left[ (\sigma_m/\bar{\sigma}) (h_0-h_f) \right]^{-m_6}
\]  \hspace{1cm} (5.10)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the initial height of the billet, \( h_f \) is the height of the billet after deformation and \( A_2 \) & \( m_6 \) are empirically determined constants.

Similarly the Figures 5.30(a) – (c) also exhibit the same relationship for all adjacent sides ratios, aspect ratios and lubricants considered for the study. This straight
Figure 5.26a  Relationship between In (Measured Radius) and In (New Geometrical Shape Factor)
Figure 5.26(b) Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = 2.285

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = 2.286

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = 2.182

Figure 5.26(c) Relationship between In (Measured Radius) and In (New Geometrical Shape Factor)
RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.9

• GREASE
○ ZINC STEARATE
□ MOLYBDENUM DISULPHIDE
△ SAE 40 OIL

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.9
a/b = 0.9

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.9

Stress Ratio Parameter, \([(\sigma_m/\sigma) (h_0-h_t)]\)

Figure 5.27(a) Variation of Measured Radius with respect to Stress Ratio Parameter

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Figure 5.27(b) Variation of Measured Radius with respect to Stress Ratio Parameter
Figure 5.27(c) Variation of Measured Radius with respect to Stress Ratio Parameter
Figure 5.28(a) Variation of Measured Radius with respect to Stress Ratio Parameter

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Figure 5.28(b) Variation of Measured Radius with respect to Stress Ratio Parameter
Figure 5.28(c) Variation of Measured Radius with respect to Stress Ratio Parameter
RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.9
Slope = -1.0

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.9
a/b = 0.9
Slope = -1.067

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.9
Slope = -1.0256

\( \ln (\text{Meas. Radius, } R_m) \)
\( \ln \left( \frac{\sigma_m}{\sigma} (h_0 - h_f) \right) \)

Figure 5.29(a) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
Figure 5.29(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = -1.33

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = -1.33

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = -2.2857

Figure 5.29(c) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)

In (Stress Ratio Parameter x 100), ln \left(\frac{c_{m}}{\theta} \right) \left( h_{o} - h_{f} \right) \times 10^{2}
line behaviour is the manifestation of power law relationship between the barrel radius and the stress ratio parameter as explained in equation (5.10).

Since the straight lines are not parallel to each other, the rate of change of barrel radius value with respect to the stress ratio parameter does exhibit a significant difference over the aspect ratios used for investigation. The effect of hydrostatic pressure on the barrel radius \( R_{ml} \) is shown in Figures 5.31(a) – (c). The barrel radius decreases exponentially with increasing value of hydrostatic stress irrespective of geometries, aspect ratios and lubricants used. The plots shown in the Figures 5.32(a) – (c) are similar in nature to plots found in Figures 5.31(a) – (c). Figures 5.33(a) – (c) show the ln-ln plot drawn between the barrel radius \( R_{ml} \) and the hydrostatic stress. It is observed that these two parameters exhibit a straight-line relationship between them. The line obtained is not parallel to one another; the rate of change of barrel radius with respect to the hydrostatic stress does exhibit a significant difference over the geometries, aspect ratios and the lubricants considered for the study. The plots shown in the Figures 5.34 (a) – (c) are similar in nature to the plots drawn between the barrel radius and the hydrostatic stress as in Figures 5.33(a) – (c).

5.5 EFFECT OF FRICTION ON BARRELING IN SQUARE AND RECTANGULAR BILLETS OF ALUMINIUM

Figure 5.35 shows the relationship between the axial strain, \( \varepsilon_z = \ln \left( \frac{h_0}{h_f} \right) \) and the hoop strain \( \varepsilon_0 \). This plot is straight line with almost same slope, irrespective of adjacent sides ratios considered. The hoop strain is calculated based on the following expressions

For square billets: \( \varepsilon_0 = \ln \left( \frac{L}{a} \right) \)  \hspace{1cm} (5.11a)

For rectangular billets: \( \varepsilon_0 = \ln \left[ \frac{(l_a + l_b)}{(a + b)} \right] \)  \hspace{1cm} (5.11b)

Figure 5.36 shows the plot drawn between the new hoop strain, \( \varepsilon_0'' \) and the
Figure 5.30(a) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL : ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO : 1.25
a/b = 0.8
Slope = - 1.23

MATERIAL : ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO : 1.00
a/b = 0.8
Slope = -1.0

MATERIAL : ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO : 0.75
a/b = 0.8
Slope = - 1.0

In (Stress Ratio Parameter x 100), ln \[\left(\frac{c_m}{\bar{s}}\right) \times 10^2\]

Figure 5.30(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = -1.11

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = -1.0

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = -1.67

Figure 5.30(c) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)

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Figure 5.31(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.31(b) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.31(c) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.32(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.32(b) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.32(c) Variation of Measured Radius with respect to Hydrostatic Stress

- Rectangular shape - minor width
- Material: Aluminium rectangular billets
- Aspect ratio: 1.25
- \( a/b = 0.6 \)

- Grease
- Zinc stearate
- Molybdenum disulphide
- SAE 40 oil

- Material: Aluminium rectangular billets
- Aspect ratio: 1.00
- \( a/b = 0.6 \)

- Material: Aluminium rectangular billets
- Aspect ratio: 0.75
- \( a/b = 0.6 \)
Figure 5.33(a) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
Figure 5.33(b) Relationship between In (Measured Radius) and In (Hydrostatic Stress)
Figure 5.33(c) Relationship between \( \ln \) (Measured Radius) and \( \ln \) (Hydrostatic Stress)
Figure 5.34(a) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.8
Slope = -2.0

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.8
SLOPE: -1.765
SLOPE: -1.67

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.8
SLOPE: -1.38
SLOPE: -1.67
SLOPE: -1.67

In (Hydrostatic Stress), ln (σm)

Figure 5.34(b) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = -3.571

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = -3.636

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = -3.57

Figure 5.34(c) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
friction factor ‘m’ for all the adjacent sides ratios selected. The plot between these two parameters establishes a straight-line relationship with different slope values for all the adjacent sides ratios selected. The new hoop strain (\(\varepsilon_0''\)) is calculated for the adjacent sides ratio \((a/b)\), which is equal to 1 based on the expression given in equation (5.1). For other adjacent sides ratios of 0.8 and 0.6, the new hoop strain is based on the expression (5.8).

Figures 5.37(a) – (b) are drawn between the measured radius of curvature of the barrel and the calculated radius of curvature. It is observed that the calculated values of the radius of curvature are in close proximity with measured values. This confirms that the barrel radius fits a circular arc.

Figures 5.38(a) – (b) are the plots drawn between the friction factor and the geometrical shape factor for different constant load factors. This relationship establishes a straight line between them. Irrespective of aspect ratios and strain levels, the rate of change of the friction factor with respect to the geometrical shape factor is not same for different adjacent sides ratios \((a'/b')\) as shown in the Figures. The friction factor decreases with the increasing value of the geometrical shape factor. The straight-line relationship between these two parameters may be expressed by a power law relationship as given below

\[
m = A_3 \ln S_5 + A_4
\]  

(5.12)

where ‘m’ is the friction factor, \(S_5\) is geometrical shape factor, and \(A_3 \& A_4\) are empirically determined constants.

Figures 5.39(a) – (b) show the relationship between the friction factor ‘m’ and the logarithmic value of measured radius. The plot is a straight-line relationship and the friction factor decreases with the increasing value of the natural logarithmic value of the
Figure 5.35 Variation of the hoop strain with respect to the axial strain
Figure 5.36 Variation of friction factor ‘m’ with respect to the new hoop strain
Figure 5.37(a) Variation of Measured Radius with respect to Calculated Radius
Figure 5.37(b) Variation of Measured Radius with respect to Calculated Radius
GEOMETRICAL SHAPE FACTOR BASED ON RADIUS MEASURED - MAJOR AXIS
MATERIAL: ALUMINIUM
1. RECTANGULAR BILLETS OF a/b = 0.6
2. RECTANGULAR BILLETS OF a/b = 0.8
3. SQUARE BILLETS OF a/b = 1.0
Load = 250 KN

Figure 5.38(a) Relationship between Friction factor and ln (New Geometrical Shape Factor)
GEOMETRICAL SHAPE FACTOR BASED ON RADIUS MEASURED - MINOR AXIS

MATERIAL: ALUMINIUM
1. RECTANGULAR BILLETS OF \( a/b = 0.6 \)
2. RECTANGULAR BILLETS OF \( a/b = 0.8 \)
3. SQUARE BILLETS OF \( a/b = 1.0 \)
Load = 250 KN

Load = 250 KN

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL
- MOLYBDENUM DISULPHIDE + SAE 40 OIL
- GRAPHITE
- ZINC STEARATE + GREASE
- GRAPHITE + SAE 40 OIL
- DRY FRICTION

Figure 5.38(b) Relationship between Friction factor and \( \ln (\text{New Geometrical Shape Factor 2}) \)
The rate of change of the friction factor \( m' \) with respect to the natural logarithmic value of the measured radius is not same for different adjacent sides ratios ('a'/b').

Figure 5.40 is a plot drawn between the friction factor \( m' \) and the natural logarithmic value of the stress ratio parameter, which shows a straight-line relationship for all the adjacent sides ratios and lubricants considered. The straight-line relationship may be expressed as given below.

\[
m = A_5 \ln \left( \frac{\sigma_m}{\bar{\sigma}} \right) (h_0 - h_f) + A_6
\]

where \( m' \) is the friction factor, \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the initial height of the billet, \( h_f \) is the height of the billet after deformation and \( A_5 \) and \( A_6 \) are empirically determined constants.

As the natural logarithmic value of the stress ratio parameter increases, the friction factor \( m' \) value decreases. The rate of change of the friction factor \( m' \) with respect to the natural logarithmic value of the stress ratio parameter is not same for various adjacent sides ratios ('a'/b').

Figure 5.41 is a plot drawn between the friction factor \( m' \) and the natural logarithmic value of the hydrostatic stress, which is a straight-line relationship. As the natural logarithmic value of the hydrostatic stress increases, the friction factor increases. The rate of change of the friction factor \( m' \) with respect to the natural logarithmic value of the hydrostatic stress is not same for different adjacent sides ratios ('a'/b').

5.6 SOME ASPECTS OF BARRELLING IN SQUARE BILLETS OF ALUMINIUM BY INTRODUCING DIE CONSTRAINT AT ONE END

The results of barrelling behaviour of aluminium square billets during extrusion forging are discussed in this Chapter. Figure 5.42 shows the plot drawn between the
Figure 5.39(a) Relationship between Friction factor and In (Measured Radius)

**Figure 5.39(a)** Relationship between Friction factor and In (Measured Radius)
RADIUS MEASURED - MINOR AXIS
MATERIAL: ALUMINIUM
1. RECTANGULAR BILLETS OF a/b = 0.6
2. RECTANGULAR BILLETS OF a/b = 0.8
3. SQUARE BILLETS OF a/b = 1.0
Load = 250 KN

Figure 5.39(b) Relationship between Friction factor and ln (Measured Radius, Rm2) mm

$SLOPE = -6.4$
$SLOPE = -1.6$
$SLOPE = -2.4$

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MATERIAL: ALUMINIUM
1. RECTANGULAR BILLETS OF a/b = 0.6
2. RECTANGULAR BILLETS OF a/b = 0.8
3. SQUARE BILLETS OF a/b = 1.0
Load = 250 KN

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL
- MOLYBDENUM DISULPHIDE + SAE 40 OIL
- GRAPHITE
- ZINC STEARATE + GREASE
- GRAPHITE + SAE 40 OIL
- DRY FRICTION

\[
\ln \left( \frac{\sigma_m}{\bar{\sigma}} \right) \ln \left[ \left( h_0 - h_f \right) \times 10^2 \right]
\]

Figure 5.40 Relationship between Friction factor and \( \ln \left( \frac{\sigma_m}{\bar{\sigma}} \right) \ln \left[ \left( h_0 - h_f \right) \times 10^2 \right] \)
MATERIAL: ALUMINIUM
1. RECTANGULAR BILLETS OF $a/b = 0.6$
2. RECTANGULAR BILLETS OF $a/b = 0.8$
3. SQUARE BILLETS OF $a/b = 1.0$
Load = 250 KN

GREASE
- O ZINC STEARATE
- □ MOLYBDENUM DISULPHIDE
- △ SAE 40 OIL
- ■ MOLYBDENUM DISULPHIDE + SAE 40 OIL
- ★ GRAPHITE
- X ZINC STEARATE + GREASE
- ▲ GRAPHITE + SAE 40 OIL
- ■ DRY FRICTION

Figure 5.41 Relationship between Friction factor and ln (Hydrostatic Stress)
axial strain, \( \varepsilon_z = \ln(h_f/h_t) \) and the new hoop strain (\( \varepsilon_{\theta}'' \)). This plot is a straight line. The slope for all aspect ratios studied is almost equal irrespective of the lubricants used. The new hoop strain (\( \varepsilon_{\theta}'' \)) is calculated based on the following expression:

\[
\varepsilon_{\theta}'' = \ln\left(\frac{(2l_b^2 + l_c^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p + \pi^2/16(D_i^2 + D_t d_t + d_t^2)h_f}{(3a^2 h_f)}\right) \tag{5.14}
\]

where \( l_b \) is the bulged length (circumferential length/4), \( l_c \) is the contact length (circumferential length/4), ‘a’ is the side of the square, \( l_{p1} \) is the side length of the pyramid at the top, \( l_{p2} \) is the side length of the pyramid at the bottom, \( h_f \) is the height of the deformed specimen, \( h_b \) is the height of the bulged portion, \( h_p \) is the height of the truncated pyramid, \( h_t \) is the height of the extruded part and \( D_i \) & \( d_t \) are diameters of the extruded part.

Using the simple theory of plasticity stresses, namely the hoop stress (\( \sigma_{\theta} \)) and the axial stress (\( \sigma_z \)) are calculated and plotted against the axial strain (\( \varepsilon_z \)) as shown in Figures 5.43(a) – (c). These stresses increase with the increasing amount of the strain. The bulged diameter expanded due to the action of secondary tensile stress. However, for any given deformation level, the increase in hoop stress due to loading is appreciably lower compared to the axial stress.

Figure 5.44 is drawn between the measured radius of curvature of the barrel and the calculated radius based on the principle of volume constancy during deformation, with the assumption that the barrel radius follows circular arc. The plot shows a straight-line relationship between the measured radius of curvature and the calculated values. This is in good agreement with the measured values irrespective of aspect ratios used and the lubricants considered for the study. This indicates that the radius of curvature follows a circular arc.
Figure 5.42 Variation of the new hoop strain with respect to the axial strain

Axial Strain, $\varepsilon_Z = \ln \left( \frac{h_0}{h_l} \right)$

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MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO : 0.75

Figure 5.43(a) Variation of stresses with respect to the axial strain

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Figure 5.43(b) Variation of stresses with respect to the axial strain
Figure 5.43(c) Variation of stresses with respect to the axial strain
Figure 5.44 Variation of Measured Radius with respect to Calculated Radius
The measured values of the radius of curvature are plotted against the new geometrical shape parameter (NGF) or factor as shown in Figure 5.45. The new concept of geometrical shape factor developed is shown in section 4.5. The straight-line behaviour suggests a power law relationship between the barrel radius and the new geometrical shape factor as expressed below:

\[ R = B_1 S_6^{m_7} \]  \hspace{1cm} (5.15)

where \( R \) is the radius of curvature of the barrel of square billet, \( S_6 \) is the new geometrical shape factor developed, \( B_1 \) and \( m_7 \) are empirically determined constants. The radii of curvature of the barrel depend on the level of the hydrostatic stress developed during axial compressive deformation for a given material. Figure 5.46(a) suggests that radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter \([\sigma_m / \bar{\sigma}] (h_0 - h_f)\). These plots are not same for the aspect ratios studied. Figure 5.46(b) is drawn to establish a relationship between the barrel radius and the stress ratio parameter on a ln-ln plot. The straight-line behaviour is the manifestation of power law relationship between the barrel radius and the stress ratio parameter as expressed below.

\[ R = B_2 [(\sigma_m / \bar{\sigma}) (h_0 - h_f)]^{m_8} \]  \hspace{1cm} (5.16)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( h_0 \) is the initial height of the cylinder, \( h_f \) is the height of the billet after deformation and \( B_2 \) & \( m_8 \) are experimentally determined constants.

From Figures 5.47(a) - (b), it is observed that the behaviour of the barrel radius with the hydrostatic stress \( (\sigma_m) \) is same as behaviour of the barrel radius with the stress ratio parameter as given in Figures 5.46(a) – (b).
Figure 5.45 Relationship between $\ln$ (Measured Radius) and $\ln$ (New Geometrical Shape Factor)
Figure 5.46(a) Variation of Measured Radius with respect to Stress Ratio Parameter
Figure 5.46(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.25

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL

Figure 5.47(a) Variation of Measured Radius with respect to Hydrostatic Stress

Hydrostatic Stress, $\sigma_m$ (Mpa)

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Figure 5.47(b) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
5.7 BARRELLING IN SQUARE BILLETS OF ALUMINIUM UNDER DISSIMILAR FRICTION

The results of barrelling behaviour of square billets under dissimilar friction are discussed. The differential lubrication is maintained by applying lubricant at one end and keeping the other end in dry friction (no lubrication). Figure 5.48 shows the plot drawn between the axial strain, \( \varepsilon_a = \ln(h_a/h_f) \) and the new hoop strain, \( \varepsilon_\theta'' \). The plot between these two parameters is a straight line with almost equal slope irrespective of aspect ratios and lubricants used for the study. The new hoop strain (\( \varepsilon_\theta'' \)) is calculated based on the following expression:

\[
\varepsilon_\theta'' = \ln \left\{ \frac{(2h_b^2 + l_e^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p}{(3a^2 h_f)} \right\}
\] (5.17)

where \( l_e \) is the bulged length, \( l_c \) is the contact length, ‘a’ is the side of the square, \( l_{p1} \) is the side length of the pyramid at the top, \( l_{p2} \) is the side length of the pyramid at the bottom, \( h_f \) is the height of the deformed specimen, \( h_b \) is the height of the bulged portion and \( h_p \) is the height of the truncated pyramid.

Using the simple theory of plasticity stresses, namely the hoop stress (\( \sigma_0 \)), the effective stress (\( \bar{\sigma} \)) and the hydrostatic stress (\( \sigma_m \)) are calculated. The hoop stress and the axial stress are plotted against the axial strain (\( \varepsilon_a \)) as shown in Figures 5.49(a) – (c). The stresses namely the axial stress (\( \sigma_a \)), the effective stress (\( \bar{\sigma} \)), the hoop stress (\( \sigma_0 \)) and the hydrostatic stress (\( \sigma_m \)) increase with the increasing amount of the strain irrespective of aspect ratios and lubricants used. It is observed that for any strain level, the increase in hoop stress is appreciably lower as compared to the axial stress. The hoop stress is of tensile in nature and barrelling occurs due to the action of secondary tensile stress.

Figure 5.50 is drawn between the measured radius of curvature of the barrel and the calculated radius based on the principle of volume constancy during deformation,
Figure 5.48 Variation of the new hoop strain with respect to the axial strain

MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.25
Slope = 0.857

MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.00
Slope = 0.857

MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 0.75
Slope = 0.875
MATERIAL: ALUMINIUM SQUARE BILLETs
ASPECT RATIO: 0.75

Figure 5.49(a) Variation of stresses with respect to the axial strain

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Figure 5.49(b) Variation of stresses with respect to the axial strain
Figure 5.49(c) Variation of stresses with respect to the axial strain
Figure 5.50 Variation of Measured Radius with respect to Calculated Radius
with the assumption that the barrel radius is circular arc. The calculated values of the radius of curvature are in close proximity with measured values and this confirms that the barrel radius fits a circular arc. The plot of radius of curvature shows a straight-line relationship.

The measured values of radius of curvature are plotted against the new geometrical shape parameter or factor as shown in Figure 5.51. The concept of new geometrical factor developed is explained in Chapter.4.6. The plot drawn between these two parameters establishes a straight-line behaviour and this relationship may be expressed by a power law relationship:

\[
R = B_3 S_7^{-m_9}
\]  \hspace{1cm} (5.18)

where \(R\) is the radius of curvature of the barrel of square billet, \(S_7\) is the new geometrical shape factor developed and \(B_3\) and \(m_9\) are empirically determined constants.

As shown in the Figure 5.52(a), the radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter \([\sigma_m/\bar{\sigma}] (h_o-h_f)\] irrespective of aspect ratios and lubricants used. These plots are not same for the aspect ratios studied. Figure 5.52(b) is drawn to establish a relationship between the barrel radius and the stress ratio parameter on a ln-ln plot. This plot establishes a straight-line relationship irrespective of aspect ratios and lubricants used. It is seen that the straight-line behavior is the manifestation of power law relationship between the barrel radius and the stress ratio parameter and it may be expressed as follows:

\[
R = B_4 (\sigma_m/\bar{\sigma}) (h_o-h_f)^{-m_{10}}
\]  \hspace{1cm} (5.19)

where \(\sigma_m\) is the hydrostatic stress, \(\bar{\sigma}\) is the representative stress, \(h_o\) is the initial height of the billet, \(h_f\) is the height of the billet after deformation and \(B_4\) & \(m_{10}\) are empirically determined constants.
MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.25
Slope = 1.96

MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 1.00
Slope = 2.00

MATERIAL: ALUMINIUM SQUARE BILLETS
ASPECT RATIO: 0.75
Slope = 2.13

\[
\ln \left( \frac{1}{2\sqrt{l_{bw}-l_{cw}}} \right) + \ln \left[ \frac{1}{2} \left( l_{p1}^2 + l_{p2}^2 + l_{p3}^2 \right) h_p \right] / \left( 2l_b^2 + l_c^2 \right)
\]

\ln \ (\text{New Geometrical Shape Factor})

Figure 5.51 Relationship between \( \ln \) (Measured Radius) and \( \ln \) (New Geometrical Shape Factor)

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Figure 5.52(a) Variation of Measured Radius with respect to Stress Ratio Parameter
Figure 5.52(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
It is observed that the behaviour of the barrel radius with respect to the hydrostatic stress ($\sigma_m$) seen in Figures 5.53(a) – (b) is same as the behaviour of the barrel radius with respect to the stress ratio parameter as shown in Figures 5.52(a)-(b).

5.8 BARRELLING IN RECTANGULAR BILLETS OF ALUMINIUM UNDER DISSIMILAR FRICTION

The results of barrelling behaviour of aluminium rectangular billets under dissimilar friction are here discussed. Figure 5.54 shows the plot drawn between the axial strain, $\varepsilon_z = \ln \left( \frac{h_o}{h_f} \right)$ and the new hoop strain, $\varepsilon_0''$. This plot is a straight-line relationship between these two parameters irrespective of aspect ratios and lubricants used. The new hoop strain ($\varepsilon_0''$) is calculated based on the following expression:

$$
\varepsilon_0'' = \ln \left\{ \frac{\left[ 2(l_a+l_b)^2+(l_ac+l_bc)^2 \right] h_b + \left[ (l_at+l bt)^2+(l_at+l bt)(l_ab+l_bb)+(l_ab+l_bb)^2 \right] }{3 \left( a+b \right)^2 h_f} \right\} \quad (5.20)
$$

where $l_a$ is the bulged side length along the minor axis, $l_b$ is the bulged side length along the major axis, $l_{ac}$ is the contact side length along the minor axis, $l_{bc}$ is the contact side length along the major axis, $h_b$ is the height of the bulged portion, $l_{at}$ is the side length of the top portion of the truncated pyramid along the minor axis, $l_{bt}$ is the side length of the top portion of the truncated pyramid along the major axis, $l_{ab}$ is the side length of the bottom portion of the truncated pyramid along the minor axis, $l_{bb}$ is the side length of the bottom portion of the truncated pyramid along the major axis, $h_{pf}$ is the height of the truncated pyramid, $h_f$ is the height of the billet after deformation, ‘a’ is the side length of rectangle along the minor axis and $h_o$ is the initial height of the rectangular billet.

Using the simple theory of plasticity stresses namely the effective stress ($\bar{\sigma}$) and the axial stress ($\sigma_z$) are calculated and plotted against the axial strain ($\varepsilon_z$) as shown in Figures 5.55(a) –(c). The stresses namely the axial stress ($\sigma_z$), the effective stress ($\bar{\sigma}$),
Figure 5.53(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.53(b) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
Figure 5.54 Variation of the new hoop strain with respect to the axial strain
Figure 5.55(a) Variation of stresses with respect to the axial strain
Figure 5.55(b) Variation of stresses with respect to the axial strain

Axial Strain, $\varepsilon_z$
Figure 5.55(c) Variation of stresses with respect to the axial strain
the hoop stress ($\sigma_\theta$) and the hydrostatic stress ($\sigma_m$) increase with the increasing amount of the strain. The effective stress is tensile in nature because during compressive deformation, the bulged diameter expanded due to the action of secondary tensile stress. However, for any given deformation level, the increase in effective stress due to loading is appreciably lower compared with the axial stress.

Figures 5.56(a) – (b) are drawn between the measured radius of curvature of the barrel and the calculated radius based on the principle of volume constancy during deformation, with the assumption that the barrel radius fits a circular arc. The calculated values of the radius of curvature are in close proximity with the measured values. The plot of radius of curvature shows a straight-line relationship and the calculated values are in good agreement with the measured values irrespective of aspect ratio and lubricants used for study.

The measured values of the radius of curvature are plotted against the new geometrical shape parameter or factor as shown in Figures 5.57(a) – (b). The new geometrical factor developed is explained in Chapter 4.7. These two parameters establish straight-line behaviour having almost same slope irrespective of aspect ratios and lubricants considered for the study. This behaviour suggests a power law relationship between the barrel radius and the new geometrical shape factor. The power law relation is expressed as follows:

$$ R = B_5 S_8^{-m_11} \quad (5.21) $$

where $S_8$ is the new geometrical shape factor and $B_5$ and $m_11$ are empirically determined constants.

Figures 5.58(a) – (b) suggest that radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter $[(\sigma_m/\sigma) (h_0-h_f)]$. These
DISSIMILAR FRICTION - RECTANGULAR SHAPE
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = 0.9375

DISSIMILAR FRICTION - RECTANGULAR SHAPE
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = 1.0417

DISSIMILAR FRICTION - RECTANGULAR SHAPE
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = 1.0714

Figure 5.56(a) Variation of Measured Radius with respect to Calculated Radius
Figure 5.56(b) Variation of Measured Radius with respect to Calculated Radius
Figure 5.57(a) Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
Slope = 2.222

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
Slope = 2.222

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
Slope = 2.222

Figure 5.57(b) Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6

- GREASE
- ZINC STEARATE
- MOLYBDENUM DISULPHIDE
- SAE 40 OIL

Figure 5.58(a) Variation of Measured Radius with respect to Stress Ratio Parameter

276
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6

Stress Ratio Parameter, \((\sigma_m/\bar{\sigma})(h_c-h_d)\)

Figure 5.58(b) Variation of Measured Radius with respect to Stress Ratio Parameter

277
plots are not same for the aspect ratios studied and lubricants used. Figures 5.59(a) – (b) are drawn to establish a relationship between the barrel radius and the stress ratio parameter on a ln-ln plot. The straight line having the different slope for all aspect ratios considered is the manifestation of power law relationship between the barrel radius and the stress ratio parameter of the following form:

$$R = B_6 \left[ \left( \frac{\sigma_m}{\bar{\sigma}} \right) \left( \frac{h_0-h_f}{h_0} \right) \right]^{-m_{12}}$$  \hspace{1cm} (5.22)

where $\sigma_m$ is the hydrostatic stress, $\bar{\sigma}$ is the representative stress and $B_6$ and $m_{12}$ are empirically determined constants.

The radii of curvature of the barrel depend on the level of the hydrostatic stress developed during the axial compressive deformation for a given metal. From Figures 5.60(a) – (b) and 5.61(a) – (b), it is observed that the behaviour of the barrel radius with the hydrostatic stress ($\sigma_m$) is same as seen through Figure 5.58(a)–(b) and 5.59(a) – (b).

5.9 BARRELLING BEHAVIOUR OF SQUARE BILLETS OF COPPER

Figure 5.62 shows the plot drawn between the axial strain, $\varepsilon_a = \ln(h_0/h_d)$ and the new hoop strain, $\varepsilon_0''$. This plot is a straight line irrespective of aspect ratio. The new hoop strain ($\varepsilon_0''$) is calculated as explained in equation (5.1).

Using the simple theory of plasticity stresses, namely the hoop stress ($\sigma_0$), the effective stress ($\bar{\sigma}$) and the hydrostatic stress ($\sigma_m$) are calculated and plotted against the axial strain ($\varepsilon_a$) as shown in figures 5.63(a) – (c). All these stresses increase with increasing amount of strain irrespective of aspect ratios. Since the hoop stress is tensile in nature during compressive deformation, the bulging occurs due to the action of secondary tensile stress. It is observed that for any given deformation level, the increase in hoop stress due to loading is appreciably lower compared to the axial strain.
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6

---

GREASE
O ZINC STEARATE
□ MOLYBDENUM DISULPHIDE
△ SAE 40 OIL

---

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6

---

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6

---

In (Stress Ratio Parameter x 100), ln [(σm/5) (h_o-h_f) x 10^2]

Figure 5.59(a) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6

SLOPE: -1.091

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6

SLOPE: -1.091

SLOPE: -1.2

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6

SLOPE: -1.333

SLOPE: -0.923

SLOPE: -1.333

\[ \ln \left( \text{Measur} \right) \text{d Radius, } R_{m2} \]
\[ \ln \left[ \left( \sigma_m / \bar{\sigma} \right) \left( h_o - h_f \right) \times 10^2 \right] \]

Figure 5.59(b) Relationship between \( \ln \) (Measured Radius) and \( \ln \) (Stress Ratio Parameter)

280
Figure 5.60(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.60(b) Variation of Measured Radius with respect to Hydrostatic Stress
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MAJOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6

SLOPE = -2.5

SLOPE = -2.22

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6

SLOPE = -2.22

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6

SLOPE = -2.727
SLOPE = -2.827

In (Hydrostatic Stress), ln (σ_m)

Figure 5.61(a) Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)

283
DISSIMILAR FRICTION - RECTANGULAR SHAPE - MINOR WIDTH
MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.25
a/b = 0.6
SLOPE = -2.727

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 1.00
a/b = 0.6
SLOPE = -1.667

MATERIAL: ALUMINIUM RECTANGULAR BILLETS
ASPECT RATIO: 0.75
a/b = 0.6
SLOPE = -3.33
SLOPE = -3.63

Figure 5.61b Relationship between ln (Measured Radius) and ln (Hydrostatic Stress)
Figure 5.62 Variation of the new hoop strain with respect to the axial strain

285
Figure 5.63(a) Variation of stresses with respect to the axial strain
MATERIAL: COPPER SQUARE BILLETS
ASPECT RATIO = 1.00

Lubricant: Dry Condition

Figure 5.63(b) Variation of stresses with respect to the axial strain
MATERIAL: COPPER SQUARE BILLET
ASPECT RATIO = 1.25

Lubricant: Dry Condition

Figure 5.63(c) Variation of stresses with respect to the axial strain

288
Figure 5.64 is drawn between the measured radius of curvature of the barrel and the calculated radius based on the principle of volume constancy during deformation, on the assumption that the barrel radius follows a circular arc. The calculated values of the radius of curvature are in close proximity with the measured values and this confirms that barrel radius fits a circular arc. The measured values of radius of curvature are plotted against the new geometrical shape parameter or factor as shown in Figure 5.65. The new geometrical factor developed as explained earlier. The plot establishes a straight-line relationship between these two parameters and this may be expressed by a power law relationship. The power law relation is expressed as given below:

\[ R = B_7 S_9^{-m_{13}} \]  

(5.23)

where \( R \) is the barrel Radius, \( S_9 \) is the new geometrical shape factor and \( B_7 \) and \( m_{13} \) are empirically determined constants.

Figure 5.66(a) suggests that the radius of curvature of the barrel decreases exponentially with increasing values of the stress ratio parameter \( [(\sigma_m/\bar{\sigma})(h_0 - h_d)] \) irrespective aspect ratios. The plots obtained for three aspect ratios are not similar. Figure 5.66(b) is drawn to establish a relationship between the barrel radius and the stress ratio parameter on a ln-ln plot. The straight-line behaviour is the manifestation of power law relationship between the barrel radius and the stress ratio parameter of the following form:

\[ R = B_8 [(\sigma_m/\bar{\sigma})(h_0 - h_d)]^{-m_{14}} \]  

(5.24)

where \( \sigma_m \) is the hydrostatic stress, \( \bar{\sigma} \) is the representative stress, \( B_8 \) & \( m_{14} \) are empirically determined constants.
MATERIAL: COPPER SQUARE BILLETS
SYMBOL  ASPECT RATIO

- 0.75
○ 1.00
□ 1.25

Slope = 0.91

Lubricant: Dry Condition

Figure 5.64 Variation of Measured Radius with respect to Calculated Radius
MATERIAL: COPPER SQUARE BILLET
SYMBOL  ASPECT RATIO
  •  0.75
  ○  1.00
  □  1.25

Slope = 2.24

Lubricant: Dry Condition

Figure 5.65 Relationship between ln (Measured Radius) and ln (New Geometrical Shape Factor)
MATERIAL: COPPER SQUARE BILLET
SYMBOL  ASPECT RATIO

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<tr>
<th>Symbol</th>
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<td>□</td>
<td>1.25</td>
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Lubricant: Dry Condition

Figure 5.66(a) Variation of Measured Radius with respect to Stress Ratio Parameter
MATERIAL: COPPER SQUARE BILLET
SYMBOL ASPECT RATIO SLOPE

- 0.75 -1.077
- 1.00 -0.75
- 1.25 -1.05

Lubricant: Dry Condition

Figure 5.66(b) Relationship between ln (Measured Radius) and ln (Stress Ratio Parameter)
Since the straight lines are not parallel to each other, the rate of change of barrel radius value with respect to the stress ratio parameter does exhibit a significant difference over the aspect ratios investigated. The radius of curvature of the barrel depends on the level of the hydrostatic stress developed during axial compressive deformation for a given metal. However, the effect of hydrostatic pressure on the barrel radius is shown in Figures 5.67(a)–(b). It is observed that the behaviour of barrel radius with respect to the hydrostatic stress ($\sigma_m$) is same as the behaviour of barrel radius with respect to the stress ratio parameter as seen through Figures 5.66(a) and 5.66(b).
MATERIAL: COPPER SQUARE BILLETS
SYMBOL  ASPECT RATIO

- 0.75
- 1.00
- 1.25

Lubricant: Dry Condition

Figure 5.67(a) Variation of Measured Radius with respect to Hydrostatic Stress
Figure 5.67(b) Relationship between $\ln$ (Measured Radius) and $\ln$ (Hydrostatic Stress)

Lubricant: Dry Condition

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<td></td>
<td>0.75</td>
<td>-2.14</td>
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<tr>
<td>![Circle Symbol]</td>
<td>1.00</td>
<td>-2.33</td>
</tr>
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<td>![Square Symbol]</td>
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<td>-2.33</td>
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MATERIAL: COPPER SQUARE BILLETS

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