8.1 Introduction

In this chapter two methods of generating very high spin states, namely,

1. The Statistical Theory of Hot Rotating nuclei (STHRN) developed by Moretto [12] and

2. The Cranked Nilsson Model (CNM) [35, 49] for hot rotating nuclei,

are compared and the details of these two methods are explained in Chapter 3. Here, the results of the calculations for the nucleus $^{186}$Hg are compared. These methods yield different results for triaxial deformations although for biaxial deformations the results are identical. Pairing correlations are included along with the collective rotation and the interplay between various degrees of freedom like deformation, angular momentum and the temperature of the system and the pairing gap parameter $\Delta$ is an important aspect of the statistical method. The results associated with the variation of the gap parameter $\Delta_P$ of protons and $\Delta_N$ of neutrons obtained from
the two methods are discussed here. It is observed that $\Delta_P$ and $\Delta_N$ vanish faster in the CNM method. Also, certain structures exhibited in the CNM are absent in the STHRN calculations.

8.2 The Lagrangian Multiplier $\gamma$ and Cranking Frequency $\omega$

The single particle levels of deformed nuclei are used in the STHRN method and the Lagrangian multiplier $\gamma$ projects out different angular momentum states of the system from the grand partition function [14, 58]. The STHRN method can be applied only for nuclei rotating around the symmetry axis since the single particle spin projection $m_z$ is a good quantum number for biaxial deformation of the system. However, this method cannot be applied for the deformed nuclei rotating perpendicular to the symmetry axis since the single particle spin projection ceases to be a good quantum number for triaxial deformation.

The rotational part of the Hamiltonian in the Cranked Nilsson Model (CNM) is diagonalised using oscillator basis [31]. For biaxial deformations, the rotational part split the states with spin projections $\pm m_j$ while in the case of triaxial deformations, it gets contributions from all $m_z$ states and only the average value $<j_z>$ is known. This value becomes zero when the spins align in the direction perpendicular to symmetry axis.

The Lagrangian multiplier $\gamma$ in the STHRN and rotational frequency $\omega$ in the CNM are identically equal for axially symmetric shapes and the values of $\gamma$ and $\omega$
are different for triaxial deformations.

The level density parameter, single neutron separation energy, single proton separation energy and collective rotational energy are extracted using the expressions in chapters 4 and 5.

In this work [16], the temperature $T$ is varied from 0 to 2 MeV and angular momentum from $0 \hbar$ to $80 \hbar$. The deformation parameters $a_0$ and $a_2$ are varied in the range $a_0 = -0.6$ to $+0.6$ and $a_2 = 0.01$ to $0.06$ for the system $^{188}$Hg. The shape parameter $\gamma = \omega$ for biaxial deformations and for triaxial deformations $\gamma \neq \omega$ where $\omega$ is the rotational frequency of the system. The values of Nilsson parameters $\kappa$ and $\mu$ are taken from Ref. [33].

8.3 Results and Discussion

Since the inputs for the two methods are the microscopic single particle levels and the single particle spins corresponding to the biaxially and triaxially deformed Nilsson harmonic oscillator potential, the results exhibit the effect of shell structure of the system at different deformations. The values of the different parameters calculated by the two methods for the nucleus $^{188}$Hg are shown in Figs 8.1 - 8.14.

In Fig. 8.1, for angular momentum $M = 30 \hbar$, Lagrangian multiplier $\gamma$ and collective rotational frequency $\omega$ are plotted as a function of temperature $T$ for various deformation parameters. When the deformation parameter $a_2 = 0$, corresponding to biaxial deformations, $\gamma$ is equal to $\omega$ for all values of $M$ and $T$. However, for
\(a_2 \neq 0, \gamma \neq \omega\), since triaxial deformation mixes states of different spin projections \(\pm m_j\). It is obvious from Fig. 8.1 that \(\gamma\) is very sensitive to the deformation parameter \(a_2\). For a change of \(a_2\) from 0.0 to 0.03, \(\gamma\) changes from 0.35 to 0.4 MeV whereas the corresponding changes in \(\omega\) values are smaller. In Fig. 8.2, for \(T = 1.0\) MeV, \(\gamma\) and \(\omega\) are plotted versus \(M\).

In Figs. 8.3 and 8.4 for \(^{186}\text{Hg}\), the collective rotational frequency \(\omega\) and \(\gamma\) are plotted versus \(M\) for temperatures \(T = 0.3\) MeV and 1.0 MeV. For prolate deformation the CNM calculations show a shape transition beyond \(\omega = 0.3\) MeV whereas the STHRN calculations show shape transition around the shape parameter \(\omega = 0.15\) MeV in addition to the one at \(\omega = 0.3\) MeV. At \(T = 1.0\) MeV, in the STHRN, this shape transition at small frequencies still exists but the ones at higher frequencies vanish in both the methods. In the case of oblate deformation both the methods yield almost the same results at high temperatures.

In Figs. 8.5 and 8.6, the corresponding rotational energies are shown as a function of collective rotational frequency. The results shown in Figs 8.5 and 8.6 are self explanatory being the same as for Figs. 8.3 and 8.4. The fluctuation at \(\omega = 0.15\) MeV, corresponding to an angular momentum of \(5\hbar\) in the case of prolate deformations, is similar to the one observed in experiments as backbending phenomena [139].

The fluctuations at \(5\hbar\) observed in Figs. 8.5 and 8.6 are also exhibited in Figs. 8.7 and 8.8 which show the level density parameter \(a\) as a function of \(M\) for \(T = 1.0\)
MeV. In Figs. 8.9 - 8.12 the neutron and proton separation energies obtained from the two methods are displayed.

In Figs. 8.13 and 8.14 the variation of the gap parameter $\Delta_P$ and $\Delta_N$ for $T = 0.3$ MeV obtained by STHRN and CNM methods for $^{186}$Hg are displayed. In these figures, curves 1 and 2 represent the $\Delta_N$ values obtained in the STHRN and CNM calculations respectively for deformations parameter values $a_0 = 0.6$ and $a_2 = 0.06$. Clearly, the structure exhibited in the CNM calculation is absent in the STHRN calculation. The former is a more microscopic calculation due to the inclusion of the coupling between the collective rotation and the single particle spins in the Hamiltonian itself. Since triaxial deformations couple states of different $M_z$, the rotational part of the Hamiltonian is affected. The nuclear structural effect on $\Delta_N$ and $\Delta_P$ are more pronounced in the CNM calculation. The spins generated are also presented in the same figures (curves 3 and 4) for direct comparison.

In conclusion, it is observed that certain parameters obtained through two methods (STHRN and CNM) of generating high spin states in $^{186}$Hg yield identical results for biaxial deformations and different results for triaxial deformations. Certain features of the structural transitions in $^{186}$Hg are explained very well by the CNM calculations rather than by STHRN method.
Figure Captions

Fig. 8.1 Collective rotational frequency for $^{186}$Hg as a function of temperature $T$ and deformation for $M = 30 \hbar$ using the STHRN and CNM calculations. The deformation parameters $a_2$ and $a_0$ are shown in the figure, the first quantity being $a_2$ and the second one being $a_0$.

Fig. 8.2 Collective rotational frequency for $^{186}$Hg as a function of angular momentum $M$ and deformation for $T = 1.0$ MeV using the STHRN and CNM calculations. The deformation parameters $a_2$ and $a_0$ are shown in the figure.

Fig. 8.3 Collective rotational frequency versus $M$ for $T = 0.3$ MeV for prolate deformation. The inset graph is for $T = 1.0$ MeV. The axes of the inset graph are same as in the main graph.

Fig. 8.4 Same as in Fig. 8.3 for oblate deformation.

Fig. 8.5 Collective rotational frequency versus collective rotational energy $E_{rot}$ for $T = 0.3$ MeV for prolate deformation. The inset graph is for $T = 1.0$ MeV. The axes of the inset graph are same as in the main graph.

Fig. 8.6 Same as in Fig. 8.5 for oblate deformation.

Fig. 8.7 Level density parameter $a$ as a function of angular momentum $M$ for $T = 1.0$ MeV using STHRN and CNM calculations for prolate deformation.

Fig. 8.8 Same as in Fig. 8.7 for oblate deformation.
Fig. 8.9 Single neutron separation energy $S_n$ versus angular momentum $M$ for $T = 1.0$ MeV using STHRN and CNM calculations for prolate deformation.

Fig. 8.10 Same as in Fig. 8.9 for oblate deformation.

Fig. 8.11 Single proton separation energy $S_p$ versus angular momentum $M$ for $T = 1.0$ MeV for prolate deformation.

Fig. 8.12 Same as in Fig. 8.11 for oblate deformation.

Fig. 8.13 Neutron gap parameter as a function of collective rotational frequency for $T = 0.3$ MeV for $^{186}$Hg. The numbers 1 and 2 on the curve correspond to the $\Delta_N$ obtained by STHRN and CNM calculations respectively. The corresponding spins generated by the two methods are shown in curves 3 and 4.

Fig. 8.14 Same as in Fig. 8.13 for proton gap parameter $\Delta_P$. 

94
Fig. 8.1

Collective rotational frequency $\omega$ (MeV)

Temperature $T$ (MeV)

- STHRN
- CNM

$^{186}$Hg

$\omega$ vs. $T$

- $\omega$ vs. $T$
- $\omega$ vs. $T$
- $\omega$ vs. $T$
- $\omega$ vs. $T$
- $\omega$ vs. $T$

Points:
- $(0.06, 0.6)$
- $(0.03, 0.6)$
- $(0.0, 0.6)$
- $(0.03, 0.6)$
- $(0.06, 0.6)$
Fig. 8.2
Fig. 8.3
Fig. 8.4
Fig. 8.5

Collective rotational frequency $\omega$ (MeV)

Collective rotational energy $E_{\text{rot}}$ (MeV)
Collective rotational energy $E_{\text{rot}}$ (MeV)

Collective rotational frequency $\omega$ (MeV)

Fig. 8.6
Fig. 8.7
Angular momentum $M$ ($\hbar$)

Level density parameter $a$ (MeV$^{-1}$)

$^{186}$Hg

OBLATE

- CNM
- STHRN

Fig. 8.8
Fig. 8.9
Fig. 8.10

Single neutron separation energy $S_n$ (MeV)

Angular momentum $M (\hbar)$

$^{186}$Hg

OBLATE
- CNM
- STHRN

Fig. 8.10
Fig. 8.11

Single proton separation energy $S_p$ (MeV)

Angular momentum $M$ ($\hbar$)

$^{186}$Hg

PROLATE
- CNM
- STHRN
Fig. 8.12

Single proton separation energy $S_p$ (MeV)

Angular momentum $M (\hbar)$
$a_z = 0.06$

$\omega_0 = 0.6$

Fig. 8.13
$$^{186}\text{Hg}$$

Proton pairing gap $\Delta_p$ (MeV)

Collective rotational frequency $\omega$ (MeV)

Angular momentum $M$ ($\hbar$)

$a_2 = 0.06$

$a_0 = 0.6$

Fig. 8.14