

APPENDIX II

CORRECTION FOR FOCAL DEPTH

A simple scheme is presented here to remove the effect of the focal depth and correction of the epicentral distance, travel time etc., so as to make each event appear to occur at the surface of the Earth.

Referring to Figure A1, let F be the focus of the earthquake at a depth d , from which a ray proceeds to emerge at an epicentral distance Δ in time T' . Let $d\Delta$ be the increment of the angular distance necessary when the focus is raised to F' at the surface of the Earth. If $i_{h\Delta}$ be the angle of incidence at the surface of the Earth stripped at the level of the focus (c.f. Figure I of Phe and Behe, 1972) and θ be the angle of refraction to the layer above it (assumed to be homogenous with an average velocity \bar{v}) and if $i_{r\Delta}$ be the angle of incidence at the surface of the Earth, then from Snell's law,

$$\frac{\sin i_{h\Delta}}{\sin \theta} = \frac{\bar{v}'}{\bar{v}} \quad 1$$

and also
$$\frac{\sin i_{r\Delta}}{\sin \theta} = \frac{R'}{R} \quad 2$$

where, \bar{v}' = velocity just below the focus, i.e., the

velocity with which the ray leaves the focus,

r' = radius of the stripped Earth

and R = radius of the Earth.

(1) Determination of θ :

Referring to the Figure A1, we can write for the triangle $SS'O$,

$$\frac{\sin i_{r\Delta}}{\sin \theta} = \frac{r'}{R}$$

$$\therefore \sin \theta = (R/r') \cdot \sin i_{r\Delta}$$

Now using the Tables of Pho and Behe (1972) for $\sin i_{r\Delta}$ for different depths of focus and Δ , θ can be calculated.

(ii) Calculation of \bar{v} :

From a standard surface focus $p - \Delta$ Table the values of Δ and T can be ascertained for a particular ray originating at the surface and penetrating to a certain depth d (\equiv depth of focus of the earthquake). Let Δ be the arcual distance traversed by a ray bottoming to a certain depth d in time T . Assuming the path of the ray to be circular, its radius is given by,

$$a = \frac{1}{2} \frac{2R(R-d) \{1 - \cos(\Delta/2)\} - d^2}{d - R \{1 - \cos(\Delta/2)\}}$$

(vide Appendix IIIB)

and the angle subtended by the ray at its centre is,

$$\phi = \frac{2}{180} \cos^{-1} \left\{ 1 - \frac{d(2R - d)}{2a(R - d + a)} \right\}$$

∴ the length of the ray path,

$$L = a \phi$$

and the average velocity of the layer,

$$\bar{v} = L/T$$

(iii) Calculation of the increment of angular distance and time :

After evaluation of θ and the average velocity the corrections for Δ and T can be made for transferring the focus of the earthquake to the surface of the Earth. Referring to the Figure A1 again we get,

$$\partial x = d \cdot \tan \theta = R \partial \Delta$$

$$\therefore \partial \Delta = (d \cdot \tan \theta) / R$$

and the corrected distance will be,

$$\Delta = \Delta' + \partial \Delta$$

So the extended length of the ray path above the level of the focus to the surface of the Earth will be,

$$FF' = d/\cos \theta$$

and the increment of the travel time is,

$$dT = FF'/v$$

.. the corrected travel time

$$T = T' + dT.$$