CHAPTER III

VELOCITY - DEPTH DISTRIBUTION

3.1 Computation of P Wave Velocity :

There are several mothods of inverting a travel time curve to get a velocity distribution at different depths which have been adopted by various workers since the begining of seismology.

Dowling and Nuttli (1964), studying for a low velocity channel in the upper mantle, used a modification of the following formulas (Bullen, 1963) -

$$\Delta = 2 \int_{r_m}^{r_o} \frac{p}{r} \frac{dr}{\sqrt{\{(r/v)^2 - p^2\}}} \qquad 3.1a$$

and

$$T = 2 \int_{r_m}^{r_o} \frac{r}{v^2} \frac{dr}{\sqrt{\{(r/v)^2 - p^2\}}} \qquad 3.1b$$

Considering the velocity function v(r) in some constant steps, i.e. considering the earth to be consisted of some concentric spherical shells within which the ray path remains straight, the expression for the epicentral distance and travel time as given in the set of equations 3.1 above were approximated by them as a summation of two series as given below.

$$\Delta = 2 \sum \frac{p}{r_i} \frac{(r_i - r_{i+1})}{\sqrt{(r_i/v_i)^2 - p^2}} \qquad 3.2a$$

and

$$T = 2 \sum \frac{r_i}{v_i^2} \frac{(r_i - r_{i+1})}{\sqrt{(r\ /v\ )^2 - p^2}} \qquad 3.2b$$

fixing the ray parameter p by determining the specific angle of incidence, i_f at the focus by the relation,

$$p = (r_f/v_f) \sin i_f \qquad 3.3$$

Taking comperatively a larger number of layers, and assigning values of velocity from a simple velocity model and adjusting them particularly for suspected anomalous velocity regions, they could identify the possible exitence of a low velocity channel from the shadow zones as evidenced from their calculated $T - \Delta$ curves.

Carder (1964) in his study of nuclear explosions in the Central Pacific determined P wave velocities using the basic principles of ray theory, within the epicentral distance range of $3^\circ - 102^\circ$ and assuming the mantle to be divided in eight 'constant velocity' concentric layers on the basis of straight line segments fitted to his $T - \Delta$ curve. Kaila et al. (1968b), Biswas and Bhattacharyya (1974) also, among others, adopted similar techniques with some modifications for studying P wave velocity structure in the upper mantle under Indian sub-continent and near Alaska respectively.

Basanova et al. (1974; 1976) used the tou - p method for inversion of the travel time taking into account the delay time function $\tau(p)$ along with the consideration of the apparent slowness characterising a velocity structure of a strictly vertically heterogeneous medium, later on used again in a geometrically modified form by Germany and Orcuit (1979). Monte-Carlo method of inversion of (T, Δ) data is also used by many (Press,1968; 1970; Wiggins,1969) for similar purposes.

The methods have their own advantages, specially when applied to a mixture of data, but for the continuous inversion of the travel time data alone, they appear to be unnecessarily complicated. Comparison between various methods have also been made (Wiggins et al.,1973), although it is not obvious how good the comparisons should be, "as methods other than Herglotz - Wiecherdt inversion gives averaged uncertainties only"(Davis and Chapman, 1975).

3.2 Herglotz - Wiechert - Bateman Integral Method :

This integral method is chiefly a refraction method following a principle of optical refraction technique and transformed from apparent surface velocity from the travel time data to a velocity depth inversion by graphical integration method (Byerly, 1956).

Hamilton's equation for wave propagation is,

$$\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 + \left(\frac{\partial T}{\partial z}\right)^2 = \frac{1}{v^2} \qquad 3.4$$

If the speed v is a function of z alone the rays with each lie in a vertical plane containing the source and the observation point, one can take X axis in this direction $\partial T/\partial x$ becomes the gradient of scalar time field created by the wave arrived at the Earth's surface. (Throughout this treatment, v will denote the true velocity and V will denote apparent velocity.)

Equation 3.4 can now be written as,

$$\left(\frac{\partial T}{\partial x}\right)^2 = \frac{1}{v^2} - \left(\frac{\partial T}{\partial z}\right)^2 \qquad 3.5$$

A solution to the above equation may be obtained by setting p ($= \partial T/\partial x$) as the ray parameter.

Equation 3.5 is to be used for the areas of the Earth's surface small enough to neglect its curvature. In this case from Snell's - Descarte's law,

$$p = \frac{\sin i}{v} = \frac{\sin i_o}{v_o} = \frac{1}{V} \qquad 3.6$$

where V (= 1/p) is the apparent speed at the surface, where the rays emerge and is obtained from the slope of the travel time curve. The angle i is between the ray and the vertical and the suffix (o) denotes the parameters at the surface of the Earth.

Now applying equation 3.5 for the larger areas on the surface of the Earth and changing to polar co-ordinates with the centre of the Earth as the origin, the equation can be written as,

$$\frac{r^2}{v^2} - r^2\left(\frac{\partial T}{\partial r}\right)^2 = \left(\frac{\partial T}{\partial \theta}\right)^2 \qquad 3.7$$

Here it is assumed that v is a function of r alone and the solution is obtained by setting p $(= \partial T/\partial\theta)$. It may then be shown that,

$$p = \frac{r \sin i}{v} = \frac{r_o \sin i_o}{v_o} = \frac{r_o}{V} \qquad 3.8$$

where i is the angle made by the ray with the vertical, i.e the radius of the Earth.

Tt follows then,

$$dT = \frac{\partial T}{\partial \theta} d\theta + \frac{\partial T}{\partial r} dr \qquad 3.9$$

From 3.7, using p,

$$T = \int p \, d\theta \pm \int \frac{dr}{r} \sqrt{(r/v)^2 - p^2} \qquad 3.10$$

To get the equation of the ray,

$$\frac{\partial T}{\partial p} = \int d\theta \mp \int \frac{p\,dr}{r\sqrt{(r/v)^2 - p^2}} \qquad 3.11$$

one may now integrate from the bottom of the ray (ref. to the Figures10a and 10b) to the surface of the Earth and obtain,

$$\Delta = 2 \int_{r}^{r_o} \frac{p}{r} \frac{dr}{\sqrt{(r/v)^2 - p^2}} \qquad 3.12$$

The difficulty in equation 3.12 is that , since it is desired to get v as a function of r, the integration cannot be performed along the ray, unless it is known which function will give v as a function of r. The travel time gives p as a function of Δ ($\equiv \theta$).

Herglotz in 1907, using Abel's transformation (Macelwane,1932; Byerly,1942) transformed the relation 3.12 to,

$$\ln (r_o/r_m) = \frac{1}{\pi} \int_{o}^{\Delta_m} \cosh^{-1} (V_m/V)\, d\Delta \qquad 3.13a$$

$$= \frac{1}{\pi} \int_{o}^{\Delta_m} \cosh^{-1} (p/p_m)\, d\Delta \qquad 3.13b$$

Now the integration is no longer along a ray as in equation 3.12 but along the Earth's surface; that is along the travel time curve (ref. to Figures 11a and 11b).

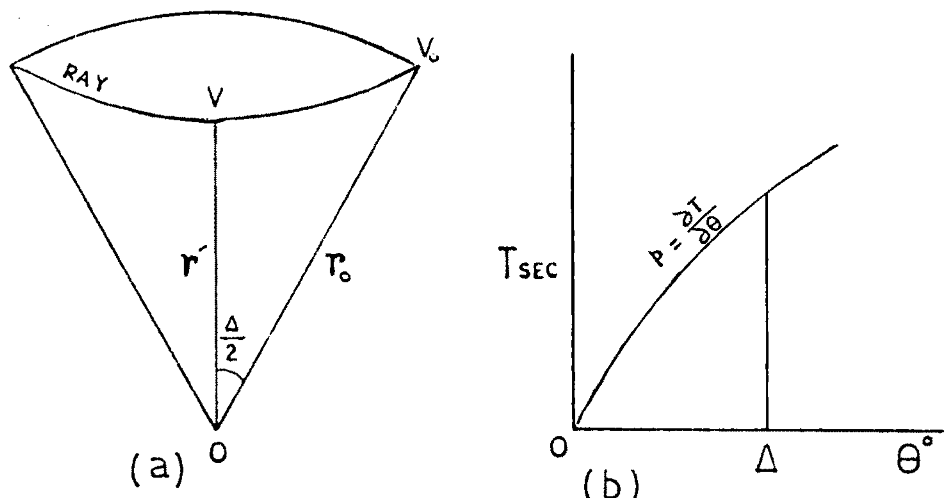

Figure 10

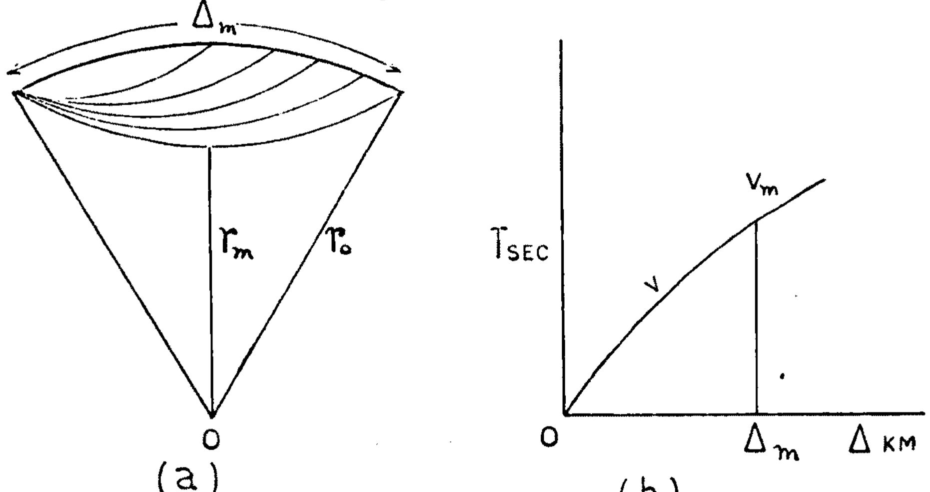

Figure 11

FIGURE 10 : (a) Ray diagram & (b) corresponding travel time curve

FIGURE 11 : (a) Ray diagram & (b) travel time curve illustrating Herglotz transformation

The apparent surface velocity V ($\equiv 1/p$) is obtained from the slope of the travel time curve as a function of arc distance. Thus the equation 3.13b can be integrated graphically or numerically by selecting a value of p_m (or V_m) corresponding to a certain Δ_m and the integration is done from the surface of the Earth to evaluate the corresponding r_m, the vertex at which the ray with the ray parameter p_{v_m} bottoms. At this corresponding depth the velocity of the wave is,

$$v_m = \frac{r_m}{r_o} \cdot V_m \qquad 3.14$$

The trouble in Herglotz method is that at the end of each integration, velocity at one depth only is obtained. To calculate the velocity again at a different depth the Earth is to be stripped and the epicentral distance Δ is to be properly reduced to bring the rays to the next selected depth again to a single point on the new surface of the stripped Earth.

This method is again applicable only when the following conditions are satisfied during the process of integration, otherwise the method fails as the argument of the integration becomes imaginary. The conditions are (Macelwane, 1932),

(i) dv/dr is -ve (velocity increases with depth) and

(ii) dv/dr is +ve but numerically less than v/r.

If at a certain level this criterion is not met and if the extent of the zone is comparatively not very wide, it is customary to assume that the region is a discontinuity for all practical purposes and the integration is performed upto that level and then the Earth is stripped to start the integration again to a new depth after proper reduction of the parameters involved in the integration.

3.3 Stripping of the Earth and Reduction of Parameters :

In the present study since the $p - \Delta$ profiles are fitted with polynomial segments in-between the assumed discontinuity levels the relation 3.13 and 3.14 are used in the following form for a selected level j .

$$\ln r_j = \ln r - \frac{1}{\pi} \int_0^{\Delta_j} \cosh^{-1} (p_{\Delta} / p_j) d\Delta_j \quad 3.15$$

and $$v_j = r_j / (r_o p_j) \quad 3.16$$

Here, r_j = radius of the Earth at the vertex of the ray bottoming at a selected depth,

r = radius of the stripped Earth

p_j = ray parameter of the ray emerging at the selected epicentral distance Δ_j

$p_{\Delta'}$ = ray parameter of the ray emerging at an epicentral distance Δ' within $0 - \Delta_j$

and, r_0 = radius of the Earth.

Now referring to Figure 12, let

$\bar{v}_\Delta$ = true velocity at the surface of the Earth corresponding to arc distance Δ measured at the surface of the Earth ($R = r_0$),

$\bar{v}'$ = true velocity at the surface of the stripped Earth,

v_r' = true velocity at the vertex of the ray($R=r_m$)

V' = apparent velocity from $T - \Delta$ curve of a ray

Now from the figure - using Snell's law,

$$\frac{\sin\theta}{\sin\phi} = \frac{\bar{v}_r}{\bar{v}'} \qquad 3.17$$

and also,

$$\sin\theta = \frac{\bar{v}_r'}{V'} \quad \text{(Macelwane, 1932)} \qquad 3.18$$

Hence from 3.17 and 3.18

$$\sin\phi = (\bar{v}'/\bar{v}_r')\sin\theta$$

$$= \bar{v}'/V'$$

$$= \bar{v}' \cdot p_\Delta \qquad 3.19$$

where, $p_\Delta = 1/V' = \frac{180}{\pi r_0}\frac{dt}{d\Delta}$ sec/Km

Hence, $$\phi = \sin^{-1}(C_1 \cdot p_\Delta) \qquad 3.20$$

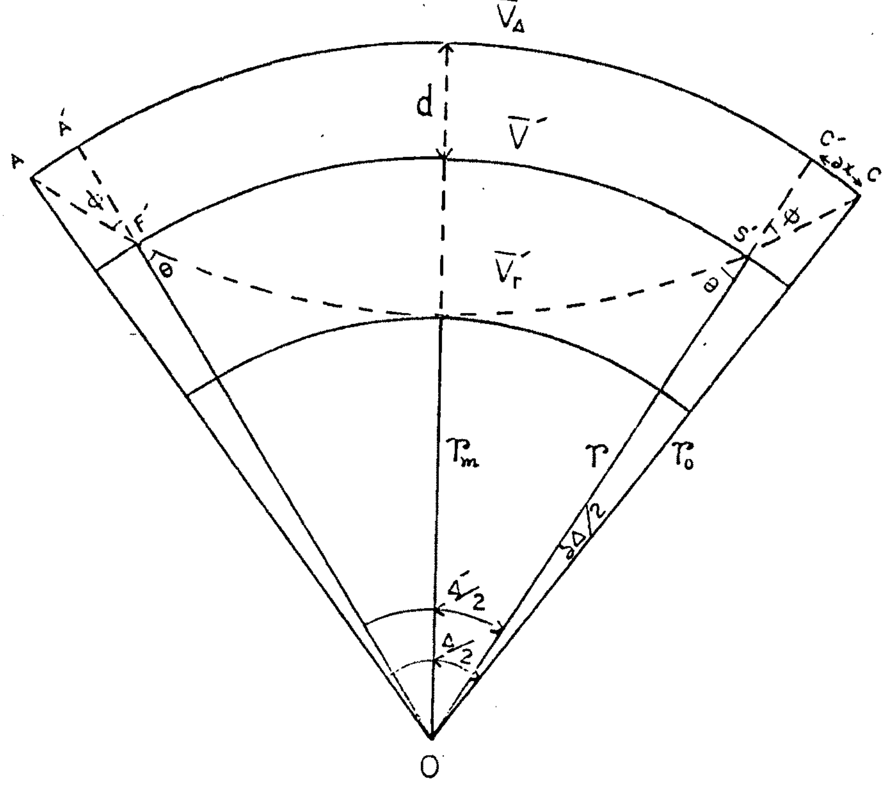

FIGURE 12 : Stripping of the Earth and reduction of parameters.

where, $C_1 = (180/\pi r_o) . \bar{v}'$, is a constant for a particular layer selected after fixing the value of $\bar{v}'$ for that layer (Appendix III).

If the upper layer is stripped the ray is assumed to originate at F' and emegge at S' on the surface of the stripped Earth covering a reduced epicentral distance Δ' corresponding to the surface of the Earth. From the Figure 12,

$$\Delta' = \Delta - 2\partial\Delta / 2 \qquad 3.21$$

It is clear from the figure that on the surface of the Earth this distance is

$$A'C' = AC - 2\partial x$$

Now, $\partial x = d \tan \phi = r_o \partial \Delta /2$ 3.22

Hence $\partial \Delta = 2 d \tan \phi / r_o$

$$= (360/\pi r_o)\ d \tan \phi$$

$$= C_2 . \tan \phi \qquad 3.23$$

Here, $C_2 = (360/\pi r_o).d$ = a constant for the assumed layer at depth d from the surface.

If Δ'_i (i = 1,2,3, ...) be the reduced arc distances on the surface of the stripped Earth corresponding to the arc distances Δ_i on the surface of the true Earth then the corresponding decrements are,

$$\partial\Delta_i = \Delta_i - \Delta'_i \qquad 3.24$$

Then the increments of $\triangle$ along the surface of the Earth will be

$$d \triangle_j = \partial \triangle_i - \partial \triangle_{i-1} \qquad 3.25$$

For computing the velocity distribution with depth, the integration in equation 3.15 has been made by using trapizoidal rule and by using the relations 3.16 to 3.25 in a computer programme

Since in this study the velocity distribution is restricted to the lower part of the mantle below the maximum depth of penetration of a ray emerging at about 30^{o} epicentral distance a standard (Herrin, 1968) P wave velocity model has been used for calculation of velocity v (Appendix IIIA) for the upper part of the mantle stripped at 750 Km depth initialy to start the integration and reduction of $\triangle$ and evaluation of the constants.

Here quite a few strippings had to be undertakin to compute the velocity - depth distribution bounded by the anomalous zones in each profile and in all the cases the upper part of the Earth had been stripped and the average velocity was calculated (Appendix III); the starting level being taken from the end point of the p - $\triangle$ profile (segment) of the previous upper layer.

The velocities within each segment have been calculated at an interval of 0.5^{o} arc distance and finally the average velocity is calculated using the raw Herglotz - Wiechert values for each profile, by polynomial method of interpolation at 50 Km depth interval. The computed Herglotz - Wiechert values and the interpolated average velocity - depth distributions for all the profiles are shown in Figures 13a and 13b. Herrin's (1968) standard surface focus average velocity distribution is also shown in each profile for comparison. The values of the interpolated average velocity - depth distribution at 50Km depth interval are presented in Table V.

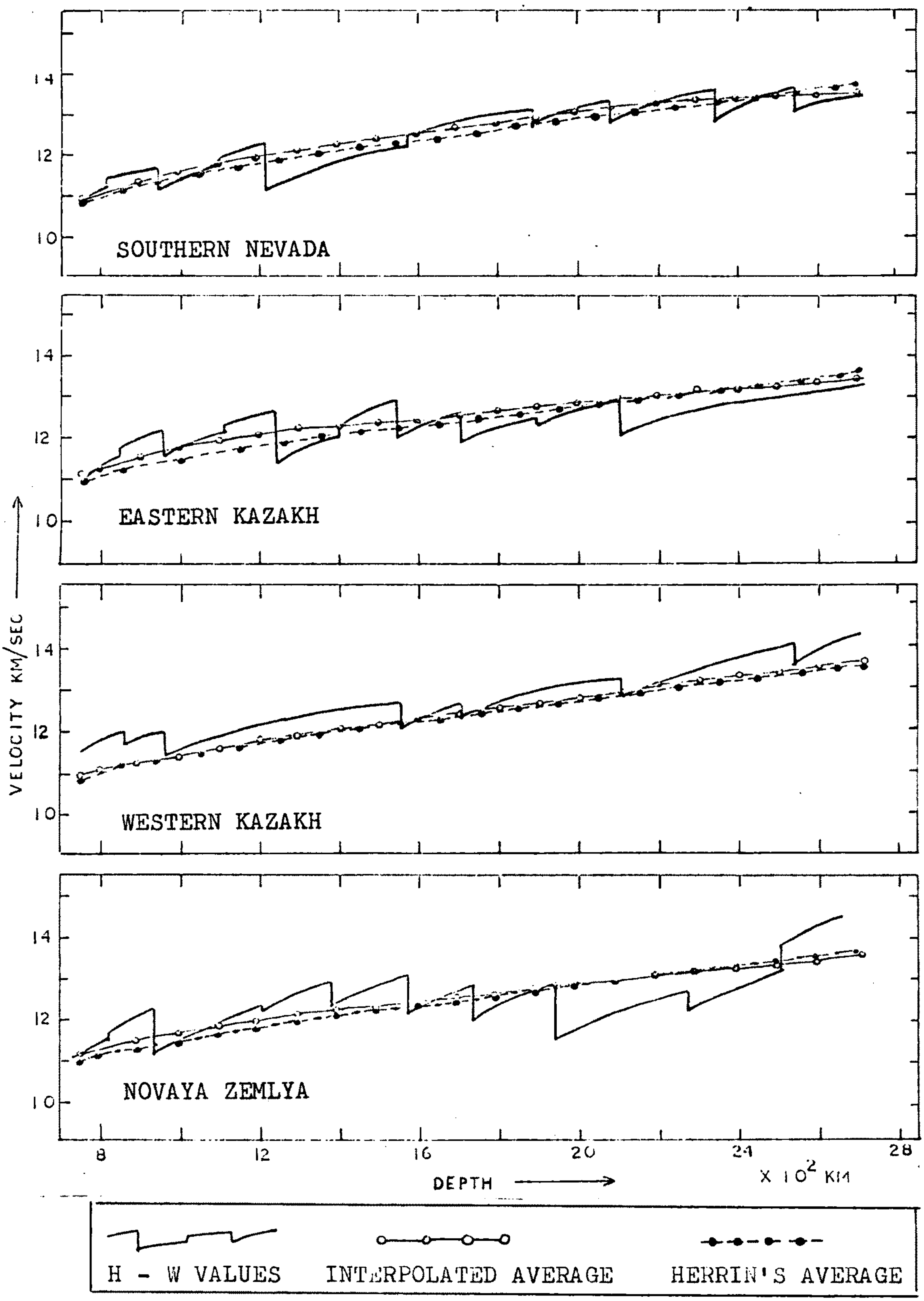

FIGURE 13a : Velocity - depth distribution for the nuclear explosion profiles, compared with Herrin's(1968) values for surface focus event.

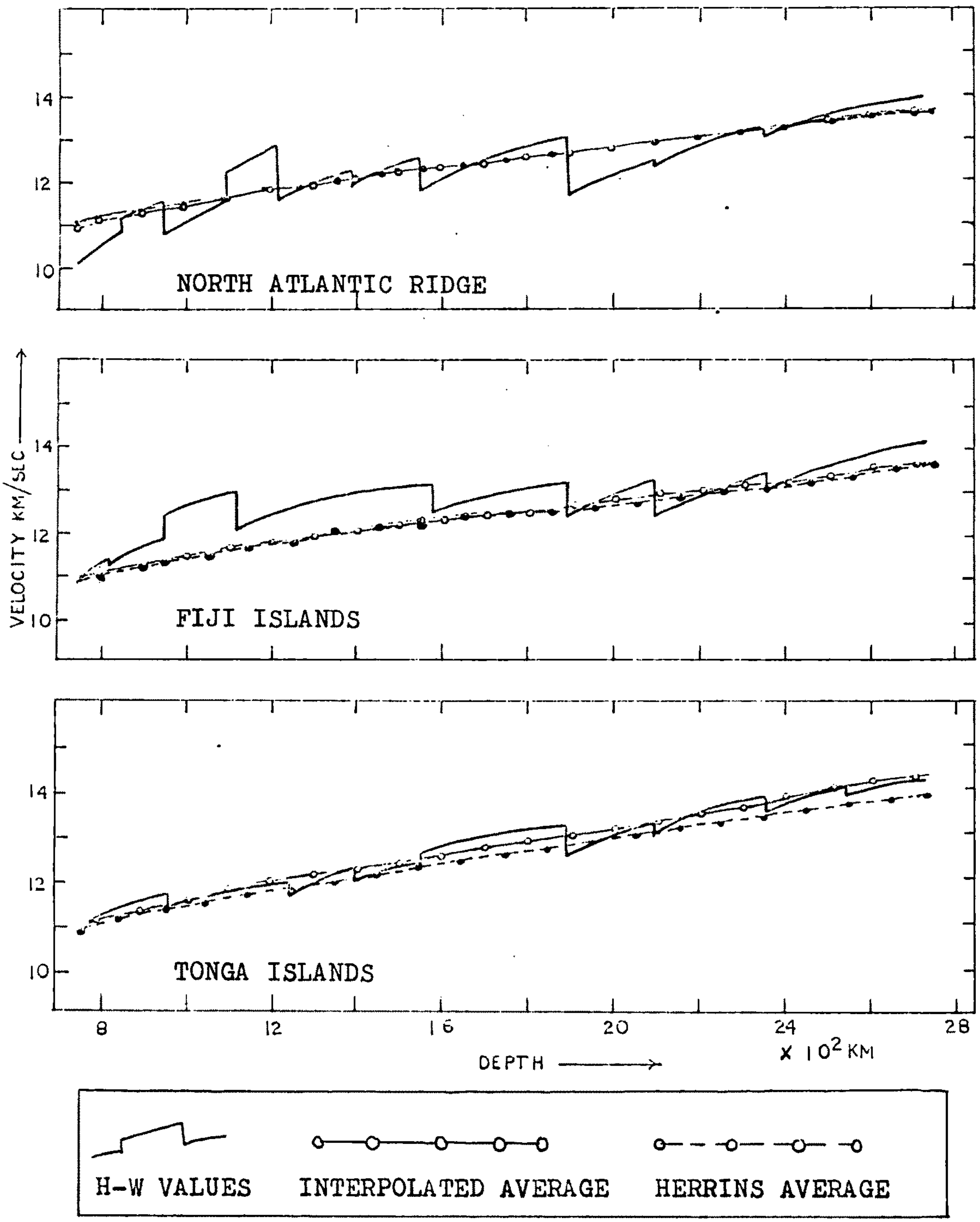

FIGURE 13b : Velocity - depth distribution for the earthquake profiles, compared with Herrin's (1968) values for surface focus event.

TABLE V : **Average velocity - depth distribution from Herglotz - Wiechert values for each profile**

Depth Km	P wave velocity in Km/sec. for -						
	Southern Nevada	Eastern Kazakh	Western Kazakh	Novaya Zemlya	North Atlantic Ridge	Fiji Islands	Tonga Islands
1	2	3	4	5	6	7	8
750	11.0545	11.3271	11.0570	11.0860	10.9752	10.9827	11.0923
800	11.1695	11.4612	11.1498	11.3847	11.0586	11.0666	11.1804
850	11.2810	11.5823	11.2402	11.6471	11.1411	11.1496	11.2674
900	11.3895	11.6512	11.3282	11.7642	11.2227	11.2617	11.3545
950	11.4949	11.7886	11.4140	12.0642	11.3034	11.3829	11.4386
1000	11.5973	11.8751	11.4976	12.2243	11.3832	11.4332	11.5328
1050	11.6968	11.9515	11.5791	12.3551	11.4621	11.4926	11.6063
1100	11.7935	12.0183	11.6585	12.4588	11.5400	11.5810	11.6890
1150	11.8863	12.0764	11.7361	12.5377	11.6171	11.6586	11.7710
1200	11.9776	12.1264	11.8117	12.5941	11.6932	11.7152	11.8525
1250	12.0654	12.1690	11.8851	12.6301	11.7684	11.8009	11.9335

continued ...

TABLE V : continued.

1	2	3	4	5	6	7	8
1300	12.1505	12.2049	11.9578	12.6481	11.8428	11.8558	12.0141
1350	12.2327	12.2347	12.0284	12.6502	11.9162	11.9497	12.0942
1400	12.3122	12.2592	12.0974	12.6387	11.9887	12.0827	12.1741
1450	12.3890	12.2790	12.1650	12.6158	12.0603	12.1648	12.2537
1500	12.4630	12.2949	12.2313	12.5838	12.1310	12.1960	12.3332
1550	12.5344	12.3075	12.2962	12.5449	12.2007	12.2162	12.4126
1600	12.6031	12.3175	12.3600	12.5014	12.2696	12.2856	12.4920
1650	12.6692	12.3256	12.4226	12.4555	12.3375	12.3540	12.5714
1700	12.7328	12.3325	12.4841	12.4094	12.4046	12.4216	12.6509
1750	12.7938	12.3388	12.5447	12.3654	12.4707	12.4882	12.7307
1800	12.8522	12.3454	12.6045	12.3257	12.5360	12.5540	12.8107
1850	12.9082	12.3527	12.6634	12.2926	12.6003	12.6188	12.8910
1900	12.9617	12.3617	12.7216	12.2683	12.6637	12.6827	12.9717
1950	13.0128	12.3728	12.7792	12.2551	12.7262	12.7457	13.0529
2000	13.0614	12.3869	12.8362	12.2551	12.7878	12.8078	13.1347
2050	13.1077	12.4045	12.8927	12.2707	12.8485	12.8690	13.2171
2100	13.1516	12.4149	12.9489	12.3041	12.9083	12.9293	13.3001

continued ...

TABLE V : continued

1	2	3	4	5	6	7	8
2150	13.1933	12.4393	13.0047	12.3575	12.9671	12.9886	13.3870
2200	13.2326	12.4861	13.0604	12.4331	13.0251	13.0471	13.4686
2250	13.2697	12.5250	13.1158	12.5332	13.0821	13.1046	13.5113
2300	13.3045	12.5711	13.1712	12.6601	13.1383	13.1613	13.6407
2350	13.3372	12.6248	13.2267	12.8160	13.1935	13.2170	13.7282
2400	13.3677	12.6870	13.2822	13.0031	13.2478	13.2718	13.8169
2450	13.3960	12.7583	13.3379	13.2236	13.3012	13.3257	13.9067
2500	13.4223	12.8394	13.3938	13.4799	13.3538	13.3788	13.9978
2550	13.4464	12.9310	13.4501	13.4840	13.4050	13.4308	14.0902
2600	13.4685	13.0338	13.5068	13.5484	13.4560	13.4820	14.1170
2650	13.4886	13.1484	13.5641	13.5952	13.5058	13.5323	14.1380
2700	13.5067	13.2756	13.6219	13.6568	13.5547	13.5817	14.1640
2750	13.5530	13.4160	13.6804	13.6971	13.6026	13.6301	14.1810