CHAPTER-IV

MATHEMATICAL ANALYSIS

4.1 PURPOSE OF STUDY

Earlier investigations reveal that the expression for the new hoop strain in terms of contact and bulge diameters have been developed using cylindrical billets. Further, the expression for the radius of curvature of the barrel has been developed in terms of height and diameters. The new hoop strain and the radius of curvature are very important and useful for forming operation and for the calculation of specific forming energy required for forming operation.

The present study is to analyse the bulging behaviour of square billets of aluminium and copper and rectangular billets of aluminium under lubricating and unlubricated conditions. It is assumed that the bulge follows the circular arc during deformation. The expressions for bulging can be obtained based on the volume constancy principle in the terms of contact width, bulge width, contact side length, bulged side length and heights before and after deformation.

4.2 EXPRESSION FOR BARREL RADIUS

Assuming that the barrel follows a circular radius of curvature [167], the expression for the radius of curvature of the barrel (R) could be found out as follows:
In Figure 4.1(a), R is the barrel radius and x is equal to \((l_{bw} - l_{cw})/2\), where \(l_{bw}\) is the bulged width and \(l_{cw}\) is the contact width of deformed specimen.

From Figure 4.1(b),

\[
\begin{align*}
AC &= BC = h/2 \\
OD &= OB = OA = R \\
OC &= OD - CD = R - x
\end{align*}
\]  
(4.1)
(4.2)
(4.3)

Since OCA and OCB are right angled triangles, from Figure 4.1(b) we can write,

\[
(OA)^2 = (OC)^2 + (CA)^2
\]  
(4.4)

By substituting \(OA\) is equal to \(R\), \(OC\) is equal to \((R-x)\) and \(CA\) is equal to \(h/2\), in the equation. (4.4). The equation becomes

\[
R^2 = (R-x)^2 + (h/2)^2
\]  
(4.5)

The equation (4.5) can be written as

\[
x^2 - 2Rx + h^2/4 = 0
\]  
(4.6)

The equation (4.6) can be written as

\[
R = (x/2) + (h^2/8x)
\]  
(4.7)

The equation (4.6) can also be modified as

\[
x = \left[ R \pm (R^2 - h^2/4)^{0.5} \right]
\]  
(4.8)

Since \(x\) cannot be greater than \(R\), therefore, the positive sign in the equation (4.8) can be ignored. The equation (4.8) becomes

\[
x = \left[ R - (R^2 - h^2/4)^{0.5} \right]
\]  
(4.9)
Figure 4.1(a) Barrelled Specimen

Figure 4.1(b) Arc of circular Barrel Radius
Once, the contact width ($l_{cw}$), the bulged width ($l_{bw}$) and the height after deformation ($h_f$) are known, the radius of curvature of the barrel surface of bulged billet can be determined.

4.3 BARRELLING ASPECT OF SOLID SQUARE BILLETS

As explained elsewhere [167], the expression for bulging can be written as follows under the condition that the bulging follows circular arc barrelling effect.

$$\frac{(\pi/12) (2l_b^2 + l_c^2)}{h_f} = (\pi/4) a^2 h_o$$

(Under volume constancy principle)

where $l_b$ is the bulged length (bulged circumference/4), $l_c$ is the contact length (contact length along the circumference/4), $h_f$ is the height of the billet after deformation, 'a' is the side length of square and, $h_o$ is the initial height of the square billet

The above equation (4.10) can be written as:

$$\frac{(h_o)}{(h_f)} = \frac{(2l_b^2 + l_c^2)/3a^2}$$

Taking natural logarithm on both sides, the equation becomes

$$\varepsilon_z = \varepsilon_0''$$

where,

$$\varepsilon_z = \ln(h_o/h_f)$$

$$\varepsilon_0'' = \ln \left((2l_b^2 + l_c^2)/3a^2\right)$$

From the equation (4.9), the expression for the radius of curvature of barrel is as follows:

$$x = R - (R^2 - h_f^2/4)^{0.5}$$

where $x = (l_{bw} - l_{cw})/2$, $R$ is the radius of the curvature of the barrel, $l_{bw}$ is the bulged width, and $l_{cw}$ is the contact width. Simplifying the expression (4.13), the expression for
the barrel radius \( R \) can be obtained neglecting \( x^2 \) term (because the quantity of the \( x \) is very less).

Therefore,

\[
R = \frac{h_t^2}{8x} \tag{4.14}
\]

Otherwise

\[
R = \frac{h_t^2}{4(l_{bw} - l_{cw})} \tag{4.15}
\]

Multiplying \( h_0 \) on both sides by the expression (4.15), the following expression can be obtained.

\[
R^{0.50} = \left[ \frac{(h_t/h_0)}{h_0} \right] \left[ \frac{h_t^2}{2(l_{bw} - l_{cw})^{0.5}} \right] \tag{4.16}
\]

From the expression (4.11) and (4.16), the barrel radius becomes as follows

\[
R^{0.50} = \frac{h_0}{2(l_{bw} - l_{cw})^{1/2}} \left( \frac{3a^2}{2l_b^2 + l_c^2} \right) \tag{4.17}
\]

The right hand side of the equation (4.17) is the new geometrical shape factor developed.

4.4 BARRELLING ASPECT OF SOLID RECTANGULAR BILLETS

As explained elsewhere [167], the expression for the bulging of rectangular billets can be written as follows under the condition that the bulging follows circular arc barrelling effect.

\[
\frac{\pi}{12} [2(2l_a + 2l_b)/\pi] + [(2l_{ac} + 2l_{bc})/\pi]^2] h_t = \frac{\pi}{4} [(2a/\pi) + (2b/\pi)] h_0 \tag{4.18}
\]

Simplifying the above equation, the following is obtained.

\[
(\pi/12) [2(l_a + l_b)^2 + (l_{ac} + l_{bc})^2] h_t = (\pi/4)(a + b)^2 h_0 \tag{4.19}
\]

(Under volume constancy principle)
where $l_a$ and $l_b$ are the bulged side lengths along the minor & major axes, $l_{ac}$ and $l_{bc}$ are the contact side lengths along the minor & major axes, $h_f$ is the height of the billet after deformation, 'a' is the side length of rectangle along the minor axis, 'b' is the side length of rectangle along the major axis and $h_0$ is the initial height of the square billet.

The above expression (4.19) can be written as follows:

\[
\left[\frac{h_0}{h_f}\right] = \left\{\frac{2(l_a + l_b)^2 + (l_{ac} + l_{bc})^2}{3(a+b)^2}\right\}^{1/3}
\] (4.20)

Taking natural logarithm on both sides, the equation becomes

\[
\varepsilon_z = \varepsilon_0''
\] (4.21)

where,

\[
\varepsilon_z = \ln\left(\frac{h_0}{h_f}\right)
\]

\[
\varepsilon_0'' = \ln \left\{\frac{2(l_a + l_b)^2 + (l_{ac} + l_{bc})^2}{3(a+b)^2}\right\}
\]

From the equation (4.9) the expression for the radius of curvature of barrel is as follows:

\[
x_1 = R_1 - \left(\frac{R_1}{2} - \frac{h_f^2}{4}\right)^{0.5}
\] (4.22)

where, $x_1 = \frac{(l_{bw1} - l_{cw1})}{2}$, $R_1$ is the radius of the curvature of the barrel in a plane parallel to major axis, $l_{bw1}$ is the bulged width parallel to the major axis and $l_{cw1}$ is the contact width parallel to the major axis.

Simplifying the expression (4.22), the expression for the barrel radius $R$ can be obtained neglecting $x_1^2$ term (because the quantity of the $x_1$ is very less)

Therefore,

\[
R_1 = \frac{h_f^2}{8x_1}
\] (4.23)

Otherwise

\[
R_1 = \frac{h_f^2}{4(l_{bw1} - l_{cw1})}
\] (4.24)
Multiplying $h_0$ on both sides by the expression (4.24), the following expression can be obtained:

$$R_1^{0.50} = \frac{[(h_f/h_0)h_0]}{[2(l_{bw1} - l_{cw1})^{0.5}]}$$  \hspace{1cm} (4.25)

From the expression (4.20) and (4.25), the barrel radius becomes as follows

$$R_1^{0.5} = \frac{h_o}{2(l_{bw1} - l_{cw1})^{0.5}} \cdot \frac{3(a + b)^2}{[2(l_a + l_b)^2 + (l_{sc} + l_{tc})^2]}$$  \hspace{1cm} (4.26)

Similarly the expression for the barrel radius in a plane parallel to the minor axis can be written as

$$R_2^{0.5} = \frac{h_o}{2(l_{bw2} - l_{cw2})^{0.5}} \cdot \frac{3(a + b)^2}{[2(l_a + l_b)^2 + (l_{sc} + l_{tc})^2]}$$  \hspace{1cm} (4.27)

where $l_{bw2}$ is the bulged width parallel to minor axis and $l_{cw2}$ is the contact width parallel to the minor axis.

The right hand side of the equations (4.26)-(4.27) are the new geometrical shape factors developed.

4.5 BARRELLING IN SOLID SQUARE BILLETS UNDER DIE CONSTRAINT AT ONE END

As explained elsewhere [170], the expression for the bulging can be written as follows under the condition that the bulging follows circular arc barrelling effect.

$$(\pi/4)a^2h_o = (\pi/12)(2l_b^2 + l_c^2)h_o + (\pi/12)(l_{p1}^2 + l_{p2}^1 + l_{p2}^2)h_p + (\pi/12)(\pi^2/16)(D_1 + D_2 + D_3 + D_4)h_t$$  \hspace{1cm} (4.28)

(Under volume constancy principle)

where $l_b$ is the bulged length (bulged circumference/4), $l_c$ is the contact length (contact length along the circumference/4), $h_f$ is the height of the billet after deformation,
'a' is the side length of square and \( h_0 \) is the initial height of the square billet.

The above equation (4.28) can be written as follows:

\[
\frac{(h_0)}{(h_f)} = \frac{[(2h_0^2 + l_c^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p + \pi^2/16(D_t^2 + D_d + d_t^2)h_b]}{(3a^2 h_f)}
\]  
(4.29)

Taking natural logarithm on both sides, the equation becomes

\[
\varepsilon_2 = \varepsilon_0''
\]  
(4.30)

where,

\[
\varepsilon_2 = \ln(h_0/h_f)
\]

\[
\varepsilon_0'' = \ln \left\{ \frac{[(2h_0^2 + l_c^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p + \pi^2/16(D_t^2 + D_d + d_t^2)h_b]}{(3a^2 h_f)} \right\}
\]

From the equation [4.9], the expression for the radius of curvature of barrel is as follows:

\[
x = R - (R^2 - h_b^2/4)^{0.5}
\]  
(4.31)

Simplifying the expression (4.31), the expression for the barrel radius \( R \) can be obtained neglecting \( x^2 \) term (because the quantity of the \( x \) is very less).

Therefore,

\[
R = h_b^2/8x
\]  
(4.32)

Otherwise

\[
R = h_b^2/4(l_{bw} - l_{cw})
\]  
(4.33)

From the expressions (4.29) and (4.33), the barrel radius becomes as follows

\[
R^{0.50} = \left( \frac{1}{2(l_{bw} - l_{cw})^{1/2}} \right) \times \left\{ \frac{3a^2 h_0 - (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p - \pi^2/16(D_t^2 + D_d + d_t^2)h_f}{(2h_0^2 + L_c^2)} \right\}
\]  
(4.34)

The right hand side of the equation (4.34) is the new geometrical shape factor developed.
4.6 BARRELLING OF SOLID SQUARE BILLETS UNDER DISSIMILAR FRICTION

As explained elsewhere [171], the expression for the bulging can be written as follows under the condition that the bulging follows circular arc barrelling effect.

\[
(\pi/4)a^2 h_0 = (\pi/12)(2l_b^2 + l_c^2)h_b + (\pi/12)(l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p \quad (4.35)
\]

(Under volume constancy principle)

where \(l_b\) is the bulged length (bulged circumference/4), \(l_c\) is the contact length (circumferential length/4), \(a\) is the side of the square (mm), \(l_{p1}\) is the side length of the pyramid at the top, \(l_{p2}\) is the side length of the pyramid at the bottom, \(h_f\) is the height of the deformed specimen, \(h_b\) is the height of the bulged portion, \(h_p\) is the height of the truncated pyramid and \(h_0\) is the initial height of the square billet.

The above equation (4.35) can be written as follows

\[
(h_0)/(h_f) = [(2l_b^2 + l_c^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p]/(3a^2 h_f) \quad (4.36)
\]

Taking natural logarithm on both sides, the equation becomes

\[
\varepsilon_z = \varepsilon_0''\quad (4.37)
\]

where,

\[
\varepsilon_z = \ln(h_0/h_f)
\]

\[
\varepsilon_0'' = \ln\{[(2l_b^2 + l_c^2)h_b + (l_{p1}^2 + l_{p1}l_{p2} + l_{p2}^2)h_p]/(3a^2 h_f)\}
\]

From the equation (4.9) the expression for the radius of curvature of barrel is as follows

\[
x = R-(R^2-h_{b0}^2/4)^{0.5} \quad (4.38)
\]

where, \(x = (l_{bw} - l_{cw})/2\), \(R\) is the radius of the curvature of the barrel, \(l_{bw}\) is the bulged width, and \(l_{cw}\) is the contact width. Simplifying the expression (4.38), the expression for
the barrel radius $R$ can be obtained neglecting $x^2$ term (because the quantity of the $x$ is very less)

Therefore,

$$R = \frac{h_b^2}{8x} \quad (4.39)$$

Otherwise

$$R = \frac{h_b^2}{4(l_{bw} - l_{cw})} \quad (4.40)$$

From the expression (4.36) and (4.40), the barrel radius becomes as follows

$$R^{0.50} = \left(\frac{1}{2}(l_{bw} - l_{cw})^{1/2}\right) \times \left\{ \left[3a^2h_b - (p_1^2 + p_1p_2 + p_2^2)h_p\right] / \left(2l_b^2 + l_c^2\right) \right\} \quad (4.41)$$

The right hand side of the equation (4.41) is the new geometrical shape factor.

4.7 BARRELLING IN SOLID RECTANGULAR BILLETS UNDER DSSIMILAR FRICTION

As explained elsewhere [171], the expression for the bulging can be written as follows under the condition that the bulging follows circular arc barrelling effect.

$$(\pi/4)(a+b)^2h_0 = \left(\pi/12\right) [2(l_a+h_b)^2 + (l_{ac} + h_{bc})^2]h_b +$$

$$(\pi/12)[(l_{at}+h_{at})^2 + (l_{at}+h_{at})(l_{ab}+h_{ab})+(l_{bb}+h_{bb})^2]h_p \quad (4.42)$$

(Under volume constancy principle)

where $l_a$ and $l_b$ are the bulged side lengths along the minor & major axes, $l_{ac}$ and $l_{bc}$ are the contact side lengths along the minor &major axes, $h_f$ is the height of the billet after deformation, ‘$a$’ is the side length of rectangle along the minor axis, ‘$b$’ is the side length of rectangle along the major axis, $h_0$ is the initial height of the square billet, $l_{at}$ and $l_{bt}$ are the side lengths along the minor & major axes of the top portion of the truncated pyramid, $l_{ab}$ and $l_{bb}$ are the side lengths along the minor & major axes of
the bottom portion of the truncated pyramid, \( h_b \) is the height of the bulged portion and \( h_p \) is the height of the truncated pyramid.

The above equation (4.42) can be written as follows

\[
(h_0)/(h_f) = \left( \frac{[2(l_a+l_b)^2+(l_ac+l_bc)^2]h_b + (\pi/12)[(l_at+l_bt)^2 + (l_ab+l_bb)^2]h_p}{(3(a+b)^2 h_f)} \right) \tag{4.43}
\]

Taking natural logarithm on both sides, the equation becomes

\[
\varepsilon_z = \varepsilon_0''
\tag{4.44}
\]

Where,

\[
\varepsilon_z = \ln(h_0/h_f)
\]

\[
\varepsilon_0'' = \ln \left\{ \frac{[2(l_a+h_b)^2+(l_ac+h_bc)^2]h_b + [(l_at+h bt)^2+(l_ab+h_bb)](l_ab+h_bb) + (l_ab+h_bb)^2 h_p}{(3(a+b)^2 h_f)} \right\}
\]

From the equation (4.9), the expression for the radius of curvature of barrel is as follows

\[
x_1 = R_1 - (R_1^2 - h_b^2/4)^{0.5}
\tag{4.45}
\]

where \( x_1 = (l_{bw1} - l_{cw1})/2 \), \( R_1 \) is the radius of the curvature of the barrel in a plane parallel to major axis, \( l_{bw1} \) is the bulged width parallel to the major axis and \( l_{cw1} \) is the contact width parallel to the major axis.

Simplifying the expression (4.45), the expression for the barrel radius \( R \) can be obtained neglecting \( x_1^2 \) term (because the quantity of the \( x_1 \) is very less)

Therefore,

\[
R_1 = h_b^2/8x_1
\tag{4.46}
\]

Otherwise
\[ R_1 = \frac{h_5^2}{4(l_{bw1} - l_{cw1})} \]  

From the expression (4.43) and (4.47), the barrel radius becomes as follows

\[ R_1^{0.50} = \frac{[3(a+b)^2 h_o - [(l_{at}+l_{bt})^2+(l_{at}+l_{bt})(l_{ab}+l_{bb})+(l_{ab}+l_{bb})^2]h_p]}{\{2[l_a+h_b]^2+(l_{ac}+l_{bc})^2\}2(l_{bw1}-l_{cw1})^{1/2}} \]  

Similarly the expression for the barrel radius in a plane parallel to the minor axis can be written as

\[ R_2^{0.50} = \frac{[3(a+b)^2 h_o - [(l_{at}+l_{bt})^2+(l_{at}+l_{bt})(l_{ab}+l_{bb})+(l_{ab}+l_{bb})^2]h_p]}{\{2[l_a+h_b]^2+(l_{ac}+l_{bc})^2\}2(l_{bw2}-l_{cw2})^{1/2}} \]  

where \( l_{bw2} \) is the bulged width parallel to the minor axis and \( l_{cw2} \) is the contact width parallel to the minor axis.

The right hand side of the equations (4.48) - (4.49) are the new geometrical shape factors developed.