Chapter 5

Structural Changes in Hot Rotating Nuclei

5.1 Introduction

The study of structural changes of nuclei at high excitation energy and large angular momentum is a topic of current interest in nuclear structure physics. The combined effect of spin and temperature has created a variety of shape transition phenomena in nuclei. When the pairing correlations in nucleus are completely destroyed beyond the temperature $T \approx 0.6$ MeV and the angular momentum $M = 30 \hbar$ [15,16,39-41], an outstanding question arises about the nature of equilibrium shape of the nucleus at these temperatures and spins. As the angular momentum increases, there is a general tendency that the nuclei with neutron range $N = 88 - 98$ and proton range $Z = 62 - 70$ should experience a transition of band spectrum from prolate to triaxial and possibly to oblate shapes in deformed rare earth nuclei or transition from weakly oblate to superdeformed shapes in neutron deficient rare earth systems [15, 16, 41]. Both the theoretical and experimental investigations of several authors about shape
transition are discussed below.

Cohen et al. [98] have predicted that the nucleus undergoes shape changes with increasing angular momentum by assuming the nucleus as a structureless and charged liquid drop subjected to Coulomb and surface forces. The behaviour of nucleus with increasing spin in the different regions of deformation was studied by Bohr and Mottelson [36]. Using a many body Hamiltonian, Faessler et al. [99] have performed microscopic calculations based on the classical model of Cohen et al. [98]. Neergard et al. [100], Anderson et al. [101] and Bengtsson et al. [49] have independent investigations on nuclei employing Strutinsky shell correction method. Calculations with Strutinsky's prescription have yielded good results for the deformation behaviour of the nuclei at high spins [36].

Further, mean field theories, both relativistic and nonrelativistic, such as Finite Temperature Hartee Fock-Bogoliubov Cranking Theory (FTHFBC) [40, 102] or finite temperature cranked Strutinsky calculations [79] have been used to study structural changes in hot rotating nuclei. From the FTHFBC description, it was observed that (i) the equilibrium phase is nearly prolate collective rotation for spins below $39 \hbar$ and below a critical temperature. However at these spins the shape becomes oblate noncollective above the critical temperature and (ii) for spins greater than $39 \hbar$, there is oblate noncollective shape at all temperatures. A shape transition from noncollective oblate shape to superdeformed or hyperdeformed collective prolate or nearly prolate (triaxial) shape has been predicted by Alhassid and Whelen.
by using Landau theory of phase transition and observed by Kicinska et al. [104]. Such kind of shape transition similar to Jacobi transition was studied in fp shell nuclei [105]. A systematic study of temperature dependence of the shapes and pairing gaps of some isotopes in the rare earth region was made in the relativistic Hartree-Fock-Bogoliubov theory [40]. The shape transition temperature of certain nuclei in this region was found to be in the domain of ~ 1.0 to 1.8 MeV [106]. Cranmer et al. [44] have reported the occurrence of a shape transition from prolate to oblate. Dudek et al. [48] have predicted shape evolution in high spin states of rare earth nuclei at $M \geq 30\hbar$ using generalised Strutinsky method. Based on STHRN Rajasekaran et al. [15, 16] have predicted a shape transition from collective prolate to oblate noncollective around the angular momentum $M \approx 50\hbar$.

Experimentally, a shape transition, i.e., transition from a prolate nucleus rotating collectively about one of its minor axes to an oblate shaped nucleus with the spin generated by single particle motion has been observed by Henss et al. [6] and Simpson [10]. The thermal response to nuclear shapes has also been studied experimentally from the shapes of Giant Dipole Resonance (GDR) built on excited states [107, 108].

### 5.2 Various Methods to Study Shape Transition

Different methods available for studying shape transition in nuclei are outlined in the following sections.
5.2.1 Finite Temperature Hartree-Fock-Bogoliubov Cranking Calculations

In the Finite Temperature Hartree-Fock-Bogoliubov Cranking calculations (FTHBC), Hamiltonian $H$ [40, 102] is written as

$$H = e - \mu_p N_p - \mu_n N_N - \omega J_x + \Gamma.$$  \hspace{1cm} (5.1)

Here, $e$, $\mu_{p(n)}$ and $\omega$ are spherical single-nucleon energy, chemical potential and angular velocity respectively. The Hartree-Fock and pair potentials are:

$$\Gamma_{ij} = \sum_{kl} <ik|U|jl> \rho_{ik},$$ \hspace{1cm} (5.2)

$$\Delta_{ij} = \frac{1}{2} \sum_{kl} <ij|v|kl> t_{kl},$$ \hspace{1cm} (5.3)

where $\rho_{ik}$ and $t_{kl}$ are the Hartree-Fock and pairing densities.

The quasi-particle occupation probabilities in terms of temperature $T$ and eigen values of FTHBC equation \textit{i.e.}, quasi particle energies $E_i$ are given by

$$f_i = \left[1 + e^{\frac{E_i}{T}}\right]^{-1}.$$ \hspace{1cm} (5.4)

The chemical potentials and the angular velocity are varied to satisfy the number constraints and spin constraints respectively.

$$< N_p > = Z, < N_N > = N,$$ \hspace{1cm} (5.5)

$$< J_x > = [I (I + 1)]^{\frac{1}{2}}.$$ \hspace{1cm} (5.6)
By the iteration method, the self-consistent solution to Eqs. (5.1) to (5.6) is obtained. The free energy is

\[ F = E - TS, \]  

(5.7)

with entropy \( S \) and energy \( E \) given by

\[ S = - \sum_i [f_i \ln f_i + (1 - f_i) \ln (1 - f_i)], \]  

(5.8)

\[ E = \langle H \rangle = \text{Tr} \left[ (e + \frac{1}{2} \Gamma) \rho + \frac{1}{2} \Delta t^+ \right]. \]  

(5.9)

The FTHFBC equation determines the values of the quadrupole deformation \( \epsilon \) and \( \gamma \) and the values of the pairing gaps \( \Delta_P \) and \( \Delta_N \). The minimization of the free energy function \( F(\beta, \gamma, \Delta_P, \Delta_N; I, T) \) defines the equilibrium or most probable state of the nucleus for the given spin and temperature.

### 5.2.2 Finite Temperature Strutinsky Method

In this method (FTSM) [25], the potential \( V(r) \) is the sum of harmonic oscillator term \( V_{H,O} \) and the correction term \( V_{corr} \).

\[ V(r) = V_{H,O} + V_{corr}, \]  

(5.10)

with

\[ V_{H,O} = \frac{1}{2} \hbar \omega_o \rho^2 \left[ 1 - \frac{2}{3} \epsilon \sqrt{\frac{4\pi}{5}} \cos \gamma Y_{20} - \frac{2}{3} \epsilon \sqrt{\frac{4\pi}{5}} \sin \gamma (Y_{23} + Y_{2-2}) \right] \]  

(5.11)

\[ V_{corr} = -k\hbar \omega_o \left\{ 2l_s + \mu (l^2 - N(N + 3)) \right\} \]  

(5.12)
The first term in Eq. (5.10) depends on the two quadrupole deformation parameters \( \epsilon \) and \( \gamma \) which decide the nuclear shape. The second term is introduced to describe the inertia properties.

The expression for the Routhian function of a nucleus at a temperature \( T \) rotating with the angular frequency \( \omega \) is

\[
R(\epsilon, \gamma; \omega, T) = E_{LD}(T = \omega = 0) + \sum_i \epsilon_i(\epsilon, \gamma, \omega)\bar{n}_i(T) - \sum_i \epsilon_i(\epsilon, \gamma, \omega = 0)\bar{n}_i(T = 0).
\]

(5.13)

Here, \( E_{LD} \) is the liquid drop component of the energy, \( \epsilon(\epsilon, \gamma; \omega) \) are the energies of single particle states and \( \bar{n}_i(T) \) is the occupation probability of the \( i \)th shell. The last term in Eq. (5.13) is the Strutinsky - smeared sum of single particle energies for \( T = 0 \). As this procedure is a direct consequence of the correct calculation of the nuclear moment of inertia, renormalization of the average moment of inertia is not necessary. On the basis of the relation

\[
\sum_{i=1}^{A} \epsilon_i(\epsilon, \gamma, \omega) - \sum_{i=1}^{A} \epsilon_i(\epsilon, \gamma, \omega = 0) = \frac{1}{2} \omega^2 J_{rig}(\epsilon, \gamma)
\]

(5.14)

Eq. (5.13) may be rewritten as

\[
R(\epsilon, \gamma; \omega, T = 0) = E_{LD}(\epsilon, \gamma, \omega = 0) - \frac{1}{2} J_{rig}(\epsilon, \gamma)\omega^2 - \delta S_{stru}(\epsilon, \gamma; \omega).
\]

(5.15)

The rigid body moment of inertia \( J_{rig} \) is valid at \( T = 0 \) with good accuracy. Here, the shell correction is written as

\[
\delta S_{stru}(\epsilon, \gamma; \omega) = \sum_{i=1}^{A} \epsilon_i(\epsilon, \gamma, \omega) - \sum_{i=1}^{A} \epsilon_i(\epsilon, \gamma, \omega).
\]

(5.16)
Routhian function given in Eqs. (5.13) and (5.15) is used for studying the rotation in cold nuclei.

The applicability of this method for nuclei at $T = 0$ can be extended to nuclei at nonzero temperatures because the heating always reduces the quantum effects and favors the quasiclassical ones.

### 5.2.3 Landau Theory of Phase Transition

Landau theory of phase transition [103] helps us to investigate the mean field shape evolution with temperature. This theory offers an economical and useful parameterization of the results of microscopic calculations and singles out a small number of combination of the parameters upon which the behaviour of the equilibrium shape depends. In this theory the most relevant results of any microscopic mean field theory of nuclear shape transition at finite $T$ and $\omega$ can be included. The free energy expression $F(T, \omega = 0, \epsilon, \gamma)$ at a given temperature $T$ and a quadrupole deformation defined by $\epsilon$ and $\gamma$ is expanded as

$$F = (T, \omega = 0, \epsilon, \gamma) = F_0 + F_2 \epsilon^2 + F_3 \epsilon^3 \cos 3\gamma + \ldots$$  \hspace{1cm} (5.17)

The expansion coefficients $F_0, F_2, F_3 \ldots$ described as temperature dependent Landau parameters are determined by least square fit with the free energy calculated by the Strutinsky method.

The free energy expression (5.17) depends upon the deformation parameters $\epsilon$ and $\gamma$. It also depends on the orientation angles relative to the rotation axis $\omega$ for
the rotating case \((\omega \neq 0)\). In the rotating case the free energy expression is

\[
F(T, \omega, \epsilon, \gamma) = F(T, \omega = 0, \epsilon, \gamma) - \frac{1}{2} J_{zz}(\epsilon, \gamma, T) \omega^2. \tag{5.18}
\]

The second term is related with the shape dependence rigid body moment of inertia.

The Landau constants are evaluated to study the shape evolution of the nuclei.

### 5.2.4 Relativistic Mean Field Theory

The Dirac equation for the nucleon in \(\sigma - \omega - \rho\) version of Relativistic Mean Field (RMF) theory [43] is

\[
[-i\alpha \cdot \nabla + V(r) + \beta(M + S(r))] \psi_i = \epsilon_i \psi_i, \tag{5.19}
\]

The vector potential \(V(r)\) is

\[
V(r) = g_\omega \omega_0(r) + g_\rho T_3 \rho_0(r) + e \left(1 - T_3 \right) A_0(r), \tag{5.20}
\]

and the scalar potential \(S(r)\) is

\[
S(r) = g_\sigma \sigma(r). \tag{5.21}
\]

The Klein-Gordon equation for the mesons and the electromagnetic fields are

\[
\{-\Delta + m_\sigma^2\} \sigma(r) = -g_\sigma \rho_3(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), \tag{5.22}
\]

\[
\{-\Delta + m_\omega^2\} \omega_0(r) = g_\omega \rho_0(r), \tag{5.23}
\]

\[
\{-\Delta + m_\rho^2\} \rho_0(r) = g_\rho \rho_3(r), \tag{5.24}
\]
\[-\Delta A_0(r) = e\rho_c(r),\]  \hspace{1cm} (5.25)

where \(\rho_x, \rho_y, \rho_3\) and \(\rho_c\) are the corresponding densities, neglecting the negative energy states. The occupational probability \(n_i\) at finite temperature in the constant pairing gap approximation (BCS) is

\[n_i = \frac{1}{2} \left[ 1 - \frac{\varepsilon_i - \lambda}{\varepsilon_i} \{1 - 2f(\varepsilon_i, T)} \right], \hspace{1cm} (5.26)\]

where \(f(\varepsilon_i, T)\) is the distribution function and \(\varepsilon_i\) is the single particle energy for the \(i^{th}\) state. The chemical potential \(\lambda\) for the protons (neutrons) is obtained from the requirement

\[\sum_i n_i = Z(N). \hspace{1cm} (5.27)\]

Free energy \(F = E - TS\) is minimized to find the equilibrium value of the quadrupole deformation \(\beta_0^0\) and the proton (neutron) pairing gaps \(\Delta_P(\Delta_N)\). The total energy

\[E(T) = \sum_i \varepsilon_i n_i + E_\sigma + E_{\sigma NL} + E_\omega + E_\rho + E_c + E_{\text{pair}} + E_{\text{cm}} - AM \hspace{1cm} (5.28)\]

where \(E_\sigma, E_{\sigma NL}, E_\omega, E_\rho, E_c\) etc., are having the usual meaning \([43]\).

Though calculations are not performed using the FTHBC, FTSM, Landau Theory and RMF theory in this work, for the sake of completeness these methods are outlined.

In the next section the results of investigations on the nuclei \(^{150}\text{Sm}, ^{152}\text{Gd}, ^{154}\text{Dy}, ^{156}\text{Er}, ^{166}\text{Er}, ^{168}\text{Yb}\) and \(^{188}\text{Hg}\) are presented and the shape transitions are predicted using statistical theory \([12-16]\). The role of deformation, spin and temperature
have been incorporated and it is found that each degrees of freedom contributes significantly in the determination of shape transition.

5.3 Determination of Structural Properties

To study the very high spins possible in the nucleus formed in collisions, the statistical theory of hot rotating nuclei [12-16] explained in Chapter 3 is employed here. The single particle level scheme for triaxially deformed harmonic oscillator described in Chapter 2 is used as an input to the partition function to generate spins for a given temperature. The shape transitions are investigated to obtain the following structural properties of the above nuclei.

1. Level density parameter,
2. Single neutron separation energy and
3. Single proton separation energy.

The level density parameter and single nucleon separation energy expressions in Chapter 4 are used here and the results are depicted in Figs. 5.1 - 5.20.

The shape evolution of $^{152}$Gd as a function of angular momentum $M$ for different temperatures is illustrated in Fig. 5.21. In Figs. 5.22 - 5.26, free energy surface curves are shown for $M = 0 \hbar, 20 \hbar, 40 \hbar, 50 \hbar$ and $60 \hbar$. The shell correction for this system is presented in Fig. 5.27. By extending the work of Ramamurthy et al. [1] for nonrotating to rotating nucleus, the shell correction as a function of angular momentum $M$, temperature $T$ and deformation degrees of freedom $(\epsilon, \gamma)$ is
extracted using the expression

\[ S^2 = 4a(M, T, \epsilon, \gamma)[E^*(M, T, \epsilon, \gamma) + \Delta E_{\text{shell}}], \]

(5.29)

where \( S \) is the entropy, \( E^* \) is the excitation energy and \( a \) is the level density parameter. The shell correction \( \Delta E_{\text{shell}} \) is the interception on the \( E^*(M, T, \epsilon, \gamma) \) axis when the large temperature values of \( S^2 \) are extrapolated towards low temperatures to cut the \( E^*(M, T, \epsilon, \gamma) \) axis. It is obvious from Fig. 5.2 that a linear behaviour of level density parameter is obtained at large temperatures.

5.4 Results and Discussion

Results are presented here for high spin hot nuclei \(^{150}\text{Sm}, \, ^{152}\text{Gd}, \, ^{154}\text{Dy}, \, ^{156}\text{Er}, \, ^{164}\text{Er},\)

\(^{168}\text{Yb} \) and \(^{188}\text{Hg} \) [16-19] using STHRN method [12-16]. Level density parameter without pairing correlations as a function of temperature and angular momentum for the nucleus \(^{150}\text{Sm} \) is presented in Fig. 5.1. Similar curves for the nuclei \(^{152}\text{Gd}, \, ^{154}\text{Dy}, \, ^{156}\text{Er}, \, ^{164}\text{Er}, \, ^{168}\text{Yb} \) and \(^{188}\text{Hg} \) are shown in Figs. 5.2 - 5.4. At zero spin the parameter \( a \) remains almost constant without any fluctuations. For a given temperature level density parameter is found to decrease with increasing spin. The fluctuations in level density parameter are different for different angular momentum states at low temperatures because the shell structure plays a major role at these temperatures. Another significant change in the level density parameter occurs when the temperature \( T > 1.0 \text{ MeV} \). At these temperatures the occupancies \( n_i \) within the active shells near the Fermi level becomes comparable. A change in \( T \) to
higher values would correspond to an appreciable contribution from higher shells. For temperature $T > 1.0$ MeV the level density parameter shows a linear behaviour for all the angular momentum considered. This behaviour of the nuclei is similar to the one obtained in Ref. [37].

In Figs. 5.5 - 5.8 results of the level density parameter as a function of angular momentum for various temperatures are displayed. These curves for various temperatures show minimum at specific spin values indicating a rearrangement of particle distribution near the Fermi level at the excitation energy considered. These minima are associated with the shape transition of the nucleus. The appearance of prominent minima for an angular momentum at $M = 50 \hbar$ for $^{150}$Sm, $^{152}$Gd and $^{154}$Dy and at $M = 55 \hbar$ for the remaining systems are interpreted as the signature for a shape transition from prolate collective to oblate noncollective. The fluctuations in the value of $a$ indicates a greater stability of the systems at high spin states on the basis of shell correction. Also, it is evident from the curves for different temperatures that the level density parameter $a$ decreases as the temperature decreases and as the angular momentum increases. A shape coexistence is predicted at high spins in $^{154}$Dy by Ma et al. [80] and a superdeformed band by Niusus et al. [80].

Figures 5.9 - 5.16 show how the single neutron and proton separation energies change with angular momentum at various temperatures for the nuclei $^{166}$Er, $^{168}$Yb, and $^{188}$Hg and also for the $N = 88$ systems such as $^{156}$Er, $^{150}$Sm, $^{152}$Gd and $^{154}$Dy. The $N = 88$ systems are lying between $N = 86$ and $N = 90$ isotones. The $N$
= 86 systems have a spherical ground state with angular momentum states built by nuclear alignments near the Fermi energy while systems with N = 90 have a prolate ground state configuration with high spin states developed by both collective rotation and nuclear alignment. Consequently, systems with N = 88 are expected to undergo shape changes as the angular momentum of the system increases beyond 30 h. In these figures the separation energy values are observed to decrease sharply for the angular momentum M = 50 h for 150Sm, 150Gd and 154Dy and for M = 55 h for the remaining systems. This sudden drop corresponds to a shape transition from prolate collective to oblate noncollective, -120° to -180° for all the above systems. It is obvious that in a rotating prolate system the Coriolis and centrifugal forces favour alignment of the individual nucleons with the rotation axis. With increasing angular momentum, the aligning nucleons polarize the nuclear potential resulting in axial symmetry about the rotational axis. The result is that the nuclei undergo a shape transition from prolate collective to oblate noncollective. The occurrence of such shape transition, a special feature of systems with N = 88 isotones has been reported by Rajasekaran et al. [15, 16] and many others [10, 44]. It is also observed that single neutron separation energy values increase rapidly with angular momentum at low temperatures. This behaviour is due to the presence of shell effects at low temperatures. Beyond 50 h the variations in the separation energy values disappear since the shell effects get washed out for T = 0.8, 1.0 and 1.2 MeV. The appearance of second peak at low temperature T = 0.4 MeV for 152Gd
and $^{154}$Dy is due to the change in the quadrupole deformation parameter $\epsilon$ in the oblate shape. At higher temperatures $T > 1.0$ MeV, the separation energy values are almost constant for angular momentum $M < 50 \hbar$.

In Figs. 5.17 - 5.20, variation of nucleon separation energy as a function of temperature is presented for various angular momenta $M$. The effects of rotation affects the separation energy values at low temperatures where shell effects play a very decisive role. At higher temperatures beyond $T > 0.8$ MeV these fluctuations disappear and the energy values become almost constant due to the absence of shell effects.

Figure 5.21 illustrates the hodograph of the deepest energy minimum of the nucleus $^{152}$Gd as a function of deformation parameters $\epsilon$ and $\gamma$ and angular momentum $M$ obtained for different temperatures 0.1, 0.3, 0.6 and 1.0 MeV. The equilibrium shape of the system is determined by minimizing the free energy with respect to deformation parameters $\epsilon$ and $\gamma$ at finite angular momentum $M$ and temperature $T$ and it is denoted by dot in this figure. It is observed from Fig. 5.21a ($T = 0.1$ MeV) that the nucleus is found to be prolate ($\epsilon = 0.2$ and $\gamma = -120^\circ$) for angular momentum range $M = 0$ to 20 $\hbar$, triaxial ($\epsilon = 0.3$ and $\gamma = -140^\circ$) for $M = 21 \hbar$ to 39 $\hbar$, again becomes prolate shape for $M = 40 \hbar$ to 49 $\hbar$ and finally reaches oblate shape with ($\epsilon = 0.3$ and $\gamma = -180^\circ$) for $M = 50 \hbar$ to 60 $\hbar$. The nucleus exhibits a similar behaviour for two different temperatures $T = 0.3$ MeV and $T = 0.6$ MeV and it is displayed in Figs. 5.21b and 5.21c. Figure 5.21d shows the behaviour of the
nucleus for $T = 1.0$ MeV. The nucleus remains at prolate shape (with $\epsilon = 0.3$ and $\gamma = -120^\circ$) for the angular momentum range $M = 0$ to $49 \hbar$ and becomes oblate shape (with $\epsilon = 0.1$ and $\gamma = -180^\circ$) for $M = 50 \hbar$ to $60 \hbar$.

The free energy surfaces of the nucleus as a function of angular momentum $M$ in $(\epsilon, \gamma)$ plane are drawn to look for a shape transition from prolate collective to oblate noncollective. Figures 5.22 - 5.26 show the shape evolution for $^{152}$Gd at spins $M = 0 \hbar, 20 \hbar, 40 \hbar, 50 \hbar$ and $60 \hbar$ for the temperature $T = 1.0$ MeV. The contour values are given in MeV and the minimum corresponds to zero. The gap between the contour lines is at the order of 1 MeV. These contour plots clearly show the shape evolution of the nucleus leading to a shape transition from collective prolate to noncollective oblate. For angular momentum range from $M = 0 \hbar$ to $40 \hbar$, Figs. 5.22 - 5.24 show the absolute minima corresponding to a collective prolate shape with deformation parameter $\epsilon = 0.3$ and shape parameter $\gamma = -120^\circ$. In Figs. 5.25 and 5.26 the absolute minima correspond to a noncollective oblate shape with deformation values $\epsilon = 0.1$ and $\gamma = -180^\circ$ for $M = 50$ to $60 \hbar$.

In Fig. 5.27 the shell correction as a function of angular momentum $M$ for $T = 0.4$ MeV is depicted for $^{152}$Gd. It is well known that the shell correction vanishes with increasing temperature for a fixed set of single particle energy levels. At the shape transition temperature due to reorganization of the single particle field, the shell correction energy is expected to be more strongly built up for deformed nuclei [43]. The variation of shell correction with angular momentum $M$ around $3 \hbar$ is due
to the change in the deformation parameter $\epsilon$ in the prolate shape. The variation around $M = 12 \hbar$ and minimum at $40 \hbar$ correspond to two consecutive backbending effects [36]. Such effects occur at non yrast line of rare earth nuclei [109].

To summarise, the statistical theory has been applied to understand the properties of some rare earth nuclei at finite temperature. Focus is made mainly on the temperature $T$ and angular momentum $M$ induced shape transition from prolate collective to oblate noncollective. Such kind of shape transitions for the nuclei $^{156}$Er, $^{166}$Er, $^{168}$Yb and $^{188}$Hg, around $M = 55 \hbar$ and for $^{150}$Sm, $^{152}$Gd and $^{154}$Dy around the angular momentum $M = 50 \hbar$ and $T > 0.6$ MeV are observed. The influence of temperature dependent effect, i.e., collapse of shell corrections has been observed around a temperature $T > 1.0$ MeV as is evident from the values of level density parameter. Single neutron and proton separation energies are very sensitive to the structural transitions i.e., collective prolate to noncollective oblate in high spin hot nuclear systems.
Figure Captions

Fig. 5.1 Level density parameter \( a \) as a function of temperature \( T \) for different angular momentum \( M \) for \(^{150}\text{Sm}\). The numbers on the curve refer to \( M \) in units of \( \hbar \).

Fig. 5.2 Same as in Fig. 5.1 for \(^{152}\text{Gd}\).

Fig. 5.3 Same as in Fig. 5.1 for \(^{154}\text{Dy}\).

Fig. 5.4 Same as Fig. 5.1 for \(^{156}\text{Er},^{166}\text{Er},^{168}\text{Yb} \) and \(^{188}\text{Hg}\).

Fig. 5.5 Level density parameter \( a \) as a function of angular momentum \( M \) for different temperatures \( T \) for \(^{150}\text{Sm}\). The numbers on the curve refer to \( T \) in MeV.

Fig. 5.6 Same as Fig. 5.5 for \(^{152}\text{Gd}\).

Fig. 5.7 Same as Fig. 5.5 for \(^{154}\text{Dy}\).

Fig. 5.8 Same as Fig. 5.5 for \(^{156}\text{Er},^{166}\text{Er},^{168}\text{Yb} \) and \(^{188}\text{Hg}\).

Fig. 5.9 Single neutron separation energy \( S_n \) with angular momentum \( M \) and temperature \( T \) for \(^{150}\text{Sm}\). The numbers on the curve refer to temperature \( T \) in MeV.

Fig. 5.10 Same as Fig. 5.9 for \(^{152}\text{Gd}\).

Fig. 5.11 Same as Fig. 5.9 for \(^{154}\text{Dy}\).

Fig. 5.12 Same as Fig. 5.9 for \(^{156}\text{Er},^{166}\text{Er},^{168}\text{Yb} \) and \(^{188}\text{Hg}\).
Fig. 5.13 Single proton separation energy $S_p$ with angular momentum $M$ and temperature $T$ for $^{150}$Sm. The numbers on the curve refer to temperature $T$ in MeV.

Fig. 5.14 Same as Fig. 5.13 $^{152}$Gd.

Fig. 5.15 Same as Fig. 5.13 $^{154}$Dy.

Fig. 5.16 Same as Fig. 5.13 $^{156}$Er, $^{166}$Er, $^{168}$Yb and $^{188}$Hg.

Fig. 5.17 Single neutron separation energy $S_n$ versus temperature $T$ for different angular momentum $M$ for $^{152}$Gd.

Fig. 5.18 Same as Fig. 5.17 $^{154}$Dy.

Fig. 5.19 Single proton separation energy $S_p$ versus temperature $T$ for different angular momentum $M$ for $^{152}$Gd.

Fig. 5.20 Same as Fig. 5.19 $^{154}$Dy.

Fig. 5.21 The shape evolution of $^{152}$Gd as a function of angular momentum $M$ for different temperatures $T$. The dot refers to the free energy minimized with respect to deformation parameters and for a particular range of angular momentum $M$. The numbers within the parenthesis denote $M$ in units of $\hbar$.

Fig. 5.22 Free energy surfaces for the nucleus $^{152}$Gd with $M = 0$ $\hbar$ in ($\epsilon, \gamma$) plane. The contour values are given in MeV and the minimum corresponds to zero. The separation between two consecutive contours is 1.0 MeV.
Fig. 5.23 Same as Fig. 5.22 M = 20 \hbar.

Fig. 5.24 Same as Fig. 5.22 M = 40 \hbar.

Fig. 5.25 Same as Fig. 5.22 M = 50 \hbar.

Fig. 5.26 Same as Fig. 5.22 M = 60 \hbar.

Fig. 5.27 Shell correction $\Delta E_{\text{shell}}$ as a function of angular momentum $M$ for $^{152}$Gd.
Level density parameter \( a \) (MeV\(^{-1}\))

Temperature \( T \) (MeV)

Fig. 5.1

\( ^{150}\text{Sm} \)
Fig. 5.2

Temperature $T$ (MeV)

Level density parameter $a$ (MeV$^{-1}$)

$^{152}$Gd
Fig. 5.3
Fig. 5.4

Temperature $T$ (MeV)

Level density parameter $a$ (MeV$^{-1}$)

- $^{188}$Hg
- $^{168}$Yb
- $^{166}$Er
- $^{156}$Er
Fig. 5.5

Level density parameter $a$ (MeV$^{-1}$) vs. Angular momentum $M$ ($\hbar$) for $^{150}$Sm.
Fig. 5.6
Fig. 5.7
Fig. 5.8
Fig. 5.9
Fig. 5.10

Angular momentum $M(\hbar)$

Single neutron separation energy $S_n$ (MeV)
Fig. 5.11

Single neutron separation energy $S_n$ (MeV) vs. Angular momentum $M (\hbar)$ for $^{154}$Dy.
Fig. 5.12

Angular momentum $M (\hbar)$

Single neutron separation energy $S_n$ (MeV)
Fig. 5.13
Fig. 5.14
Fig. 5.15
Fig. 5.16

Angular momentum $M$ ($\hbar$)

Single proton separation energy $S_p$ (MeV)

- $^{168}$Hg
- $^{168}$Yb
- $^{166}$Er
- $^{156}$Er
Fig. 5.17

Single neutron separation energy $S_n$ (MeV)

Temperature $T$ (MeV)
Fig. 5.18
Fig. 5.19

Single proton separation energy $S_p$ (MeV)

Temperature $T$ (MeV)

$^{152}$Gd
Fig. 5.20

Temperature T (MeV)

Single proton separation energy $S_p$ (MeV)

$^{154}$Dy
Fig. 5.21
Fig. 5.22
Fig. 5.23
Fig. 5.24
Fig. 5.25

\[ M = 50 \hbar \]

\[ \gamma = -120^\circ \]

\[ \gamma = -180^\circ \]

\[ \varepsilon \sin \gamma \]

\[ \varepsilon \cos \gamma \]

\[ ^{152}\text{Gd} \]
Fig. 5.26

\[ M = 60 \hbar \]

\[ \gamma = -120^\circ \]

\[ \gamma = -180^\circ \]
Fig. 5.27

Angular momentum $M (\hbar)$

Shell correction $\Delta E_{\text{shell}}$ (MeV)

$T = 0.4$ MeV

$^{152}\text{Gd}$