CHAPTER 2

TOTALLY SEQUENTIAL CORDIAL GRAPHS

2.1 Introduction

The concept of cordial labeling has its origin from Cahit [9] in 1987. In 2002, Ibrahim Cahit [16] introduced totally sequential cordial labeling which is a weaker version of simply sequential labeling defined by Bange and Barkauskas [6, 7] and studied extensively by Slater [38].

2.1.1 Definition

According to Cahit [16], totally sequential cordial labeling is defined as follows:

Suppose $G = (V, E)$ is a simple graph. The graph $G$ is called totally sequential cordial (TSC) if there is a total mapping $f: V \cup E \rightarrow \{0, 1\}$ such that for each edge $uv$, $f(uv) = |f(u) - f(v)|$ and the condition $|f(0) - f(1)| \leq 1$ holds, where $f(0) = v_f(0) + e_f(0)$ and $f(1) = v_f(1) + e_f(1)$ and $v_f(i)$, $e_f(i)$ are respectively, the number of vertices and edges labeled with $i$ where $i \in \{0, 1\}$.

In other words, suppose $G = (V, E)$ is a simple graph. A vertex labeling $f: V \rightarrow \{0, 1\}$ induces an edge labeling $f^*: E \rightarrow \{0, 1\}$ defined by $f^*(xy) = |f(x) - f(y)|$. Let $v_0$ and $v_1$ be the number of vertices labeled with ‘0’ and ‘1’ respectively. Let $e_0$ and $e_1$ be the number of edges labeled with ‘0’, and ‘1’ respectively. Such a labeling is cordial if both $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$. A graph is called a cordial graph if it has a cordial labeling. Let $t_0 = v_0 + e_0$ and $t_1 = v_1 + e_1$. If $|t_0 - t_1| \leq 1$ then the labeling is called a totally sequential cordial labeling. A graph with a totally sequential cordial labeling is called a totally sequential cordial graph.
Cahit [16] proved that every cordial graph is totally sequential cordial, the cycle $C_n$, and the trees are totally sequential cordial, the wheel $W_n$ is totally sequential cordial for all $n > 3$, the complete bipartite graph $K_{m,n}$ is totally sequential cordial for all $m, n \geq 1$ and the friendship graph $F_n$ is totally sequential cordial for all $n \geq 1$. Also he gave certain conditions for a complete graph $K_n$ to be totally sequential cordial and he conjectured that cubic graphs other than $K_4$ are totally sequential cordial.

However, Cahit’s result does not hold in some cases where he stated positively. In this chapter we modify his results for the classes of graphs to hold good.

Cahit proved that the complete graph $K_n$ is totally sequential cordial if and only if

a). $\sqrt{k}$ has an integer value for $n = 4k+1$, $k \geq 1$,

b). $\sqrt{4k+1}$ has an integer value for $n = 4k+2$,

c). $\sqrt{4k+1}$ has an integer value for $n = 4k$,

d). $\sqrt{k+1}$ has an integer value for $n = 4k+3$.

However, $K_6$, $K_{13}$, $K_{22}$, $K_{33}$, $K_{46}$ etc. are totally sequential cordial, they do not satisfy any of the above conditions. Hence the modified conditions which are satisfied by the above complete graphs are given.

In the next section of this chapter, we prove that the graphs $K_{m,n}$, tree $T_n$ and $P_n^2$ are totally sequential cordial 1. Also, we modify the conjecture given by Cahit as a cubic graph with $n$ vertices is totally sequential cordial if $n \not\equiv 4(\text{mod } 8)$. 

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2.2 Some totally sequential cordial graphs

Example 2.2.1 The cycle $C_6$ is not cordial but it is totally sequential cordial by the labeling given in the following Figure.

![Totally sequential cordial labeling on $C_6$](Fig. 2.1)

Example 2.2.2 The cordial labeling for $C_7$ which is also a totally sequential cordial labeling is given in Fig. 2.2.

![Cordial labeling on $C_7$.](Fig. 2.2)
Example 2.2.3 \( P_7^2 \) has a totally sequential cordial labeling as shown in Fig. 2.3.

Example 2.2.4 The totally sequential cordial labeling for the ladder \( P_6 \times P_2 \) is given as follows.

Theorem 2.2.5 Every cordial graph is totally sequential cordial.

Proof. Let \( G \) be a cordial graph with \( n \) vertices and \( m \) edges. Then there exists a cordial labeling \( f \) of \( G \) such that \( |v_0 - v_1| \leq 1 \) and \( |e_0 - e_1| \leq 1 \). Let \( t_0 = v_0 + e_0 \) and \( t_1 = v_1 + e_1 \).

If either \( n \) or \( m \) or both are even then either \( v_0 = v_1 \) or \( e_0 = e_1 \) or both hold.

Clearly, \( |t_0 - t_1| \leq 1 \). Hence the cordial labeling \( f \) is also a totally sequential cordial labeling.
Thus $G$ is totally sequential cordial.

If both $n$ and $m$ are odd then $v_0 - v_1 = -1$ or $1$ and $e_0 - e_1 = -1$ or $1$.

**Case 1:** $v_0 - v_1 = -1$ and $e_0 - e_1 = -1$.

Clearly $t_0 - t_1 = -2$. Consider the complementary cordial labeling $\bar{f}$.

Then $\bar{f}(v) = 1 - f(v)$, for all $v \in V$ and the edge labels are invariant under $\bar{f}$.

Therefore $v_0 - v_1 = 1$ and $e_0 - e_1 = -1$.

Hence $G$ is totally sequential cordial.

**Case 2:** $v_0 - v_1 = 1$ and $e_0 - e_1 = 1$.

Clearly $t_0 - t_1 = 2$. Consider the complementary cordial labeling $\bar{f}$.

Then $\bar{f}(v) = 1 - f(v)$, for all $v \in V$ and the edge labels are invariant under $\bar{f}$.

Therefore $v_0 - v_1 = -1$ and $e_0 - e_1 = 1$.

Hence $G$ is totally sequential cordial.

**Case 3:** $(v_0 - v_1 = -1$ and $e_0 - e_1 = 1)$ or $(v_0 - v_1 = 1$ and $e_0 - e_1 = -1)$.

Clearly, $t_0 - t_1 = 0$.

Hence in all the cases, the graph $G$ is totally sequential cordial.

\[\blacksquare\]

**Corollary 2.2.6** Every totally sequential cordial graph need not be cordial.
Example 2.2.7 Cahit proved that $C_n$ is cordial if and only if $n \neq 2 \pmod{4}$

But $C_n$ is totally sequential cordial for all $n > 2$.

2.3 Totally sequential cordial labeling for cycles and paths

Theorem 2.3.1 Let $f$ be any binary vertex labeling of the cycle $C_n$. Then the number of edges labeled with 1 is always even.

Proof. Let $u_1, u_2, \ldots, u_n$ be the cycle $C_n$ and $f$ be a binary vertex labeling of $C_n$.

Let $e(1)$ be the number of edges labeled 1.

Case 1: All the vertex labels are 0.

Then each edge has a label 0.

Therefore, $e(1) = 0$, which is even.

Case 2: All the vertex labels are 1.

Then, all the edge labels are 0.

Therefore, $e(1) = 0$, which is even.

Case 3: The vertex labels assume both 0 and 1.

Assume $f(u_1) = 1$. Let $i$ be the least positive integer for which $f(u_i) = 0$.

Then, $f(u_1) = f(u_2) = \cdots = f(u_{i-1}) = 1$. Here the edge $u_{i-1}u_i$ gets the label 1.

If $f(u_j) = 0$ for all $j > i$ then the edge $u_nu_1$ gets the label 1.
Hence $e(1) = 2$, which is even.

If not, $f(u_j) = 1$ for some $j > i$ then let $k$ be the least positive integer greater than $i$ for which $f(u_k) = 1$.

Then, $f(u_i) = f(u_{i+1}) = \ldots = f(u_{k-1}) = 0$ and $f(u_k) = 1$.

Therefore the edge $u_{k-1}u_k$ gets the label 1 and the edge $u_{i-1}u_i$ also gets the label 1

Hence a sum of 2 is contributed to $e(1)$.

Proceeding like this, we get $e(1)$ is even.

\[\square\]

**Theorem 2.3.2** The cycle $C_n$ is a totally sequential cordial graph for all $n > 2$.

**Proof.** Cahit showed that a unicyclic graph is cordial unless it is $C_{4k+2}$. Let $n = 4r + i$, where $i = 0, 1, 2, 3$ and for some $r \in \mathbb{N}$. Let $L_{4r}$ denote the labeling $00110011 \ldots 0011$. Also this labeling is modified by adding symbols at one end or other (or both).

Thus $L_{4r}001$ denotes $00110011 \ldots 0011001$.

The totally sequential cordial labeling we use for $C_n$ is given in the following Table.

<table>
<thead>
<tr>
<th>$n=4r+i$</th>
<th>Labeling of $C_n$</th>
<th>$v_0$</th>
<th>$v_1$</th>
<th>$e_0$</th>
<th>$e_1$</th>
<th>$v_0 - v_1$</th>
<th>$e_0 - e_1$</th>
<th>$t_0 - t_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 0$</td>
<td>$L_{4r}$</td>
<td>$2r$</td>
<td>$2r$</td>
<td>$2r$</td>
<td>$2r$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

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Table 2.1

The last column of this table shows that $|t_0 - t_1| = 0 \leq 1$.

Hence $C_n$ is totally sequential cordial for all $n > 2$.

Theorem 2.3.3 The path $P_n$ is a totally sequential cordial graph for all $n$.

Proof. The totally sequential cordial labeling that we use is given as follows. Let $n = 4r+i$, where $i= 0, 1, 2, 3$ and for some natural number ‘$r$’.

Let $L_{4r}$ denote the labeling 00110011...0011. Also this labeling is modified by adding symbols at one end or other (or both).

Thus $01L_{4r}$ denotes 0100110011...0011.

<table>
<thead>
<tr>
<th>i = 1</th>
<th>$1L_{4r}$</th>
<th>2r</th>
<th>2r+1</th>
<th>2r+1</th>
<th>2r</th>
<th>-1</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>i = 2</td>
<td>$11L_{4r}$</td>
<td>2r</td>
<td>2r+2</td>
<td>2r+2</td>
<td>2r</td>
<td>-2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>i = 3</td>
<td>$L_{4r}001$</td>
<td>2r+2</td>
<td>2r+1</td>
<td>2r+1</td>
<td>2r+2</td>
<td>1</td>
<td>-1</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 2.2

From the last column we observe that, $|t_0 - t_1| \leq 1$.

Hence $P_n$ is totally sequential cordial for all $n$.

2.4. Totally sequential cordial labeling for the complete graph $K_n$

Theorem 2.4.1 The complete graph $K_4$ is not totally sequential cordial.

Proof. All possible vertex labelings satisfying the condition that $v_0 = 1$ and $v_1 = 3$ are [0111], [1101], [1110].

All possible vertex labelings with $v_0 = 3$ and $v_1 = 1$ are [0001], [0010], [0100], [1000].

For all the above labelings, $e_0 = 3$ and $e_1 = 3$.

Therefore, $|t_0 - t_1| = 2$ not less than or equal to 1.

All possible vertex labelings with $v_0 = v_1 = 2$ are [0011], [1001], [1100], [0110], [0101], [1010].

For these labelings, $e_0 = 2$ and $e_1 = 4$. 

<table>
<thead>
<tr>
<th>$i$ = 1</th>
<th>$1L_{4r}$</th>
<th>2$r$</th>
<th>2$r+1$</th>
<th>2$r$</th>
<th>2$r$</th>
<th>-1</th>
<th>0</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$ = 2</td>
<td>01$L_{4r}$</td>
<td>2$r+1$</td>
<td>2$r+1$</td>
<td>2$r$</td>
<td>2$r+1$</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$i$ = 3</td>
<td>001$L_{4r}$</td>
<td>2$r+2$</td>
<td>2$r+1$</td>
<td>2$r+1$</td>
<td>2$r+1$</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>
Therefore, \( t_0 - t_1 = 0 - 2 = -2 \)

Hence \( K_4 \) is not totally sequential cordial.

**Example 2.4.2** The complete graph \( K_5 \) is not cordial but it is totally sequential cordial by the labeling given in the following Fig. 2.5.

![Fig. 2.5](image_url)

The edge labels are 011111000. Therefore \( t_0 - t_1 = -1 \).

Hence \( K_5 \) is totally sequential cordial.

**Example 2.4.3** The complete graph \( K_8 \) is totally sequential cordial.
Fig. 2.6

The complete graph $K_8$ has 8 vertices and 28 edges

Using the vertex labeling $11100000$, the corresponding edge labels are

0011111011111111110000000000.

Here $v_0 = 5$, $v_1 = 3$, $e_0 = 13$, $e_1 = 15$.

Therefore, $t_0 - t_1 = 18 - 18 = 0$.

Hence $K_8$ is totally sequential cordial.

The generalized conditions for the complete graph $K_n$ to be totally sequential cordial is given in the following theorem.
Theorem 2.4.4 The complete graph $K_n$ is totally sequential cordial if and only if

\begin{enumerate}
  \item[i)] $\sqrt{n+1}$ is an integer, when $n \equiv 0(\text{mod } 4)$
  \item[ii)] $\sqrt{\frac{n-1}{4}}$ or $\sqrt{\frac{n+3}{4}}$ is an integer, when $n \equiv 1(\text{mod } 4)$
  \item[iii)] $\sqrt{n-1}$ or $\sqrt{n+3}$ is an integer, when $n \equiv 2(\text{mod } 4)$
  \item[iv)] $\sqrt{\frac{n+1}{4}}$ is an integer, when $n \equiv 3(\text{mod } 4)$
\end{enumerate}

**Proof.** The complete graph $K_n$ has $n$ vertices and $nC_2$ edges. Let $f$ be a totally sequential cordial labeling of $K_n$. Then $|t_0 - t_1| \leq 1$.

Also, $t_0 + t_1 = n + \frac{n(n-1)}{2} = \frac{n(n+1)}{2} = \begin{cases} 
\text{even if } n \equiv 0,3(\text{mod } 4) \\
\text{odd if } n \equiv 1,2(\text{mod } 4) 
\end{cases}$.

We consider two cases:

**Case 1:** $t_0 + t_1$ is even.

Then $n \equiv 0,3(\text{mod } 4)$. Since $f$ is a totally sequential cordial labeling of $K_n$, $t_0 = t_1$.

Under this labeling $f$, the complete graph $K_n$ can be decomposed as: $K_n = K_p \cup K_r \cup K_{p,r}$, where $K_p$ is the sub-complete graph of $K_n$ whose vertices are labeled with 1’s; $K_r$ is the sub-complete graph of $K_n$ whose vertices are labeled with 0’s and $K_{p,r}$ is the complete bipartite sub-graph of $K_n$ with the bipartition $V(K_p) \cup V(K_r)$ in which the edges labeled with all 1’s.

Then $n = p + r$.

Clearly, for the labeling $f$, we write
\[ t_1 = p + rp \text{ and } t_0 = pC_2 + r + rC_2. \]

Using \( t_0 = t_1 \), we get, \( (r - p)^2 - 3p + r = 0. \)

Put, \( p = n - r \) in the above equation, we get

\[ 4r^2 - 4(n - 1)r + n^2 - 3n = 0 \]

Solving this for \( r \), we obtain \( r_{1,2} = \frac{(n-1)\pm\sqrt{n+1}}{2} \)

This \( r_{1,2} \) will represent the order of the sub-complete graph \( K_r \). In order to have integer values for \( r_{1,2} \), for \( n \equiv 0 \text{ (mod 4)} \), \( \sqrt{n+1} \) is an integer.

Therefore, \( K_n \) is totally sequential cordial if and only if \( \sqrt{n+1} \) is an integer.

Also, for \( n \equiv 3 \text{ (mod 4)} \), \( K_n \) is totally sequential cordial if and only if \( \sqrt{\frac{n+1}{4}} \) is an integer.

**Case 2:** \( t_0 + t_1 \) is odd.

Clearly \( n \equiv 1, 2 \text{(mod 4)} \)

Since \( f \) is a totally sequential cordial labeling of \( K_n \), there arise two cases \( t_1 > t_0 \) or \( t_0 > t_1 \).

**Sub case 2. (i):** \( t_1 > t_0. \)

Then \( t_1 = t_0 + 1. \)

By the same decomposition of case 1, we write,

\[ t_1 = p + rp \text{ and } t_0 = pC_2 + r + rC_2 \quad \text{(1)} \]

Also, \( p = n - r \quad \text{(2)} \)
Using all these equations, we get the quadratic equation,

\[ 4r^2 - 4(n-1)r + n^2 - 3n + 2 = 0 \]

Solving this for \( r \) we get,

\[ r_{1,2} = \frac{(n-1) \pm \sqrt{(n-1)}}{2} \]

This gives the order of the sub-complete graph \( K_r \). In order to have integer values for \( r_{1,2} \), for 

\( n \equiv 1(\text{mod } 4) \) and \( n \equiv 2(\text{mod } 4) \) respectively \( \sqrt{\frac{n+1}{4}} \) and \( \sqrt{n-1} \) must be an integer.

**Sub case 2. (ii):** \( t_0 > t_1 \).

We can take \( t_0 = t_1 + 1 \) \hspace{1cm} (3)

By using equations (1) and (2), equation (3) becomes

\[ 4r^2 - 4(n-1)r + n^2 - 3n - 2 = 0 \]

Solving this for \( r \) we get,

\[ r_{1,2} = \frac{(n-1) \pm \sqrt{(n+3)}}{2} \]

This gives the order of the sub-complete graph \( K_r \).

It can easily be seen that,

When \( n \equiv 1(\text{mod } 4) \), \( K_n \) is totally sequential cordial if and only if \( \sqrt{\frac{n+1}{4}} \) is an integer and

When \( n \equiv 2(\text{mod } 4) \), \( K_n \) is totally sequential cordial if and only if \( \sqrt{n+3} \) is an integer.

Thus from case (2) we observe that
$K_n$ is totally sequential cordial if and only if $\sqrt{\frac{n-1}{4}}$ or $\sqrt{\frac{n+3}{4}}$ is an integer, when $n \equiv 1 \pmod{4}$

And for $n \equiv 2 \pmod{4}$, $K_n$ is totally sequential cordial if only if $\sqrt{n-1}$ or $\sqrt{n+3}$ is an integer.

Hence the theorem follows.

2.5. Totally sequential cordial labeling for square of paths

Example 2.5.1 $P_3^2$ is totally sequential cordial by the following labeling.

![Fig. 2.7](image)

Example 2.5.2 $P_5^2$ is totally sequential cordial by the following labeling.

![Fig. 2.8](image)
Theorem 2.5.3 The second power of path $P_n$, is totally sequential cordial for all $n > 1$.

Proof. Let $P_n^2$ denote the second power of path $P_n$ with $n$ vertices and $2n - 3$ edges.

Let $M_{2r}$ denote the labeling $0101\cdots01$.

The totally sequential cordial labeling that we use is given in the following table.

<table>
<thead>
<tr>
<th>$n=2r+i$, $i = 0, 1$</th>
<th>Labeling of $P_n^2$</th>
<th>$v_0$</th>
<th>$v_1$</th>
<th>$e_0$</th>
<th>$e_1$</th>
<th>$v_0 - v_1$</th>
<th>$e_0 - e_1$</th>
<th>$t_0 - t_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i = 0$</td>
<td>$M_{2r}$</td>
<td>$r$</td>
<td>$r$</td>
<td>$2r-2$</td>
<td>$2r-1$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$i = 1$</td>
<td>$M_{2r,0}$</td>
<td>$r+1$</td>
<td>$r$</td>
<td>$2r-1$</td>
<td>$2r$</td>
<td>$1$</td>
<td>$-1$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

Table 2.3

From the last column we observe that $|t_0 - t_1| \leq 1$.

Hence $P_n^2$ is totally sequential cordial.

2.6. Totally sequential cordial labeling for trees and $K_{m,n}$

Example 2.6.1 The totally sequential cordial labeling for $T_5$ is given below.
Example 2.6.2  $T_7$ is totally sequential cordial by the following labeling.

Theorem 2.6.3  Trees are totally sequential cordial.

**Proof.** It is well known that trees are cordial.

Then there exists a binary vertex labeling $f$ such that $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$, the induced edge labels are computed by $f(e) = |f(u) - f(v)|$, for all $e = uv \in E$.

Let $f$ be a cordial labeling of an $n$ vertex tree $T_n$.

We have two cases to consider:

**Case 1:** $n$ is even.
Then $|v_0 - v_1| \leq 1$ gives $v_0 = v_1$.

For a tree, number of edges $q = n-1$ which is odd. Therefore, $|e_0 - e_1| \leq 1$ implies $|e_0 - e_1| = 1$.

Now $|t_0 - t_1| = |(v_0 - v_1) + (e_0 - e_1)| \leq |v_0 - v_1| + |e_0 - e_1| = 0 + 1 = 1$.

Hence $T_n$ is totally sequential cordial.

Case 2: $n$ is odd

Then $q = n-1$ which is even. Since $T_n$ is cordial, we have $|v_0 - v_1| = 1$ and $e_0 = e_1$.

Now $|t_0 - t_1| = |(v_0 - v_1) + (e_0 - e_1)| \leq |v_0 - v_1| + |e_0 - e_1| = 1 + 0 = 1$.

Hence $T_n$ is totally sequential cordial.

Thus a cordial labeling $f$ is also a totally sequential cordial labeling for a tree.

Example 2.6.4 $K_{2,2}$ is totally sequential cordial by the following labeling.

![Fig. 2.11](image-url)
Example 2.6.5 $K_{3,2}$ is totally sequential cordial by the following labeling

$$
\begin{array}{c c c c}
0 & 1 & 0 \\
0 & & 1 \\
0 & & & 1 \\
\end{array}
$$

Fig. 2.12

Theorem 2.6.6 The complete bipartite graph $K_{m,n}$ is totally sequential cordial for all $m, n \geq 1$.

Proof. Cahit [9] proved that $K_{m,n}$ is cordial for all $m$ and $n$.

Let $f$ be a cordial labeling for $K_{m,n}$. Then $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$.

The graph $K_{m,n}$ has $m + n$ vertices and $mn$ edges.

Consider the following three cases:

Case 1: $m$ and $n$ are even.

Then $m + n$ is even and $mn$ is even.

Therefore, $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$ gives $v_0 = v_1$ and $e_0 = e_1$.

Now $t_0 - t_1 = 0$. Hence $K_{m,n}$ is totally sequential cordial.

Case 2: Both $m$ and $n$ are odd.
Then $m + n$ is even and $mn$ is odd. Since $K_{m,n}$ is cordial, $v_0 = v_1$ and $|e_0 - e_1| = 1$.

Now $|t_0 - t_1| \leq |v_0 - v_1| + |e_0 - e_1| \leq 1$.

Thus $K_{m,n}$ is totally sequential cordial.

**Case 3:** ($m$ is even and $n$ is odd) or ($m$ is odd and $n$ is even).

Then $m + n$ is odd and $mn$ is even.

Cordiality of $K_{m,n}$ gives $|v_0 - v_1| = 1$ and $e_0 = e_1$.

Here, $|t_0 - t_1| \leq |v_0 - v_1| + |e_0 - e_1| \leq 1$. Thus $K_{m,n}$ is totally sequential cordial.

Hence the theorem.

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Cahit [16] gave the conjecture that “Cubic graphs other than $K_4$ are totally sequential cordial”. Z. Liu and B. Zhu [33] proved that a 3-regular graph of order $n$ is cordial if and only if $n \equiv 4 \pmod{8}$. In Theorem 2.2.5, we prove that every cordial graph is totally sequential cordial.

Thus we have the following theorem.

**Theorem 2.6.7** A cubic graph of order $n$ is totally sequential cordial if $n \not\equiv 4 \pmod{8}$. 