CHAPTER-2

APPROXIMATE RESISTANCE EQUATION USED IN THE ANALYSIS OF PRESSURE VARIATION IN SURGE TANK AND PRESSURE CONDUIT

2.1 INTRODUCTION:

A brief review of resistance formulae for use in conduit flow is presented at the beginning of this chapter. The Darcy-Weisbach friction factor $f$ is used to assess the resistance to flow.

Resistance equations for pipe flow as based on modern concept of Prandtl-Karman\(^1,2\) form of logarithmic resistance laws are preferred. The Explicit solutions of the Colebrook-White\(^3\) equation developed by Jain\(^4\) and Barr\(^5,6\) have been assessed with the help of a small computer program as described in this chapter. The percentage error of the above explicit solutions for full range of laminar to turbulent flow conditions are determined in the program. Barr's explicit solution presents slightly less error than does the Jain's solution. Therefore, the former explicit solution is preferred for use in the author's work due to its simplicity.

2.2 REVIEW OF FRICTION FACTOR FORMULA:

Darcy-Weisbach formula of friction factor $f$ is expressed as:

$$\frac{1}{\sqrt{f}} = \frac{V}{\sqrt{sgD}} \quad \text{............... (2.1)}$$

Equation (2.1) above does not consider overtly the roughness and the viscosity of the liquid. Implicity thus is allowed for in
choice of value of \( f \), which then allows \( V \) to be found given \( s \), or vice versa. To obtain a more general formula which considers the above two parameters explicitly, recourse has to be made to a more modern concept.

Modern concepts are based on the Prandtl's boundary layer theory and Von Karman's turbulence theory. These have profoundly effected the present concept of resistance to flow in pipe. Nearly a century prior to Prandtl, the non dimensional resistance coefficient \( f \) as given in equation (2.1) was first introduced by Weisbach, following interaction with the work of Darcy.

By 1932, Prandtl\(^1\) had deduced a formula for \( f \) in smooth pipes as a function of Reynolds number \( R \). This could be rearranged to give the form shown below, with empirical constants 2 and 2.51 evaluated from Nikuradse's\(^6\) experimental data.

\[
\frac{1}{\sqrt{f_{\text{smooth}}}} = 2 \log_{10} \left[ \frac{R \sqrt{f}}{2.51} \right] \quad \ldots \quad \ldots \quad (2.2)
\]

By the next year (1933), Nikuradse\(^6\) had shown that in a pipe of uniform diameter, if walls are rough and Reynolds number is sufficiently high, \( f \) becomes independent of \( R \) and depends only on relative roughness values. He evaluated a formula for \( f \) in terms of ratio of diameter \( D \) of the pipe to the diameter size \( k \) of the sand grains that he had used to roughen the inside of the pipe. The formula can be re-arranged to:

\[
\frac{1}{\sqrt{f_{\text{rough}}}} = 2 \log_{10} \left[ \frac{2.71 D}{k} \right] \quad \ldots \quad (2.3)
\]

Nikuradse's data have served as the basis for many subsequent analyses of frictional resistance in pipes and in open channels for smooth, transitional and fully rough turbulent flow. However, Nikuradse used uniform sand grains for roughness and this
produced an increase in $f$ with Reynolds number $R$ over a particular range of partly rough flow.

In 1937, Colebrook and White\textsuperscript{3} investigated the same range using nonuniform roughness. It was found that $f$ then decreased throughout the partly rough flow region. In 1939, combining equation (2.2) and (2.3), Colebrook gave the following familiar transition formula for commercial pipe:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \frac{k}{3.71D} + \frac{2.51}{R \sqrt{f}} \right] \quad \ldots \quad (2.4)$$

Thus the Colebrook-White (C-W) transition formula covers the above transition for smooth to rough turbulent flow, and is one of the important and popular formulae in resistance to flow in pipes.

### 2.3 EXPLICIT APPROXIMATIONS FOR COLEBROOK-WHITE (C-W) EQUATION:

Direct applications of C-W equation is implicit for the evaluation of $f$. For field engineer, direct use of this equation is tedious and time consuming. Explicit forms of this equation are proposed by Barr\textsuperscript{5,6}, Jain\textsuperscript{4} and a few others. The development by Barr and Jain are of recent origin. These two equations are:

$$\frac{1}{\sqrt{f}} = -2 \log_{10} \left[ \frac{k}{3.71D} + \frac{5.1286}{R^{0.89}} \right] \quad \text{(by Barr)} \ldots (2.5)$$
2.4 ASSESSMENT OF BARR'S AND JAIN'S APPROXIMATIONS:

A small computer program has been developed to assess the validity in using one of the above equations to evaluate friction factor $f$. The computer program is included in the end of this chapter. Full range of Reynolds numbers from $2 \times 10^3$ to $10^7$ are taken to cover the turbulent flow stage up to fully turbulent. Relative roughness $[k/D]$ values are also changed from $10^{-6}$ to $10^{-1}$. Departure of friction factor values by these two equations from C-W equations for above flow stages and roughness numbers are calculated. The average percentage errors in both the equations are presented in table 2.1. The table 2.2 shows that average percentage error in Barr's explicit C-W equation is smaller than Jain's equation for all values of $[k/D]$. The maximum in Barr's equation is $-0.5096\%$ whereas maximum in Jain's equation comes out to be $-0.6072\%$. Both the equations give almost small equal error but preference is given to the Barr's equation as all the percentage errors are always smaller than Jain's equation.

2.5 ASSESSMENT OF FRICITION FACTOR AT LOW REYNOLDS NUMBER:

In the above section 2.4, it has been indicated that the friction factor $f$ is assessed for the range of Reynolds number from $2.5 \times 10^3$ to $10^7$ in Barr's and Jain's equations. These two equations do not consider the assessment of friction factor below the above range. When Reynolds number is less than 1500, the following well known Poiseuille equation is used.

\[
f = \frac{0.3164}{R^{0.25}} \hspace{1cm} \text{.............. (2.7)}
\]
If Reynolds number falls between $2.5 \times 10^3$ to 1500, a constant friction factor values equal to 0.04 has been adopted. This assessment has been adopted based on recommendations made by various investigators who have already worked extensively in the assessment of friction factor in full range of Reynolds number or flow conditions from purely laminar to fully turbulent.

2.6 CONCLUSION:

Barr's explicit solution of C-W equation is preferred and will be used in subsequent chapters to evaluate friction factor. This is due to its simplicity and the lesser percentage error. It is also concluded from the analysis that field engineers may confidently use any one of the above explicit equations in the computation of $f$ for commercial pipe. This analysis is presented to justify the use of Barr's explicit equation to evaluate friction factor at every time step of numerical solution's of nonlinear equations of continuity and momentum. Application can be to both the most practical situations of analysis of surge tank conditions and of pressure fluctuation in pipes generally because the equation covers the full range of turbulent flow conditions.

It may be mentioned this type resistance equation was successfully used in the case of open channel flow where pipe diameter is replaced by the equivalent diameter of the open channel i.e. four times the hydraulic radius. Das$^9$, Barr and Das$^9,10$ had successfully applied this type of logarithmic resistance law in dam-break and bore reflection problems in open channels, where the resistance law was the generalised C-W approximation of Barr which takes care of different types of roughness with different transition routes. Vardy and Brown$^{47,48}$ also obtained a strong...
variation of $f$ from turbulent to laminar transient flow in smooth pipes.

Considering all the above facts, the author has decided to use the equation (2.5) and (2.7) for friction factor evaluation.

The influence of friction factor and its variation in controlling the pattern of damping of the pressure surge will be displayed in the next chapter.
### Table 2.1 Numerical p.c. error assessment of explicit C-W equation

Values of friction factor $f$ in the following formulae

(when $k/D= .0000001$ )

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<th>Reynolds no.</th>
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<th>Jain</th>
<th>C-W</th>
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p.c. error in the formula of

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Values of friction factor $f$ in the following formulae
(when $k/D = .000001$)

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p.c. error in the formula of

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Values of friction factor $f$ in the following formulae
(when $k/D = 0.00001$)

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p.c. error in the formula of $k/D$

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Values of friction factor $f$ in the following formulae
(when $k/D = 0.0001$)

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</tr>
<tr>
<td>80000</td>
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<td>0.02282</td>
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</tr>
<tr>
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<td>0.02232</td>
<td>0.02217</td>
</tr>
<tr>
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<td>0.02075</td>
<td>0.02073</td>
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</tr>
<tr>
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<td>0.02017</td>
<td>0.02015</td>
<td>0.02006</td>
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<td>0.02001</td>
<td>0.01993</td>
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<tr>
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<td>0.01976</td>
<td>0.01973</td>
</tr>
<tr>
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<td>0.01970</td>
<td>0.01969</td>
<td>0.01967</td>
</tr>
<tr>
<td>10000000</td>
<td>0.01968</td>
<td>0.01967</td>
<td>0.01965</td>
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</table>

p.c. error in the formula of

<table>
<thead>
<tr>
<th>$k/D$</th>
<th>Reynolds no.</th>
<th>Barr</th>
<th>Jain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001000</td>
<td>2500</td>
<td>-1.635108</td>
<td>-2.767428</td>
</tr>
<tr>
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<td>10000</td>
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<td>30000</td>
<td>-0.423283</td>
<td>-0.521278</td>
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<td>-0.394539</td>
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<tr>
<td>0.001000</td>
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<td>-0.188510</td>
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<tr>
<td>0.001000</td>
<td>7000000</td>
<td>-0.142390</td>
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<tr>
<td>0.001000</td>
<td>10000000</td>
<td>-0.110294</td>
<td>-0.061781</td>
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</tbody>
</table>
Values of friction factor $f$ in the following formulae (when $k/D = 0.001$)

<table>
<thead>
<tr>
<th>Reynolds no.</th>
<th>Barr</th>
<th>Jain</th>
<th>C-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
<td>0.05551</td>
<td>0.05595</td>
<td>0.05390</td>
</tr>
<tr>
<td>4000</td>
<td>0.05027</td>
<td>0.05054</td>
<td>0.04906</td>
</tr>
<tr>
<td>6000</td>
<td>0.04693</td>
<td>0.04710</td>
<td>0.04595</td>
</tr>
<tr>
<td>8000</td>
<td>0.04509</td>
<td>0.04520</td>
<td>0.04421</td>
</tr>
<tr>
<td>10000</td>
<td>0.04390</td>
<td>0.04398</td>
<td>0.04310</td>
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<td>0.04028</td>
<td>0.04028</td>
<td>0.03979</td>
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<td>0.03941</td>
<td>0.03905</td>
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<td>0.03862</td>
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<td>0.03872</td>
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<td>300000</td>
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<td>0.03817</td>
<td>0.03807</td>
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<tr>
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<td>0.03802</td>
<td>0.03800</td>
<td>0.03796</td>
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<td>0.03798</td>
<td>0.03796</td>
<td>0.03793</td>
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p.c. error in the formula of

<table>
<thead>
<tr>
<th>k/D</th>
<th>Reynolds no.</th>
<th>Barr</th>
<th>Jain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010000</td>
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<td>-3.810372</td>
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<tr>
<td>0.010000</td>
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<td>-3.017450</td>
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<td>-2.509448</td>
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<tr>
<td>0.010000</td>
<td>8000</td>
<td>-1.978929</td>
<td>-2.229534</td>
</tr>
<tr>
<td>0.010000</td>
<td>10000</td>
<td>-1.851963</td>
<td>-2.034599</td>
</tr>
<tr>
<td>0.010000</td>
<td>30000</td>
<td>-1.233619</td>
<td>-1.225905</td>
</tr>
<tr>
<td>0.010000</td>
<td>50000</td>
<td>-0.961423</td>
<td>-0.920134</td>
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<td>0.010000</td>
<td>80000</td>
<td>-0.740770</td>
<td>-0.684722</td>
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<tr>
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<td>-0.648590</td>
<td>-0.588958</td>
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<tr>
<td>0.010000</td>
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<tr>
<td>0.010000</td>
<td>700000</td>
<td>-0.172032</td>
<td>-0.113605</td>
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<tr>
<td>0.010000</td>
<td>1000000</td>
<td>-0.131823</td>
<td>-0.075006</td>
</tr>
<tr>
<td>0.010000</td>
<td>3000000</td>
<td>-0.056603</td>
<td>-0.003648</td>
</tr>
<tr>
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<td>7000000</td>
<td>-0.028895</td>
<td>0.022227</td>
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<td>10000000</td>
<td>-0.021688</td>
<td>0.02891</td>
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</table>
Values of friction factor $f$ in the following formulae (when $k/D = 0.01$)

<table>
<thead>
<tr>
<th>Reynolds no.</th>
<th>Barr</th>
<th>Jain</th>
<th>C-W</th>
</tr>
</thead>
<tbody>
<tr>
<td>2500</td>
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<td>0.11171</td>
<td>0.10782</td>
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<tr>
<td>4000</td>
<td>0.10810</td>
<td>0.10819</td>
<td>0.10551</td>
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<tr>
<td>6000</td>
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<td>0.10420</td>
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<tr>
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<td>0.10353</td>
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<tr>
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<td>0.10441</td>
<td>0.10313</td>
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<tr>
<td>30000</td>
<td>0.10261</td>
<td>0.10254</td>
<td>0.10205</td>
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<tr>
<td>50000</td>
<td>0.10221</td>
<td>0.10213</td>
<td>0.10183</td>
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<tr>
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<td>0.10197</td>
<td>0.10188</td>
<td>0.10171</td>
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<tr>
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<td>0.10188</td>
<td>0.10180</td>
<td>0.10167</td>
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<td>300000</td>
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<td>0.10156</td>
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<tr>
<td>700000</td>
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<td>0.10153</td>
</tr>
<tr>
<td>1000000</td>
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<td>0.10152</td>
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<td>0.10143</td>
<td>0.10151</td>
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</table>

p.c. error in the formula of

<table>
<thead>
<tr>
<th>k/D</th>
<th>Reynolds no.</th>
<th>Barr</th>
<th>Jain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.100000</td>
<td>2500</td>
<td>-3.394860</td>
<td>-3.607816</td>
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<tr>
<td>0.100000</td>
<td>4000</td>
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<td>-2.542878</td>
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<td>6000</td>
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<td>0.100000</td>
<td>8000</td>
<td>-1.493206</td>
<td>-1.482317</td>
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<td>0.100000</td>
<td>10000</td>
<td>-1.266677</td>
<td>-1.237252</td>
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<td>30000</td>
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<td>-0.288936</td>
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<td>0.100000</td>
<td>80000</td>
<td>-0.252958</td>
<td>-0.171757</td>
</tr>
<tr>
<td>0.100000</td>
<td>100000</td>
<td>-0.211494</td>
<td>-0.129725</td>
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<tr>
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<td>300000</td>
<td>-0.086589</td>
<td>-0.004556</td>
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<tr>
<td>0.100000</td>
<td>700000</td>
<td>-0.042996</td>
<td>0.038365</td>
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<tr>
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<td>1000000</td>
<td>-0.031954</td>
<td>0.049134</td>
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<tr>
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<td>0.067724</td>
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<td>7000000</td>
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<td>0.073921</td>
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<tr>
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<td>10000000</td>
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<td>0.075463</td>
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</table>

Table 2.2 Average error in the explicit C-W formula of Barr and Jain

<table>
<thead>
<tr>
<th>Average error in the formula of</th>
<th>Barr</th>
<th>Jain</th>
<th>k/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.109499</td>
<td>-0.131991</td>
<td>0.000001</td>
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</tr>
<tr>
<td>-0.148314</td>
<td>-0.221788</td>
<td>0.000010</td>
<td></td>
</tr>
<tr>
<td>-0.241859</td>
<td>-0.414848</td>
<td>0.000100</td>
<td></td>
</tr>
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<td>-0.594272</td>
<td>-0.812226</td>
<td>0.001000</td>
<td></td>
</tr>
<tr>
<td>-1.089950</td>
<td>-1.215207</td>
<td>0.010000</td>
<td></td>
</tr>
<tr>
<td>-0.873685</td>
<td>-0.847582</td>
<td>0.100000</td>
<td></td>
</tr>
</tbody>
</table>
Hence
Average error in all k/D and Reynolds nos. in Barr's and Jain's
equations are:

Barr's equation = -0.5095965
Jain's equation = -0.6072737

REM COMPUTER PROGRAM TO ASSESS THE EXPLICIT C-W EQUATIONS.
AND TO GENERATE THEREFROM THE Table 2.1 AND Table 2.2

DIM REY(20), FB(20), FJ(20), FAAA(20), FAA(20), FA(20), ERB(20), ERJ(20)
DIM B(20), J(20), D(20)
N = 15
DK = .0000001
C = 1 / 2.302585
FOR I = 1 TO N
READ REY(I)
NEXT I
DATA 2500, 4000, 6000, 8000, 10000, 30000, 50000, 80000, 100000, 300000, 700000
DATA 1000000, 3000000, 7000000, 10000000
DKD = DK * 10
PRINT "Table 2.1 Numerical p.c. error assessment of explicit C-W equation"
PRINT "Values of friction factor f in the following formulae following formulae (when k/D = n; DKD: * )"
IF DKD > .1 GOTO 750
PRINT"
PRINT " Reynolds no. Barr Jain C-W"
FOR I = 1 TO N
FB(I) = 1 / (-2 * C * LOG((DKD / 3.71) + (5.1286 / (REY(I) ** .89)))) ** 2
FJ(I) = 1 / (1.14 - 2 * C * LOG(DKD + (21.25 / (REY(I) ** .9)))) ** 2
NEXT I
FOR I = 1 TO N
FAA(I) = FB(I)
FAA(I) = FAAA(I)
FA(I) = 1 / (-2 * C * LOG((DKD / 3.71) + (2.51 / (REY(I) * (FAA(I)) ** .5)))) ** 2
FAAA(I) = FA(I)
IF FA(I) - FAA(I) > .0001 THEN GOTO 290
IF FA(I) - FAA(I) < -.0001 THEN GOTO 290
NEXT I
FOR I = 1 TO N
PRINT USING "##########"; REY(I); FB(I); FJ(I); FA(I)
PRINT"
FOR I = 1 TO N
ERB(I) = ((FA(I) - FB(I)) / FA(I)) * 100
ERJ(I) = ((FA(I) - FJ(I)) / FA(I)) * 100
NEXT I
PRINT**
18

PRINT " k/D Reynolds no. Barr Jain"
480 PRINT ""
490 FOR I = 1 TO N
500 PRINT USING "##.#######"; DKD;
510 PRINT USING "#####"; REY(I);
520 PRINT USING "####M#.####M"; ERB(I); ERJ(I)
530 NEXT I
540 FOR I = 1 TO N
550 ERBB = ERBB + ERB(I)
560 ERJJ = ERJJ + ERJ(I)
570 NEXT I
580 ERBB = ERBB / 15
590 ERJJ = ERJJ / 15
600 M = M + 1
610 B(M) = ERBB
620 J(M) = ERJJ
630 DK = DK * 10
640 D(M) = DK
650 IF DK > 9.000001E-02 GOTO 750
700 PRINT ""
710 PRINT ""
720 PRINT ""
730 PRINT " Values of friction factor f in the following formulae"
735 PRINT " when k/D=", DKD
740 GOTO 180
750 PRINT ""
760 PRINT " Table 2.2 Average error in the explicit C-W formula of Barr and Jain"
770 PRINT ""
780 PRINT " Average error in the formula of"
790 PRINT " Barr Jain k/D"
800 PRINT " --- --- ---"
810 FOR I = 1 TO M
820 PRINT USING "######ff.####M"; B(I); J(I); D(I)
830 PRINT ""
840 NEXT I
850 FOR I = 1 TO M
860 BA = BA + B(I)
870 FA = FA + J(I)
880 NEXT I
890 BA = BA / 6
900 FA = FA / 6
910 PRINT
920 PRINT "Hence"
930 PRINT "Average error in all k/D and Reynolds nos. in Barr's and Jain's equations are:" 
940 PRINT " Barr's equation=", BA
950 PRINT " Jain's equation=", FA
960 PRINT ""
970 PRINT ""
980 END