CHAPTER-7

NUMERICAL SOLUTION OF CONTINUITY AND MOMENTUM EQUATIONS OF WATER HAMMER PRESSURE IN CONDUIT WITHOUT A SURGE TANK

7.1 INTRODUCTION:

In this phase of the work, the situation studied is the occurrence of water hammer pressure in a long pipe line without a surge tank. The pipe is connected upstream to an infinite reservoir. Downstream, just close to the power house, a closing valve is fitted without a surge tank. Water hammer pressure in the pipe in this situation can be dangerous and its analysis is complex. Therefore, this part of study is important also from practical point of view. Again in this situation the equation of continuity and momentum are nonlinear. The author has developed means of solution of these equations by two different numerical methods. His computer models have been developed in both FORTRAN and Basic. The two methods of solution are the Method of Characteristics (MoC) and the Lax diffusive explicit finite difference system. As demonstrated here, identical results were obtained by these two methods. The results are presented graphically in the forms of computer plots. Damping of pressure and discharge with the pipe after valve closure are clearly shown in the plots. As the velocity within the pipe fluctuates from higher to very lower values, assessment of friction is made by same equation as is used in previous chapters.
7.2 BASIC EQUATION IN PRESSURE CONDUIT WITHOUT SURGE TANK:

The figure 7.1 shows the steady state condition of flow.

Referring to figure and again considering the continuity and momentum in the unsteady system, the basic differential equations applying to closure of valve at \( x = L \) are given by Parmakian\(^{29} \), Choudhury\(^{32} \), Wylie\(^{34} \), and Streeter\(^{33} \) as:

\[
\frac{\partial H}{\partial t} + \frac{a^2}{gA} \frac{\partial Q}{\partial x} = 0 \quad \cdots \quad \cdots \quad \cdots (7.1)
\]

\[
\frac{\partial H}{\partial x} + \frac{1}{gA} \frac{\partial Q}{\partial t} + \frac{f}{2gDA^2} Q|Q| = 0 \quad \cdots \quad \cdots \quad \cdots (7.2)
\]

The above two equations are applicable to both horizontal and slopping pipe lines.

7.3 REVIEW OF THE SOLUTIONS OF THE ABOVE BASIC EQUATIONS:

Before, the advent of computer, there was predominance of graphical methods of solution of water hammer pressure problems. Review of the literature on this method of solution shows that Schnyder\(^{35,36} \) and Bergeron\(^{37,39} \) developed graphical methods independently in the 1930's. In those days the equations (7.1) and (7.2) could not be solved analytically or numerically due to non linearity. Schnyder-Bergeron diagrams for pressure surge analysis aided the approximate solution of the surge problems of the above situation. Angus\(^{40} \) advocated that the Schnyder-Bergeron diagram was indispensable for design engineers.

Allievi\(^{39} \) gave classical solutions of the governing equations. This was a solution for pressure head \( H \) only. Pickford\(^{21} \) compared
the graphical and Allievi's classical solutions. Pickford was of the opinion that graphical methods are better than Allievi's solution. This was because the graphical method gives solutions for both pressure H and for discharge Q at different points of the pipe line at different time. Moreover, the classical solution of Allievi does not give accurate solutions to be used for practical purposes due to neglect of friction. It gives only the trend and type of solution. Jaeger also gave a similar type of classical solution. Again, the values of his solutions is reduced due to the neglect of friction. This is the main parameter of damping of pressure and discharge. Allievi also presented a graphical diagram known as Allievi's pressure-surge diagram which is a three parameter plot. For practical design this graphical treatment has been found.

The foregoing graphical and analytical methods have been replaced by digital computer methods since the beginning of 1960's. Various methods of solutions of the nonlinear equations (7.1) and (7.2) are available. Streeter used the Method of Characteristics to solve the equations.

Simpson and Wylie investigated the variation, magnitude and shape of short duration pressure pulses for a simple reservoir valve system connected by a hypothetical frictionless pipe.

Shimada and Okushima gave a new numerical model and a solution technique for water hammer. They proposed series solution methods
Figure 7.1 Schematic illustration of steady state flow when the valve is open.
Figure 7.2 Schematic illustration of steady state flow when the valve is open
and Newton-Raphson methods. Further, the validity of their solutions are examined by numerical computations in which system parameters are varied widely.

Chatterjee\textsuperscript{46} solved the equations by Method of Characteristics.

Burman\textsuperscript{55} investigated water hammer in a coaxial pipe system.

7.4. CONCLUSION WITH PREVIOUS SOLUTION:

Thus the review on previous work shows that a significant amount of research on this problem has already been undertaken using graphical, classical and numerical methods. In none of the solutions was it reported that there had been achieved complete damping of pressure head $H$ and discharge $Q$ after instantaneous or partial closure. Besides, laboratory model studies to these solution have not been taken due to difficulty in installation of the necessary length of pipe line in the laboratory. In none of the numerical solutions was friction factor evaluated at every time step of damping. Constant friction factor values have been taken in the solutions although velocity within the pipe may be close to zero at the time of change over from positive to negative or from negative to positive discharge values.

Therefore, it was decided to develop two numerical methods of solution with approximate C-W equation of Barr\textsuperscript{47}. The objective would be to display complete damping of pressure head $H$ and discharge $Q$ while adjusting friction factor values as appropriate at every step of damping. Change of friction values with increasing time would also be displayed. The solutions so developed could be assessed by comparison with the available solution of Streeter\textsuperscript{12}. 
7.5 THE PROPOSED NUMERICAL SOLUTIONS:

The partial differential equations (7.1) and (7.2) are nonlinear. Thus exact solution is not possible. Therefore, theoretical analysis of these nonlinear equations were undertaken by two different numerical methods. The methods adopted by the author are Methods of Characteristics (MoC) and the Lax Wandroff explicit finite difference (for simplicity LAX) scheme. Streeter\(^{42}\) advocated that MoC has some advantages for the solution of transient flows in pipe. Choudhury\(^{7}\) claimed the Lax explicit method yields satisfactory results in nonlinear partial differential solutions with smaller time steps provided initial and boundary conditions are purely imposed.

Although smaller time steps apparently would increase the volume of computation at time, much iteration is saved leading to a net decrease in time needed and the method is simple and very easy to handle. Therefore, the author has chosen these two methods of solution of the equations.

7.6 METHOD OF CHARACTERISTICS (MoC):

Equation (7.2) is multiplied by an unknown multiplier to obtain:

\[
\lambda \frac{dH}{dx} + \frac{\lambda}{gA} \frac{dQ}{dt} = -\frac{\lambda f}{2gDA^2} Q \left| Q \right| \ldots \ldots \ldots \ldots \ldots (7.3)
\]

Equations (7.2) and (7.3) are added. Then rearranging the terms:

\[
\left( \frac{dH}{dt} + \lambda \frac{dH}{dx} \right) + \frac{\lambda}{gA} \left( \frac{dQ}{dt} + \frac{a}{\lambda} \frac{dQ}{dx} \right) = -\frac{\lambda f}{2gDA} Q \left| Q \right| \ldots \ldots \ldots (7.4).
\]
The discharge \( Q \) and pressure head \( H \) are function of \( x \) and \( t \), i.e.

\[
Q = Q(x, t) \quad \text{and} \quad H = H(x, t)
\]

Now total derivative \( Q \),

\[
dQ = \frac{\partial Q}{\partial t} dt + \frac{\partial Q}{\partial x} dx
\]

Dividing by \( dt \),

\[
\frac{dQ}{dt} = \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} \left( -\frac{dx}{dt} \right) \quad \ldots \quad (7.5)
\]

Similarly,

\[
\frac{dH}{dt} = \frac{\partial H}{\partial t} + \frac{\partial H}{\partial x} \left( -\frac{dx}{dt} \right) \quad \ldots \quad (7.6)
\]

Comparing \((7.4)\) and \((7.5)\), \( \lambda \) may be written as:

\[
\lambda = \frac{dx}{dt} = \frac{a'}{\lambda}
\]

\[
\therefore \ a' = \lambda
\]

\[
\therefore \ a = \pm \lambda
\]

\[
\therefore \ \frac{dx}{dt} = \pm a
\]

\[
\therefore \ \frac{dx}{dt} = + a \quad \ldots \ldots \quad (7.7)
\]

\[
\therefore \ \frac{dx}{dt} = - a \quad \ldots \ldots \quad (7.8)
\]

These are the two positive and negative characteristics equations.

\[
\frac{dx}{dt} = a
\]

Again comparing \((7.4)\), \((7.5)\) and \((7.6)\) when \( \lambda = a \), i.e. \( \frac{dx}{dt} = a \) along positive characteristic \((7.4)\) becomes

\[
\frac{dH}{dt} + \frac{a}{gA} \frac{dQ}{dt} = - \frac{af}{2gDA^2} Q |Q|
\]

\[
\Rightarrow dH + \frac{a}{gA} dQ = - \frac{af}{2gDA^2} Q |Q| dt \quad \ldots \ldots \ldots \quad (7.9)
\]
Integrating equation (7.9) along AP in figure 7.3,

\[ \int_{A}^{P} \frac{dH}{gA} + \frac{dQ}{Q} = -\frac{af}{2gDA} Q|Q_i| \int_{t_i}^{t_f} dt \]

\[ \Rightarrow (H_P - H_A) + \frac{a}{gA} (Q_P - Q_A) = -\frac{af}{2gDA} Q_a|Q_a| (t_f - t_i) \text{ along a positive characteristic AP} \]

\[ \Rightarrow H_P = H_A - \frac{a}{gA} Q_P - \frac{a}{gA} Q_A - \frac{af \Delta t}{2gDA} Q_a|Q_a| \quad \ldots (7.10) \]

Similarly along negative characteristics when \( \frac{dx}{dt} = -a \), i.e., \( \lambda = -a \)

\[ \Rightarrow H_P = H_B + \frac{a}{gA} Q_P - \frac{a}{gA} Q_B + \frac{af \Delta t}{2gDA} Q_B|Q_B| \quad \ldots (7.11) \]

Adding (7.10) and (7.11), it is obtained:

\[ H_P = \frac{1}{2}(H_A + H_B) + \frac{1}{2} \frac{a}{gA} (Q_A - Q_B) + \frac{1}{2} \frac{af \Delta t}{2gDA} \left[ Q_P|Q_P| - Q_A|Q_A| \right] \ldots (7.12) \]

Subtracting (7.11) from (7.10), there is obtained:

\[ Q_P = \frac{1}{2} (Q_A + Q_B) + \frac{gA}{2a} (H_A - H_B) - \frac{af \Delta t}{4gDA} \left[ Q_A|Q_A| - Q_B|Q_B| \right] \ldots (7.13) \]

From the above equations (7.12) and (7.13), from known values of H and Q at points A and B from initial conditions, values are calculated at P i.e., at next time step, are calculated.

7.6.1. Finite difference four equations (7.12) and (7.13):

The finite difference grid along positive characteristics (i.e., c' and c") are shown in figure 7.3. Equation (7.10) is valid along c' having the shape of \( \frac{\Delta t}{\Delta x} = \frac{1}{a} \) and equation (7.11) is valid along c" having the slope of \( \frac{\Delta t}{\Delta x} = -\frac{1}{a} \). Combination of these equations yields solution for pressure H at P by equation (7.12) and for discharge Q by equation (7.13).
To convert these two equation to finite difference equations one proceeds as follows:

\[ H_{p} = H_{k}^{j+1}, \quad H_{a} = H_{k-1}^{j+1}, \quad H_{a} = H_{k-1}^{j+1}, \quad Q_{f} = Q_{k}^{j+1}, \quad Q_{a} = Q_{k-1}^{j+1}, \quad Q_{a} = Q_{k-1}^{j+1}, \quad \ldots \quad (7.14) \]

Putting (7.14) in (7.12) and (7.13), the finite difference form of equations are written as:

\[ H_{k}^{j+1} = \frac{1}{2} (H_{k-1}^{j} + H_{k+1}^{j}) + \frac{a}{2gA} (Q_{k-1}^{j} - Q_{k+1}^{j}) + \frac{af\Delta t}{4gDA} (Q'_{k-1}^{j+1} - Q'_{k-1}^{j} - Q'_{k+1}^{j+1} - Q'_{k+1}^{j}) \ldots (7.15) \]

And,

\[ Q_{k}^{j+1} = \frac{1}{2} (Q_{k-1}^{j} - Q_{k+1}^{j}) + \frac{gA}{2a} (H_{k-1}^{j} - H_{k+1}^{j}) - \frac{f\Delta t}{4gDA} (Q'_{k-1}^{j+1} - Q'_{k-1}^{j} - Q'_{k+1}^{j+1} - Q'_{k+1}^{j}) \ldots (7.16) \]

Thus equations (7.15) and (7.16) are the finite difference forms of equation (7.12) and (7.13). With help of these equations, and from known initial conditions, solution for \( H \) and \( Q \) can be advanced with increase of time step \( \Delta t \).
Figure 7.3 Finite difference grid in x-t plane by MoC
7.6.2. Methods of calculation of discharge Q and pressure II:

The above two equations (7.15) and (7.16) are employed to calculate the transient pressure head and discharge at fixed and equally spaced interior sections throughout the pipe. In figure 7.3, Q and H are known at each of the grid intersections along the abscissa axis at \( t = t_0 \) i.e., initial time. The conditions or values of Q and H at P at time \( t_i = t_0 + \Delta t \) are obtained by the above equations. Similarly Q and H at other interior grid points at the same time \( t_i \) can be calculated. After having calculated Q and H at all interior points at time \( t_i \), boundary values at upstream and downstream points are imposed by the methods or equations described below. Then values at \( t = t_i \), becomes the initial values to calculate Q and H for the next time \( t_i = t_i + \Delta t \). Again the boundary conditions are imposed. Solutions thus are advanced in time until the time \( t_{\text{final}} \) is reached.

7.6.3. Boundary conditions before complete closure at \( X = L \):

The conditions or values at \( X = L \) i.e., at the valve after first step are \( Q_{i+1} \) and \( H_{i+1} \). To evaluate these values, principle of orifice during valve closure time is used. By this principle discharge \( Q_{i+1} \) under dead of \( H_{i+1} \) will be

\[
Q_{i+1} = (C_d A_v) \sqrt{2gH_{i+1}}
\]

where \( C_d \) is the coefficient of discharge and \( A_v \) is the area at the valve at the time \( j+1 \) i.e., after time \( \Delta t \). the term \((C_d A_v)\) will be a known function of time.

Now finite difference equation along \( c' \) at right hand boundary i.e., at \( X = L \) is taken as:
Equations (7.17) and (7.18) have two unknowns $Q_{l}^{j+1}$ and $H_{l}^{j+1}$ and solving these two simultaneous equations,

$$Q_{l}^{j+1} = \frac{(C_{d}A_{v})^{2}a}{A} [1 + \frac{2(H_{l}^{j} - \Delta x) + \frac{af\Delta t}{2gA^{2}}Q_{l}^{j} - \frac{af\Delta t}{2gA^{2}}Q_{l}^{j+1}}{2} \frac{Q_{l}^{j} - \frac{af\Delta t}{2gA^{2}}Q_{l}^{j+1}}{Q_{l}^{j}}]$$

and,

$$H_{l}^{j+1} = \frac{a}{gA}Q_{l}^{j} - \frac{af\Delta t}{2gA^{2}}Q_{l}^{j+1}$$

Thus equations (7.19) and (7.20) are used to calculate boundary values at $X = L$ up to to previous time step of complete valve closure.

When the valve is completely closed, $A_{v}$ becomes zero, hence equation (7.19) becomes

$$Q_{l}^{j+1} = 0$$

Putting $Q_{l}^{j+1} = 0$, pressure at $X = L$, after complete closure, is obtained from equation (7.20) i.e.,

$$H_{l}^{j+1} = H_{l}^{j} - \frac{a}{gA}Q_{l}^{j} - \frac{af\Delta t}{2gA^{2}}Q_{l}^{j+1}$$

7.6.4 Upstream boundary condition at $X = 0$, i.e. at reservoir:

At the upstream end, i.e., at reservoir end, the reservoir is assumed to infinite. Therefore, backward and forward flows through conduit do not have any effect on such an infinite reservoir and pressure head remains constant i.e.,

$$H_{X=0}^{j+1} = H_{0}$$
and discharge is computed by equation (7.24) along negative characteristic line i.e. along c^- line as:

$$Q_{x-0}^{j+1} = \frac{gA}{a} (H_0 - H_{j+1}^{1} + Q_{j+1}^{1} - \frac{a f \Delta t}{2gDA} Q_{j+1}^{0} - \frac{2gDA}{2} |Q_{j+1}^{0}|) \quad \ldots \quad (7.24)$$

Thus boundary values are at upstream reservoir are calculated by equation (7.23) and (7.24).

### 7.7 LAX DIFFUSIVE FINITE DIFFERENCE METHOD:

Choudhury\(^3^2\) has analysed the efficacy of numerical solutions. From this experience, he has advocated that Lax diffusive explicit finite difference scheme yield satisfactory results when integration time step is small. Besides, the method is simple, and easy to handle. Based on this conclusion of Choudhury\(^\_3\) the Lax explicit method was adopted for the second of the author's numerical solutions of the equations: (7.1) and (7.2).

#### 7.7.1: Finite difference equations of Lax method:

Equation (7.1) is written in finite difference form as:

$$\frac{H_{k+1}^{j+1} - H^{*}}{\Delta t} + \frac{a_2}{gA} \frac{Q_{k+1}^{j+1} - Q_{k-1}^{j+1}}{2\Delta x} = 0 \quad \ldots \quad (7.25)$$

Where, $$H^{*} = \frac{1}{2} (H_{k-1}^{j} + H_{k+1}^{j}) \quad \ldots \quad (7.26)$$

Putting $$H^{*}$$ in (7.25) and simplifying

$$H_{k+1}^{j+1} = \frac{1}{2} (H_{k-1}^{j} + H_{k+1}^{j}) - \frac{\Delta t a^2}{gA} \frac{1}{2\Delta x} (Q_{k+1}^{j} - Q_{k-1}^{j}) \quad \ldots \quad (7.27)$$

Equation (7.2) is written in finite difference form as:

$$\frac{1}{2\Delta x} (H_{k+1}^{j+1} - H_{k-1}^{j+1}) + \frac{1}{gA} \frac{Q_{k+1}^{j+1} - Q^{*}}{\Delta t} + \frac{f}{2gDA} Q_{j}^{*} \quad |Q_{j}^{*}| = 0 \quad \ldots \quad (7.28)$$

Where, $$Q^{*} = \frac{1}{2} (Q_{k+1}^{j} + Q_{k-1}^{j}) \quad \ldots \quad (7.29)$$
Putting $Q^*$ in (7.28) and simplifying

$$Q_k^{j+1} = \frac{1}{2} (Q_{k-1}^j + Q_{k+1}^j) - \frac{gA\Delta t}{2Ax} (H_{k-1}^j - H_{k-1}^j) - \frac{f\Delta t}{8gA} .$$

Thus the equations (7.27) and (7.30) are the finite difference equations to be used to calculate pressure head $H$ and discharge $Q$ in different cross-sections with increasing time step.

7.7.2. Method of calculation and boundary condition:

The method of calculation at interior points follows the same lines as for MoC, now with the help of equations (7.27) and (7.30). The boundary conditions at both upstream and downstream sections are imposed as for the procedures and equations used in MoC. Calculation of pressure heads $H$ and discharge $Q$ are advanced with increasing time, which has already been explained in dealing with MoC.

7.8. COMPUTER PROGRAM FOR SOLUTIONS OF PRESSURE HEAD $H$ AND DISCHARGE $Q$ BY THE TWO PROPOSED METHODS:

Computer program has been developed both for the methods of MoC and for the Lax diffusive explicit finite difference scheme. These give solution for discharges and pressures at different cross-sections at different time intervals after the valve closure i.e., during unsteady transient flow situations. The computer program has been included in this chapter. To start calculation by the equations already described by the two methods, the hydraulic and geometric parameters of figures 7.1 and 7.2 are taken as inputs. A particular problem is chosen to illustrate the solution.
of the equations. The length and diameter of the pipe are taken to be 12,000 feet and 2 feet receptively.

The steady discharge and steady head at reservoir are taken as 20 feet³/sec and 600 feet. The steady flow friction factor is 0.02. Steady heads at pipe sections 1, 2, 3, 4 and 5 are 600, 582.5, 565.0, 547.5 and 530.0 feet respectively. The Δx of the pipe is taken to be 3000 feet. The velocity of pressure wave a is assumed to be 3000 ft./sec. The Δt of solution is 1 second so that (Δx/Δt) = a. The time of valve closing is chosen to be 4 seconds which is equivalent to instantaneous because it is smaller than (2h/a). The roughness k of the pipe is calculated from above data to be 0.002162 m. The resistance equation to calculate friction factor f at every location at any time is the same explicit C-W equation of Barr⁵,⁶ i.e., equation (2.5). The next step is to calculate the discharges and pressures at interior points by equation (7.16) and (7.15) for MoC and equations (7.27) and (7.30) for the Lax method. After calculations at interior points, boundary values are calculated by the equations (7.17) to (7.24). Thus the values of Q and H at time t₁ = t₁+Δt at different cross-sections are obtained. These values are converted to initial values for the next time t₂ = t₁+Δt and computations of Q, H and f at interior and boundary points are made by the same process as is described above. Again these values are initialised for the next time step t₁ = t₂+Δt, Thus the solution is advanced in time up to required time of t_final.

Along with the calculation by the computer program, all the calculated values are stored. Computer graphic techniques are incorporated in the programs and the results are simultaneously plotted and presented in graphic modes. This is for discharges, pressures and friction factor against corresponding time.
At the beginning, computer plots of discharges $Q$ and pressure heads $H$ at different cross-sections of the pipe upto 100 seconds by the MoC are presented in figure 7.4 and 7.5. Even in this small time 100 seconds damping of $Q$ and of $H$ in all the sections are quite noticeable. Similar plots of $Q$ and $H$ are presented for increasing time upto 300 seconds and 600 seconds in figures 7.6, 7.7, 7.8 and 7.9. Damping of both the parameters in these figures are then very much pronounced. Solution for discharges both by MoC and Lax methods at cross-section numbers 1, 2, 3 and 4 are shown in figures 7.10 to 7.13. These figures show the general similarity of the solutions by the two methods. Peak of discharge by the Lax method are slightly greater. Solutions for pressures, again by both the methods, are presented from figures 7.14 to 7.17 in pipe cross-section numbers 2, 3, 4, and 5. In these figures also pressure peaks by Lax are slightly less than by MoC.

Variations of friction factor $f$ with increasing time at different cross-sections are presented in figure 7.18 to 7.21. Figure 7.22 is the superimposition of figures 7.18 to 7.21. These plots show how friction varies with increasing time of solution. Abrupt increases in friction factor values are displayed in these figures when flow direction begins to change. A comparison of friction values by the two methods are plotted in figure 7.23. Both the methods give the same pattern of variation of friction factor.
Figure 7.4 Plot of discharge vs time t at all pipe sections by MoC.
Figure 7.5 Plot of pressure H Vs time t at all pipe sections by MoC
Figure 7.6 Plot of discharge $Q$ vs time $t$ at all pipe cross-sections by MoC
Figure 7.7 Plot of pressure head $H$ Vs time $t$ at all pipe sections by MoC
Figure 7.8 Plot of discharge $Q$ Vs time $t$ at all pipe sections by MoC
Figure 7.9 Plot of pressure $H$ Vs time $t$ at all pipe sections by MOC
Figure 7.10 Plot of discharge Q vs time t at a pipe cross-sections no.1 by MoC and Lax diffusive F.D.(Explicit) Methods.
Figure 7.11 Plot of discharge \( Q \) vs time \( t \) at a pipe cross-sections no.2 by MoC and Lax diffusive F.D. (Explicit) Methods.
Figure 7.12 Plot of discharge $Q$ Vs time $t$ at a pipe cross-sections no.3 by MoC and Lax diffusive F.D.(Explicit) Methods.
Figure 7.13 Plot of discharge $Q$ vs time $t$ at a pipe cross-sections no. 4 by MoC and Lax diffusive F.D.(Explicit) Methods.
Figure 7.14  Plot of pressure $H$ Vs time $t$ at pipe cross-section no 2 by Lax diffusive F.D.(explicit) and MoC methods
Figure 7.15: Plot of pressure $H$ vs. time $t$ at pipe cross-section no 3 by Lax diffusive F.D. (explicit) and MoC methods.
Figure 7.16 Plot of pressure \( H \) Vs time \( t \) at pipe cross-section no 4 by Lax diffusive F.D.(explicit) and MoC methods
Figure 7.17  Plot of pressure $H$ Vs time $t$ at pipe cross-section no 5 by Lax diffusive F.D. (explicit) and MoC methods
Figure 7.18  Plot of friction factor $f$ Vs Time $t$ at pipe cross-section no. 1 by MoC
Figure 7.19  Plot of friction factor $f$ Vs Time $t$ at pipe cross-section no. 2 by MoC
Figure 7.20  Plot of friction factor $f$ Vs Time $t$ at pipe cross-section no. 3 by MoC
Figure 7.21 Plot of friction factor $f$ vs time $t$ at pipe cross-section no. 4.
Figure 7.22  Plot of friction factor $f$ Vs Time $t$ at all pipe cross-sections
by MoC
Figure 7.23 Plot of Friction factor $f$ Vs Time $t$ at pipe cross-section 1 by Lax method.
Figure 7.24 Comparison of discharge Q vs time t at reservoir section between Streeter's solution and author's by M.O.C.
7.9. PRESENTATION OF RESULTS BY COMPUTER PLOTS IN BOTH THE METHODS:

At the beginning, computer plots of discharges $Q$ and pressure heads $H$ at different cross-sections of the pipe up to 100 seconds by the MoC are presented in figure 7.4 and 7.5. Even in this small time 100 seconds damping of $Q$ and of $H$ in all the sections are quite noticeable. Similar plots of $Q$ and $H$ are presented for increasing time up to 300 seconds and 600 seconds in figures 7.6, 7.7, 7.8 and 7.9. Damping of both the parameters in these figures are then very much pronounced. Solution for discharges both by MoC and Lax methods at cross-section numbers 1, 2, 3 and 4 are shown in figures 7.10 to 7.13. These figures show the general similarity of the solutions by the two methods. Peak of discharge by the Lax method are slightly greater. Solutions for pressures, again by both the methods, are presented from figures 7.14 to 7.17 in pipe cross-section numbers 2, 3, 4, and 5. In these figures also pressure peaks by Lax are slightly less than by MoC.

Variations of friction factor $f$ with increasing time at different cross-sections are presented in figure 7.18 to 7.21. Figure 7.22 is the superimposition of figures 7.18 to 7.21. These plots show how friction varies with increasing time of solution. Abrupt increases in friction factor values are displayed in these figures when flow direction begins to change. A comparison of friction values by the two methods are plotted in figure 7.23. Both the methods give the same pattern of variation of friction factor.
7.10 COMPARISON OF AUTHOR'S SOLUTION WITH STREETER'S SOLUTION:

Streeter\textsuperscript{42} solved the same nonlinear equation by different methods. He presented his solutions up to only 50 seconds. The results presented by Streeter\textsuperscript{42} have been adopted to assess the author's results in a comparative plot which has been presented in figure 7.24. The plot shows a distinctly satisfactory simulation. Peak values of discharge between streeter and the author are almost the same. Slight difference does occur in frequency, but this only during initial few seconds. With increase of time, frequency coincides but residual difference remain in peak values. Streeter solution gives somewhat higher peaks. This is probably due to use of constant friction factor adopted by Streeter and the variable friction factor value adopted by the author with the use of Barr's equation. On the whole, this comparison gives a satisfactory verification of the author’s solution.

7.11 CONCLUSIONS AND DISCUSSIONS OF RESULTS PRESENTED:

The author’s study was conducted by the developing of numerical solutions of the nonlinear two equations (7.1) and (7.2). This was computerised, as the graphical representation of results. The graphical evidence is clearcut and significant. The two explicit numerical methods i.e., Method of Characteristics (MoC) and the Lax method are very easy to handle by the author's computer program. The computer program presented can be handled by any field engineer, even with little background knowledge of computer programming and graphics. It is presented, therefore, in the simple basic language where computer graphics can be introduced together with the calculations to present the results directly.
Positive and negative flows, rise and fall of pressure within the pipe line are satisfactorily presented. Damping of these two parameters with increase of time is distinct in the plots. Discharge decreases about 5 times in 100 seconds, about 10 times in 300 seconds and about 20 times in 600 seconds. Little irregular undulations discharges at peak values at cross-section number 1 i.e., at reservoir sections (figure 7.10) is observed which is practically quite expected as the backward and forward flows at this reservoir section meet the constant head infinite reservoir. Such fluctuations are quite possible in this boundary points of the pipe. Plots of discharges in interior sections are smooth, as can be observed in figures 7.11 to 7.13.

The case of pressure plots at valve ends, i.e., in figure 7.17, is similar. At the valve end in both the methods fluctuation of pressure head at peak values are observed. It is not unexpected that these should be same unsteady peak values at valve ends due to sudden obstruction. Plots of pressure at the interior points i.e., as shown in figures 7.14 to 7.17, are smooth.

The study of pressure rise is a very important aspect for design of the conduit. The maximum pressure rise for this particular problem occurs at the valve and increases to about 1.9 times the static pressure. It falls to be close static level pressure in 600 seconds. Of course, the occurrence of maximum rise and fall with time will differ depending on hydraulic, geometrical parameter and valve closing time. Maximum pressure rise for design of the pipe line can obtained once the hydraulic, geometrical parameters and closing time are given as inputs to the Computer Program.

It appears from the plots that the MoC is little better than Lax method. Lax method gives slightly higher values of discharges at peak and slightly lower values of pressure at peaks. Although the two methods give almost nearer values, Lax method has slight
tendency to produce error when solution time increases indefinitely. Therefore, author suggests that if for those following generally similar approach, the explicit MoC is best.

The study of variation of friction factor $f$ with time is presented in figures 7.18 to 7.22 and is new in its nature. In most of the earlier studies by previous investigators, friction factor is assumed to constant. That this is not so as predicted in all the foregoing figures. The average friction increases 5 times in 450 seconds and it suddenly shoots up to a very high values even 1000 times at the time of change of flow direction.

It is believed that in transient flow stage a quick change of velocity takes place. At the time of change of velocity direction, Reynolds number becomes very small. Friction factor value shoots up at this stage. Therefore, this new line of variation of friction factor is explored and expected rise and shooting up its values are predicted from figure 7.18 to 7.23. The part of study can suggest the future investigators not to use constant friction factor values which may lead to wrong solutions. Resistance to flow is an important phase of any analysis and therefore, it is always advisable to assess the resistance at every stage of flow. These findings of increase and shooting up of friction values are very interesting and should always be taken care of. The comparative plot of author’s solution with Streeter’s solution in figure 7.24 has displayed a satisfactory verification of the author’s numerical solutions. The little difference in this comparative plot may be due to the resistance equation used by the author.
COMPUTER PROGRAM FOR SOLUTION OF UNSTEADY EQUATIONS IN
CHAPTER 7 BY MoC AND LAX METHODS.
DIM H(5,2),Q(5,2),QD(5,350),HD(5,350),FF(5,2),TD(1,350),FP(5,350)
SCREEN 2
KEY OFF:CLS
LINE (39,20)-(519,140),2,B
FOR I=1 TO 480 STEP 4
PSET (39+I,40),2
PSET (39+I,50),2
PSET (39+I,60),2
PSET (39+I,70),2
PSET (39+I,80),2
PSET (39+I,90),2
PSET (39+I,100),2
PSET (39+I,110),2
PSET (39+I,120),2
PSET (39+I,130),2
NEXT I
LOCATE 21,2:PRINT"Figure 7.10 Plot of discharge Q Vs time t at a pipe cross-sections no. 1 by MoC"
LOCATE 6,45:PRINT"M.o.C."
LOCATE 23,2:PRINT"and Lax diffusive F.D.(Explicit) Methods."
LOCATE 6,54:PRINT"Lax"
FOR I=1 TO 120 STEP 2
PSET (99,20+I),2
PSET (159,20+I),2
PSET (219,20+I),2
PSET (279,20+I),2
PSET (339,20+I),2
PSET (399,20+I),2
PSET (459,20+I),2
PSET (519,20+I),2
REM PSET (579,20+I),2
REM PSET (639,20+I),2
REM PSET (589,20+I),2
NEXT I
LOCATE 19,78:PRINT"100"
LOCATE 19,65:PRINT"80"
LOCATE 19,50:PRINT"60"
LOCATE 19,35:PRINT"40"
LOCATE 19,20:PRINT"20"
LOCATE 19,18:PRINT"50"
LOCATE 19,6:PRINT"0"
LOCATE 17,67:PRINT"Time t in secs"
LINE (39,80)-(519,80),2
LOCATE 3,3:PRINT"30"
LOCATE 11,3:PRINT"0"
LOCATE 18,2:PRINT"-30"
LOCATE 6,3:PRINT"-20"
LOCATE 8,3:PRINT"10"
LOCATE 16,2:PRINT"-20"
LOCATE 13,2:PRINT"-10"
LOCATE 1,3:PRINT"Q *
LOCATE 2,3:PRINT"cfs"
TTT=79
560  DELTA=1
570  G=32.19
580  AC=3000
590  D=2!
600  PL=12000
610  HS=600
620  RK=.002162
630  VIS=.000001*2.54*2.54*12*12
640  PI=3.14159
650  A=(PI/4)*D^2
660  N=5
670  C=1/2.302585
680  P=1
690  KK=1
700  FOR K=1 TO N
710  Q(K,1)=20
720  FP(K, KK)=.02
730  NEXT K
740  H(1,1)=600
750  H(2,1)=582.5
760  H(3,1)=565
770  H(4,1)=547.5
780  H(5,1)=530
790  FOR K=1 TO N
800  QD(K, KK)=Q(K, 1)
810  HD(K, KK)=H(K, 1)
820  REM PRINT QD(K, KK), HD(K, KK)
830  NEXT K
840  VCT=4
850  CD=.62
860  AV=Q(N, 1)/(CD*(2*G*H(N, 1))^.5)
870  DV=(4*AV/PI)^.5
880  DV1=DV-(DELTA*DV)/VCT
890  AV1=(PI/4)*DV1^2
900  T1=H(N-1, 1)+(AC/(G*A))*Q(N-1, 1)
910  VEL=Q(N, 1)/A
920  REY={ABS(VEL)*D}/VIS
930  F=1/(-2*C*LOG((RK/(3.71*D))+(5.1286/REY^.89))^2
940  T2=(AC*F*DELTA)/(2*G*D*A^2)
950  T3=T2*ABS(Q(N-1, 1))*Q(N-1, 1)
960  T4=AC^2*CD^2*AV1^2
970  T5=T4/(G*A^2)
980  T6=(2*(T1-T3))/T5
990  T11=(AC*CD^2*AV1^2)/A
1000 Q(N, 2)=T11*((1+T6)^.5-1)
1010 HT1=H(N-1, 1)
1020 HT2=(AC/(G*A))*Q(N-1, 1)
1030 HT3=((AC*F*DELTA)/(2*G*D*A^2))*(ABS(Q(N-1, 1))*Q(N-1, 1))
1040 HT4=(AC/(G*A))*Q(N, 2)
1050 H(N, 2)=HT1+HT2-HT3-HT4
1060 H(N, 1)=H(N, 2)
1070 Q(N, 1)=Q(N, 2)
1080 KK=KK+1
1090 FOR K=1 TO N
1100 QD(K, KK)=Q(K, 1)
HD(K,KK)=H(K,1)
FP(K,KK)=F
REM PRINT QD(K,KK),HD(K,KK)
NEXT K
QT1=.5*(Q(N-2,1)+Q(N,1))
VEL=Q(N,1)/A
REY=(ABS(VEL)*D/VIS)
F=1/(2*C*LOG(RK/(3.71*D))+(5.1286/REY^.89))^.2
QT2=(G*A)/(2*AC)*(H(N-2,1)-H(N,1))
QT3=(F*DELTA)/(4*D*A)*((ABS(Q(N-2,1))*Q(N-2,1))+(ABS(Q(N-2,1))*Q(N,1))
Q(N-1,2)=QT1+QT2-QT3
HHT1=.5*(H(N-2,1)+H(N,1))
HHT2=(AC/(2*G*A))*(Q(N-2,1)-Q(N,1))
HHT3=(AC*F*DELTA)/(4*G*D*A^2)
HHT4=HHT3*(ABS(Q(N,1))*Q(N,1)-ABS(Q(N-2,1))*Q(N-2,1))
H(N-1,2)=HHT1+HHT2+HHT4
DV2=DV-((2*DELTA)*DV)/VCT
AV2=(PI/4)*DV^2
T1=H(N-1,1)+(AC/(G*A))*Q(N-1,1)
T2=(AC*F*DELTA)/(2*G*D*A^2)
T3=T2*ABS(Q(N-1,1))*Q(N-1,1)
T4=AC*2*CD*2*AV2^2
T5=T4/(G*A^2)
T6=(2*(T1-T3))/T5
T11=(AC*CD*2*AV2*2)/A
Q(N,2)=T11*(1+T6)^.5-1
HT1=H(N-1,1)
HT2=(AC/(G*A))*Q(N-1,1)
HT3=((AC*F*DELTA)/(2*G*D*A^2))*(ABS(Q(N-1,1))*Q(N-1,1))
HT4=(AC/(G*A))*Q(N,2)
H(N,2)=HT1+HT2+HT3+HT4
FOR K=N-1 TO N
Q(K,1)=Q(K,2)
H(K,1)=H(K,2)
NEXT K
KK=KK+1
FOR K=1 TO N
QD(K,KK)=Q(K,1)
HD(K,KK)=H(K,1)
REM PRINT QD(K,KK),HD(K,KK)
NEXT K
KK=KK+1
FOR K=N-2 TO N-1
QK1=.5*(Q(K-1,1)+Q(K+1,1))
QK2=((G*A)/(2*AC))*(H(K-1,1)-H(K+1,1))
FF(K,2)=1/(2*C*LOG((RK/(3.71*D))+(5.1286/(((ABS(Q(K+1,1)/A)*D)/VIS)^.89)))^.2
FP(K,KK)=FF(K,2)
QK3=((FF(K,2)*DELTA)/(4*D*A))*((ABS(Q(K-1,1))*Q(K-1,1))+(ABS(Q(K+1,1))*Q(K+1,1))
Q(K,2)=QK1+QK2-QK3
HHK1=.5*(H(K-1,1)+H(K+1,1))
HHK2=(AC/(2*G*A))*(Q(K-1,1)-Q(K+1,1))
HHK3=(AC*FF(K,2)*DELTA)/(4*D*A^2)
HHH4=HHK3*(ABS(Q(K-1,1))*Q(K+1,1)-ABS(Q(K-1,1))*Q(K-1,1))
H(K,2)=HHK1+HHK2+HHK4
1660 NEXT K
1670 FP(N,KK)=FF(N-1,2)
1680 FP(1,KK)=F
1690 FP(2,KK)=F
1700 DV3=DV-((3*DELTA)*DV)/VCT
1710 AV3=(PI/4)*DV3^2
1720 T1=H(N-1,1)+(AC/(G*A))*Q(N-1,1)
1730 T2=(AC*FF(4,2)*DELTA)/(2*G*D*A^2)
1740 T3=T2*ABS(Q(N-1,1))*Q(N-1,1)
1750 T4=AC^2*CD^2*AV3^2
1760 T5=T4/(G*A^2)
1770 T6=(2*(T1-T3))/T5
1780 T11=(AC*CD^2*AV3^2)/A
1790 Q(N,2)=T11*((1+T6)^.5-1)
1800 HT1=H(N-1,1)
1810 HT2=(AC/(G*A))*Q(N-1,1)
1820 HT3=((AC*FF(4,2)*DELTA)/(2*G*D*A^2))*(ABS(Q(N-1,1))*Q(N-1,1))
1830 HT4=(AC/(G*A))*Q(N,2)
1840 H(N,2)=HT1+HT2-HT3-HT4
1850 FOR K=N-2 TO N
1860 Q(K,1)=Q(K,2)
1870 H(K,1)=H(K,2)
1880 NEXT K
1890 FOR K=1 TO N
1900 QD(K,KK)=Q(K,1)
1910 HD(K,KK)=H(K,1)
1920 REM PRINT QD(K,KK),HD(K,KK)
1930 NEXT K
1940 KK=KK+1
1950 FOR K=N-3 TO N-1
1960 QK1=.5*(Q(K-1,1)+Q(K+1,1))
1970 QK2=((G*A)/(2*AC))*(H(K-1,1)-H(K+1,1))
1980 FF(K,2)=1/(-2*C*LOG((RK/(3.71*D))+(5.1286/(((ABS(Q(K+1,1)/A)*D)/VIS)^.89)))*2
1990 FP(K,KK)=FF(K,2)
2000 QK3=((FF(K,2)*DELTA)/(4*D*A))*((ABS(Q(K-1,1))*Q(K-1,1))+((ABS(Q(K+1,1))*Q(K+1,1))
2010 Q(K,2)=QK1+QK2-QK3
2020 HHK1=.5*(H(K-1,1)+H(K+1,1))
2030 HHK2=(AC/(2*G*A))*Q(K-1,1)-Q(K+1,1))
2040 HHK3=(AC*FF(K,2)*DELTA)/(4*G*D*A^2)
2050 HHK4=HHK3*(ABS(Q(K+1,1))*Q(K+1,1)-ABS(Q(K-1,1))*Q(K-1,1))
2060 H(K,2)=HHK1+HHK2+HHK4
2070 NEXT K
2080 Q(N,2)=0
2090 FP(N,KK)=0
2100 FP(1,KK)=FF(N-3,2)
2110 HT1=H(N-1,1)+(AC/(G*A))*Q(N-1,1)
2120 HT2=((AC*FF(4,2)*DELTA)/(2*G*D*A^2))*(ABS(Q(N-1,1))*Q(N-1,1))
2130 H(N,2)=HT1-HT2
2140 FOR K=N-3 TO N
2150 Q(K,1)=Q(K,2)
2160 H(K,1)=H(K,2)
2170 NEXT K
2180 FOR K=1 TO N
2190 QD(K,KK)=Q(K,1)
2200 HD(K,KK)=H(K,1)
2210 REM PRINT QD(K,KK),HD(K,KK)
2220 NEXT K
2230 KK=KK+1
2240 FOR K=N-3 TO N-1
2250 QK1=.5*(Q(K-1,1)+Q(K+1,1))
2260 QK2=((G*A)/(2*AC))*(H(K-1,1)-H(K+1,1))
2270 FF(K,2)=1/(2*C*LOG((RK/(3.71*D))+(5.1286/((ABS(Q(K,1)/A))*D)/VIS*.89)))^2
2280 FP(K,KK)=FF(K,2)
2290 QK3=((FF(K,2))/(4*D*A))*((ABS(Q(K-1,1))*Q(K-1,1))+(ABS(Q(K+1,1))*Q(K+1,1)))
2300 Q(K,2)=QK1+QK2-QK3
2310 HHK1=.5*(H(K-1,1)+H(K+1,1))
2320 HHK2=(AC/(2*G*A))*(Q(K-1,1)-Q(K+1,1))
2330 HHK3=(AC*FF(K,2))/(4*G*D*A^2)
2340 HHK4=HHK3*(ABS(Q(K+1,1))*Q(K+1,1)-ABS(Q(K-1,1))*Q(K-1,1))
2350 H(K,2)=HHK1+HHK2+HHK4
2360 NEXT K
2370 FP(1,KK)=FP(N-3,2)
2380 FP(5,KK)=0
2390 Q(N,2)=0
2400 Q11=600-H(2,1)+AC/(G*A)*Q(2,1)-AC*FF(3.2)*DELTA/(2*G*D*A^2)*((ABS(Q(2,1))*Q(2,1))
2410 Q(1,1)=Q11/(AC/(G*A))
2420 HT1=H(N-1,1)+(AC/(G*A))*Q(N-1,1)
2430 HT2=((AC*FF(4,2)*DELTA)/(2*G*D*A^2))*((ABS(Q(N-1,1))*Q(N-1,1))
2440 H(N,2)=HT1-HT2
2450 FOR K=2 TO N
2460 Q(K,1)=Q(K,2)
2470 H(K,1)=H(K,2)
2480 NEXT K
2490 FOR K=1 TO N
2500 QD(K,KK)=Q(K,1)
2510 HD(K,KK)=H(K,1)
2520 REM PRINT QD(K,KK),HD(K,KK)
2530 NEXT K
2540 IDC=5*DELTA
2550 KK=KK+1
2560 IF P=2 GOTO 2710
2570 FOR K=2 TO 4
2580 QK1=.5*(Q(K-1,1)+Q(K+1,1))
2590 QK2=((G*A)/(2*AC))*(H(K-1,1)-H(K+1,1))
2600 FF(K,2)=1/(2*C*LOG((RK/(3.71*D))+(5.1286/((ABS(Q(K,1)/A))*D)/VIS*.89)))^2
2610 FP(K,KK)=FF(K,2)
2620 QK3=((FF(K,2)*DELTA)/(4*D*A))*((ABS(Q(K-1,1))*Q(K-1,1))+(ABS(Q(K+1,1))*Q(K+1,1)))
2630 Q(K,2)=QK1+QK2-QK3
2640 HHT1=.5*(H(K-1,1)+H(K+1,1))
2650 HHT2=(AC/(2*G*A))*(Q(K-1,1)-Q(K+1,1))
2660 HHT3=(AC*FF(K,2)*DELTA)/(4*G*D*A^2)
2670 HHT4=HHT3*(ABS(Q(K+1,1))*Q(K+1,1)-ABS(Q(K-1,1))*Q(K-1,1))
2680 H(K,2)=HHT1+HHT2+HHT4
2690 NEXT K
2700 GOTO 2770
2710 FOR K=2 TO 4
2720 FF(K,2)=1/(2*C*LOG((RK/(3.71*D))+(5.1286/((ABS(Q(K,1)/A))*D)/VIS*.89)))^2
2730 FP(K,KK)=FF(K,2)
2740 Q(K,2)=.5*(Q(K-1,1)+Q(K+1,1))-(G*A*DELTA)/(2*AC)*(H(K+1,1)-H(K-1,1))-
2750 ((FF(K,2)*DELTA)/(8*D*A))*(Q(K-1,1)+Q(K+1,1))*ABS(Q(K-1,1)+Q(K+1,1))
2750 \[ H(K,2) = 0.5 \times (H(K-1,1) + H(K+1,1)) \times \frac{AC/(2*G*A*DELTA)}{2*G*A*DELTA} \times (Q(K+1,1) - Q(K-1,1)) \]
2760 \[ \text{NEXT } K \]
2770 \[ Q(N,2) = 0 \]
2780 \[ HT1 = H(N-1,1) \times \frac{AC/(G*A)}{Q(N-1,1)} \]
2790 \[ HT2 = \frac{(AC \times FF(4,2) \times DELTA)}{(2*G*DA^2)} \times ((ABS(Q(N-1,1)^2) \times Q(N-1,1)) \]
2800 \[ H(N,2) = HT1 - HT2 \]
2810 \[ Q(1,1) = Q(1,1)/(AC/(G*A)) \]
2820 \[ \text{FOR } K = 2 \text{ TO } N \]
2830 \[ Q(K,1) = Q(K,1) \]
2840 \[ H(K,1) = H(K,2) \]
2850 \[ \text{NEXT } K \]
2860 \[ FP(1,KK) = FP(2,KK) \]
2870 \[ FP(5,KK) = 0 \]
2880 \[ IDC = IDC + DELTA \]
2890 \[ \text{FOR } K = 1 \text{ TO } N \]
2900 \[ QD(K,KK) = Q(K,1) \]
2910 \[ HD(K,KK) = H(K,1) \]
2920 \[ \text{REM PRINT } KK.QD(K,KK),HD(K,KK) \]
2930 \[ \text{NEXT } K \]
2940 \[ \text{IF } IDC < TTT \text{ GOTO } 2990 \]
2950 \[ KK = KK + 1 \]
2960 \[ \text{GOTO } 2710 \]
2970 \[ \text{GOTO } 2570 \]
2980 \[ \text{GOTO } 2570 \]
2990 \[ \text{REM PRINT } \text{TIME...Q(1,1),Q(2,1),Q(3,1),Q(4,1),Q(5,1)} \]
3000 \[ \text{PRINT} \]
3010 \[ \text{PRINT} \]
3020 \[ \text{PRINT} \]
3030 \[ \text{FOR } KK = 2 \text{ TO } M \]
3040 \[ TD(1,1) = 0 \]
3050 \[ M = KK \]
3060 \[ \text{PRINT} \]
3070 \[ \text{PRINT} \]
3080 \[ \text{FOR } KK = 1 \text{ TO } M \]
3090 \[ \text{REM PRINT USING } \text{TIME...Q(1,1),Q(2,1),Q(3,1),Q(4,1),Q(5,1)} \]
3100 \[ \text{PRINT} \]
3110 \[ \text{PRINT} \]
3120 \[ \text{PRINT} \]
3130 \[ \text{PRINT} \]
3140 \[ \text{PRINT} \]
3150 \[ \text{PRINT} \]
3160 \[ \text{PRINT} \]
3170 \[ \text{PRINT} \]
3180 \[ \text{PRINT} \]
3190 \[ \text{PRINT} \]
3200 \[ \text{PRINT} \]
3210 \[ \text{PRINT} \]
3220 \[ \text{PRINT} \]
3230 \[ \text{PRINT} \]
3240 \[ \text{PRINT} \]
3250 \[ P = P + 1 \]
3260 \[ \text{IF } P = 3 \text{ GOTO } 3300 \]
3270 \[ \text{GOTO } 690 \]
3280 \[ \text{LOCATE } 22,1; \text{PRINT**} \]
3290 \[ \text{END} \]