APPENDIX III

A. Calculation of the Average Velocity in a Layer (Method I):

Reffering to Figure A2, let a ray AB travelling at an average velocity $\bar{v}$ meet a layer at B of higher velocity $\bar{v}'$ and get totally reflected to emerge at C on the surface of the Earth. Let T be the travel time of the ray and Δ the epicentral distance. From the geometry of the figure, AC = RΔ. Let d be the depth of B from the surface of the Earth. Then,

$$\tan\theta = \frac{AN}{NB} = \frac{R\sin(\Delta/2)}{R\cos(\Delta/2) - r'}$$

$$= \frac{R\sin(\Delta/2)}{R\cos(\Delta/2) - (R - d)}$$

Again $\quad AB = AN \operatorname{cosec}\theta = R\sin(\Delta/2).\operatorname{cosec}\theta$

$$\therefore \quad \bar{v} = \frac{2AB}{T} \quad \frac{2R\sin(\Delta/2).\operatorname{cosec}\theta}{T} \qquad 1.$$

B. Calculation of the Average Velocity, Path Length and Transit Time of a Ray in Layered Earth :

Reffering to the Figure A3 let a ray leave its source S and travel through the Earth in time T and emerge

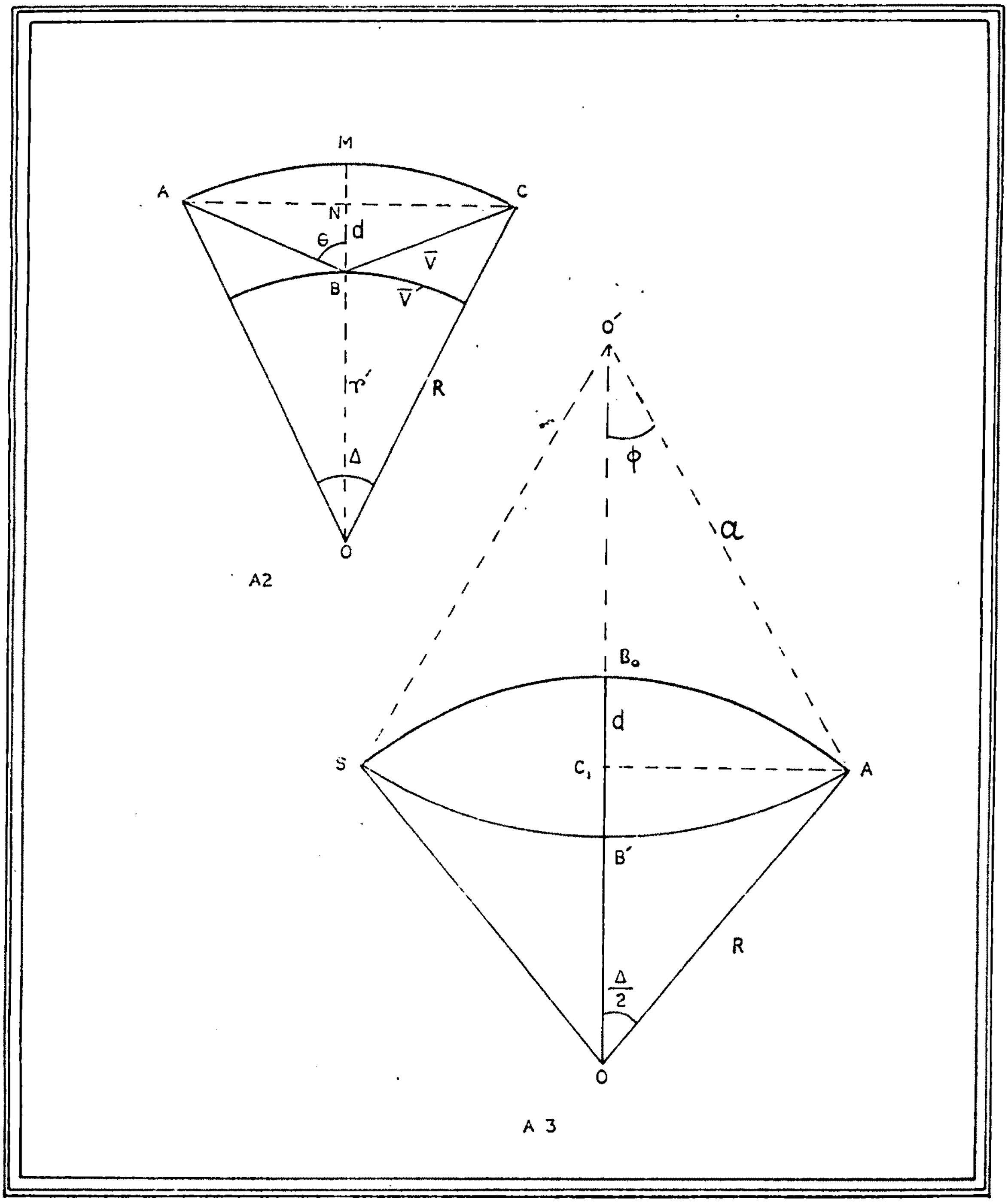

FIGURE A2 : Calculation of average velocity .

FIGURE A3 : Calculation of average velocity, path length and transit time of a ray.

at a point A on the surface of the Earth at an epicentral distance .

Now the velocity function for producing a circular ray is given as,

$$c = a - br^2 \qquad \text{(Officer, 1972, p 233)}$$

or $$\frac{dc}{dr} = -2b$$

$$= \text{a constant.}$$

(i) Calculation of average velocity (Method II) :

Assuming the above relation to be valid let a ray SB'A passing through a layer have depth of penetration d (Figure A3). If O' be the centre of curvature of the ray path, then the radius of the path is,

$$a = O'A = O'S = O'B'$$

Now from the geometry of the figure,

$$B_oC_1 = OB_o - OC_1$$

$$= R\left[1 - \cos(\Delta/2)\right] \qquad 2.$$

$$AC_1^2 = (2a - C_1B')C_1B' \qquad 3.$$

and $$C_1B' = d - B_oC_1 \qquad 4.$$

Now from equations 3 and 4,

$$R^2\sin^2(\Delta/2) = (2a - d + B_oC_1)(d - B_oC_1)$$

or $$2a(d - B_oC_1) = R^2\sin^2(\Delta/2) + (d - B_oC_1)^2$$

Now using 2. and simplifying,

$$2a\left[d - R\{1-\cos(\Delta/2)\}\right] = 2R(R-d)\left[1-\cos(\Delta/2)\right] + d^2$$

∴ the radius of the ray path,

$$a = \frac{1}{2}\left[\frac{2R(R-d)\{1-\cos(\Delta/2)\} + d^2}{d - R\{1 - \cos(\Delta/2)\}}\right] \qquad 5.$$

If the ray now subtends an angle 2ϕ (rad) at its centre, then the length of the ray path is given as,

$$L = 2a\phi \qquad 6.$$

and the average velocity for the entire layer will be,

$$\bar{v} = \frac{2a\phi}{T} \qquad 7.$$

(ii) <u>Path length in a layer</u> :

Before using the geometrical calculations given above in a multilayered Earth, let the superscript to a symbol denote the ray number and the subscript the layer number the ray passes through. (No superscript is used in case of the radius of the ray and its depth of penetration d, where a subscript denotes the ray number only).

Now for a ray passing through n layers of the Earth, penetrating to a depth d_n (Figure A4) the radius of the nth ray can be given from equation 5 as,

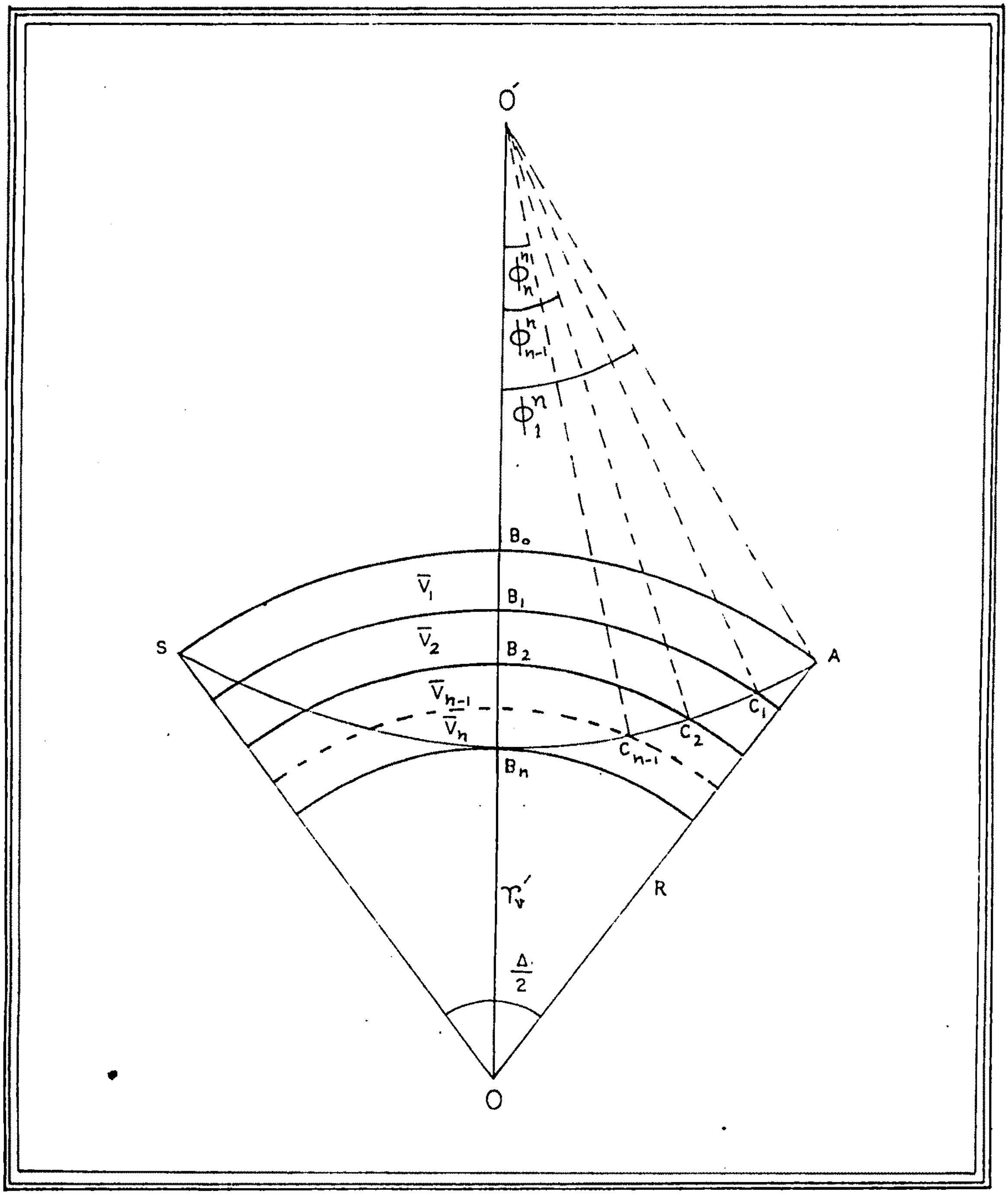

FIGURE A4 : Calculation of average velocity, transit time and path length in a multilayered Earth.

$$a_n = \frac{1}{2}\left[\frac{2R(R - d_n)\left\{1 - \cos(\Delta_n/2)\right\} + d_n^2}{d_n - R\left\{1 - \cos(\Delta_n/2)\right\}}\right] \qquad 8.$$

Reffering to the Figure A4, $OO' = A = R + a_n - d_n$

where, $B_oB_n = d_n$, $B_oB_{n-1} = d_{n-1}$ etc.

If $2\phi_n^n$ be the angle subtended at the centre O' by the segment of the ray within the nth layer, then -

$$\cos\phi_n^n = \frac{a_n^2 + A^2 - (R - d_{n-1})^2}{2a_nA} \qquad 9.$$

$$= 1 - \frac{(d_n - d_{n-1})\, 2R - (d_n + d_{n-1})}{2a_n(a_n + R - d_n)}$$

$$= X_n^n \quad \text{(say)} \qquad 10.$$

$$\therefore \qquad \phi_n^n(\text{rad}) = \frac{\pi}{180}\cos X_n^n \qquad 11.$$

Hence the length of the arc within the nth layer is given as,

$$L_n^n = 2a_n\phi_n^n \qquad 12.$$

Similarly using the Figure A4, the total length of the ray segment within the (n-1)th layer is,

$$L^{n}_{n-1} = 2a_n\,(\phi^{n}_{n-1} - \phi^{n}_{n}) \qquad 13.$$

$$= 2a_n\,\phi^{n}_{n-1} \quad \text{and so on.}$$

(iii) Calculation of transit time and average velocity in different layers :

Starting from the top most layer the average velocity in the first layer can be calculated either from relation (1) in Appendix IIA or from the relation (7) in Appendix IIB. Hence, if the travel time of the first ray in the first layer be T_1 (or Δt^1_1 in matrix 4.17 or t^1_1 in 4.24) then from 8,

$$a_1 = \frac{1}{2}\left[\frac{2R(R - d_1)\left\{1 - \cos(\Delta_1/2)\right\} + d_1^2}{d_1 - R\left\{1 - \cos(\Delta_1/2)\right\}}\right]$$

from 10, $\phi^1_1 = \frac{\pi}{180}\cos^{-1} X^1_1$,

from 12, $L^1_1 = 2a_1\,\phi^1_1$, and from 7,

$$\bar{v}_1 = \frac{2a_1\phi^1_1}{T_1}$$

Now for the second ray, half of the angle subtended by the ray segment within the second layer at its centre is given as, (from equation 10),

$$\phi_2^2 = \frac{\pi}{180} \cos^{-1} X_2^2$$

and the length of the path within the second layer is,

$$L_2^2 = 2a_2 \phi_2^2$$

and the total length of the path of the second ray within the first layer is (from 13),

$$L_1^2 = 2a_2 \partial \phi_1^2 \qquad \because \partial \phi_1^2 = \phi_1^2 - \phi_2^2$$

Hence the transit time of the second ray in the first layer is,

$$\triangle t_1^2 \text{ or } t_1^2 = L_1^2 / \bar{v}_1 ,$$

and the transit time of the second ray within the first layer will be,

$$\triangle t_2^2 \text{ or } t_2^2 = T_2 - \triangle t_1^2$$

where T_2 is the travel time of the second ray emerging at epicentral distance $\triangle_2$ from the $T - \triangle$ curve.

So the average velocity in the second layer will be,

$\bar{v}_2 = L_2^2/t_2^2$ and so on for the third ray.

Thus the transit times required for the travel time matrix 4.17 and/or 4.26 for the ith ray travelling through the kth layer (viz. $\triangle t_k^i$ or t_k^i) and the average velocity $\bar{v}_k$ can be computed following the procedure as above. A computer programme used for the purpose is given in Appendix I.

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