CHAPTER 2

PROPAGATION OF LIGHT IN A FIBRE WAVEGUIDE

2.1 THE WAVE NATURE OF LIGHT

The perception of light is the mean by which the world can be viewed. The way everything is seen or visualized is due to the wave nature of light. The manner by which light travels can be seen by looking at the surface waves of water. Till the seventeenth century, it was believed that light consisted of a stream of luminous particles emitted by luminous sources. However later works by Maxwell in 1864 showed that light is an electromagnetic wave and so it is polarized in nature. This polarization effect also demonstrated that light waves are transverse in nature (i.e. wave motion in perpendicular to the direction in which light travels). The wave theory of light accounts for all phenomena related to the transmission of light. From the viewpoint of wave motion of light, the electromagnetic wave radiated by a small optical source is represented by a train of spherical *wavefronts* with the source at the centre as shown in Figure 2.1. A wave front is defined as “the locus of all points in the wave train which have the same phase” [23, p.17].

When the wavelength of light is much smaller than the object or opening (which it encounters), the wave fronts appear as straight lines to the object or opening. The light wave is then represented by a plane wave and the direction of travel is indicated by a light ray drawn perpendicular to the wave front.

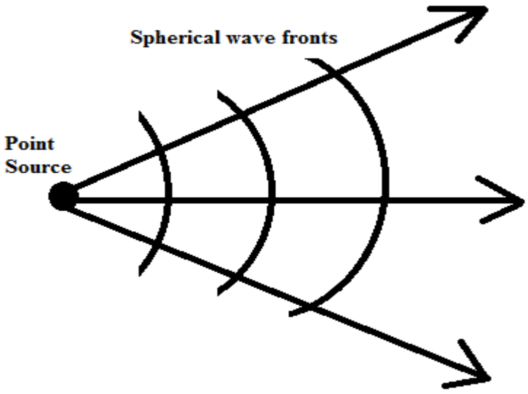

Fig. 2.1: Optical wave fronts from point source

2.2 WAVE THEORY OF OPTICAL WAVEGUIDE

It was after mid-1960 that the idea of a communication system based on propagation of light within circular dielectric waveguides was considered. The propagation of light through an optical waveguide requires the solution of the Maxwell equation [23] which is discussed in section 2.2.2. Light travels within a circular dielectric waveguide by total internal reflection, the detail of which is discussed in section 2.2.1.

2.2.1 Propagation of Light in Optical Fibre Waveguide

An Optical fibre waveguide can be considered as a single solid dielectric cylinder called as the core having a refractive index of refraction n_1. The core is surrounded by a solid dielectric called the cladding having refractive index n_{cl} that is less than n_1. In addition to the core and the cladding the fibre may be encapsulated in an elastic, abrasion-resistant plastic material called as the buffer. The buffer adds mechanical strength to the fibre and shields the fibre from small geometrical irregularities, distortions, or roughness of adjacent surfaces [23]. The geometrical structure of an optical fibre wave guide is shown in Figure 2.2.

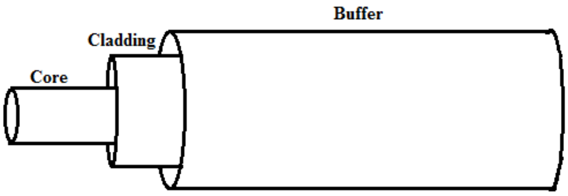

Figure 2.2: Geometrical structure of an optical fibre waveguide

The variation in the material composition of the core give rise to two commonly types of fibres. If the refractive index of the core is uniform throughout and undergoes an abrupt (step) change at the cladding, then it is called a step index fibre. On the other hand if the refractive index of the core varies as a function of the radial distance from the center, then it is called graded index fibre.

The propagation of light along an optical fibre (waveguide) can be described in terms of a set of guided electromagnetic waves called the *modes* of the wave guide. These guided modes are trapped within the waveguide. Each guided mode is a pattern of electric and magnetic lines that is repeated along the fibre at intervals equal to the wavelength. The step and the graded index are divided into Single-mode and Multimode fibres depending on the number of modes supported by them. A single- mode fibre can sustain only one mode of propagation, whereas a multimode fibre can support hundreds of modes. Now each modes in a multimode fibre can propagate at slightly different velocity i.e. the modes in a given optical pulse arrive at the fibre end at different times thus causing the pulse to spread out in time as it travels along the fibre. This effect is known as Intermodal dispersion [23].

A light ray can propagate along the length of a cylindrical waveguide as long as the ray is incident within the acceptance cone[1] semi angle of the fibre which in turn is dependent on the refractive index of the core and cladding of the fibre [24] as shown in Fig. 2.3. The light should enter the fibre at an incident angle θ_a such that the internal reflection angle ϕ is greater than the critical angle θ_C. The light will be contained within the fibre and propagate to the far end due to total internal reflection. If the condition is not satisfied than the light will leak into the cladding.

[1] Acceptance cone is the maximum angle at which light must be incident at the core-cladding interface so that the angle is greater than the critical angle at the interface.

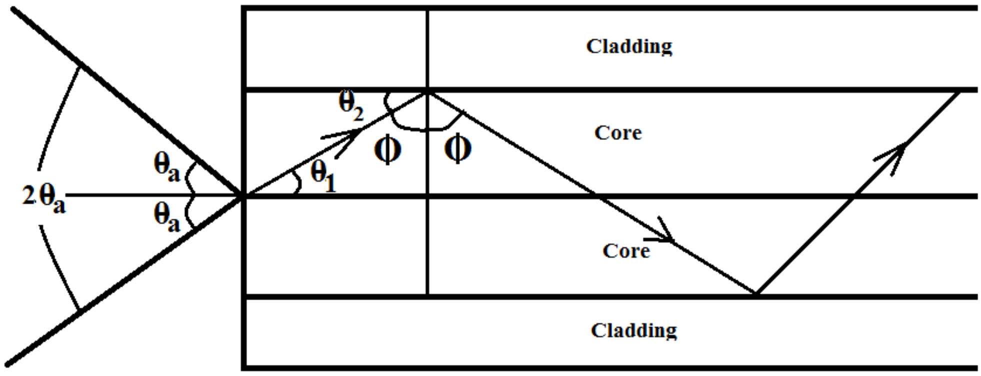

Fig. 2.3: The acceptance cone and propagation of light in an optical fibre

The ray enters the fibre at an angle outside the cone and it will leave the core and will leave the fibre itself. The term Numerical Aperture (NA) is used as a figure of merit and is defined as the Sine of the largest angle contained within the cone of acceptance.

The Numerical Aperture is given by –

$$NA = \sqrt{n_1^2 - n_{cl}^2} \tag{2.1}$$

where $n_1$and n_{cl} represents the refractive index of the core and cladding of the fibre (single mode or multimode) respectively.

The normalized difference Δ between the core and cladding is given by-

$$\Delta = \frac{n_1 - n_{cl}}{n_{cl}} \tag{2.2}$$

Thus Numerical Aperture (NA) can now be given as [24]-

$$NA = n_1\sqrt{2\Delta} \tag{2.3}$$

However the propagation of light through a circular waveguide can also be approximated with the help of a parameter called the normalized frequency or V number which is defined as-

$$V = \left(2\pi/\lambda_0\right) \times a_0 n_1 \sqrt{2\Delta} \tag{2.4}$$

Where a_0 is the core radius of the waveguide (optical fibre), λ_0 is the free-space wavelength, Δ is the relative core-cladding index difference

If $V>>10$, the geometrical-optics results can explain the propagation effect in optical fibre, whereas for $V<10$, the geometrical-optics results cannot explain the propagation effect. Thus to have a general framework for an optical waveguide for any arbitrary V number, it is necessary to take the Maxwell equations [24].

2.2.2 The Maxwell Equation

The Maxwell equation is used to derive relationship between the electric and magnetic fields in an optical waveguide. Assuming a linear, isotropic dielectric material having no current and free charges, the equations take the form [23]-

$$\nabla \times E = -\frac{\partial B}{\partial t} \tag{2.5}$$

$$\nabla \times H = \frac{\partial H}{\partial t} \tag{2.6}$$

$$\nabla . D = 0 \tag{2.7}$$

$$\nabla . B = 0 \tag{2.8}$$

Where $D = \epsilon E$ and $B = \mu H$, ϵ is the permittivity and μ is the permeability of the medium. E and H represents the electric and magnetic field respectively.
Taking the curl of equation (2.5) and using equation (2.6), we get

$$\nabla \times (\nabla \times \mathrm{E}) = -\mu \frac{\partial}{\partial \mathrm{t}} (\nabla \times \mathrm{H}) = -\epsilon\mu \frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} \tag{2.9}$$

Using vector identity, equation (2.9) becomes

$$\nabla \times (\nabla \times \mathrm{E}) = \nabla(\nabla . \mathrm{E}) - \nabla^2 \mathrm{E} \tag{2.10}$$

Now using equation (2.7), equation (2.9) becomes

$$\nabla^2 \mathrm{E} = \epsilon\mu \frac{\partial^2 \mathrm{E}}{\partial \mathrm{t}^2} \tag{2.11}$$

Similarly taking the curl of equation (2.6), it can be shown that

$$\nabla^2 H = \epsilon\mu \frac{\partial^2 H}{\partial t^2} \tag{2.12}$$

Equation (2.11) and (2.12) are the standard wave equation.

Solving the Maxwell equation shows that the optical fibre waveguide has an infinite continuum of radiation modes that are not trapped in the core and guided by the fibre but are still solutions of the same boundary value problem. The radiation fields results from the optical power that is outside the fibre acceptance angle refracted out of the core. As the radius of the cladding is finite, some of the radiation gets trapped in the cladding. As core and cladding modes propagate along the fibre, mode coupling occurs between cladding modes and higher order core modes. A diffusion of power back and forth between the core and cladding modes occur which results in loss of power in the core modes [23].

There are three types of modes in an optical fibre viz. the bound mode, refracted mode and leaky mode. An optical Fibre can switch between the different modes following a boundary condition which depends on the propagation constant β given by

$$n_{cl}k_0 < \beta < n_1 k_0$$

Where n_1 and n_{cl} are the core and cladding refractive indices and $k_0 = \frac{\omega}{c} = \frac{2\pi}{\lambda_0}$, k_0 is called the wave number.

The boundary between truly guided modes and leaky modes is defined by the cut off condition $\beta = n_{cl}k_0$. If β is less than $n_{cl}k_0$, then power leaks out from the core to the cladding.

2.3. ATTENUATION DUE TO CURVATURE LOSS IN A MULTIMODE OPTICAL FIBRE

Multimode fibres are the types of fibres which allow many modes to propagate through the waveguide. These fibres can be step index fibres and graded index fibres. Multimode step index fibres have got large core diameters and large numerical apertures to facilitate efficient coupling to incoherent light sources such as light emitting diodes (LED's). They are manufactured from multi-component glass compound or doped silica. On the other hand, multimode graded index fibres are also constructed from glass compound or doped silica but of higher purity to reduce losses. These fibres have diameter less than Multimode step index fibres. When the multimode step or graded index fibres are bent, then they suffer from radiation losses [25]. The radiation loss in optical fibre bends is based on the studies made by Miller and Marcatili and modified by Shevchenko. Their analysis lead to a straight forward formula for finding the attenuation loss in multimode fibre due to bending (α_B) in terms of propagation constant (β), which is not far from cut-off as given by Gloge [26].

$$\alpha_B = \frac{2\gamma^2(0)}{\beta} exp\left[-2\int_{a_0}^{r_0} \gamma(r)dr\right] \tag{2.13}$$

where
$$\gamma^2(r) = \frac{\beta^2 R'^2}{(r+R')^2} - n_{cl}^2 k_0^2 \tag{2.14}$$

γ is the propagation constant of a wave in a waveguide.

and
$$r = {R'\beta}\big/{n_{cl}k_0} - R' \tag{2.15}$$

R' is the curvature radius, r is the distance of propagation of the wave in the fibre guide and a_0 is the core radius

For a straight fibre, the mode field decreases as $exp[-\gamma(0)(r - a_0)]$ in the cladding at a distance r from the guide axis produces a loss for this mode proportional to $exp[-2\gamma(0)(r - a_0)]$

When a multimode fibre is put at a sharp bend (Macrobend) with a fixed radius of curvature, light rays are lost into cladding, which results in power loss and thus attenuation. For a slight bend, the loss in the optical power is extremely small and unobservable. However, as the radius of curvature decreases the loss increases exponentially until at a certain critical radius the fibre suffers radiation loss at bend/ curves along the path. The distributed modes in a bend optical fibre [23] can be represented as shown in Figure 2.4

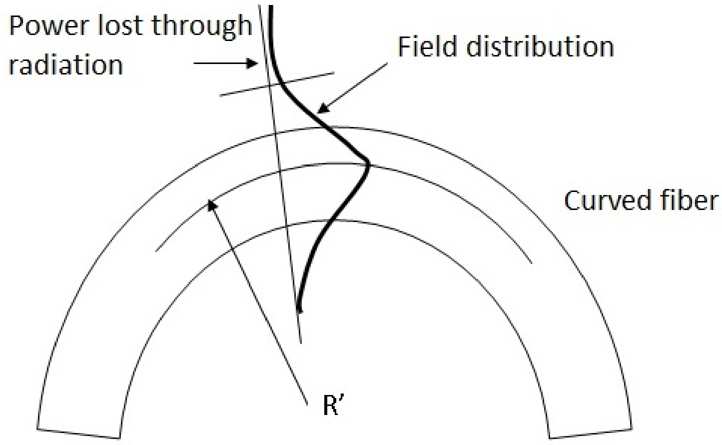

Fig. 2.4: Schematic representation of radiation loss of a mode at a fibre bend

When a fibre is bent, the fraction of the mode field in the bent fibre that travels along the periphery of the circular arc in the cladding may at some stage be required to travel faster than the local plane wave velocity in order to maintain equiphase- fronts at radial planes. This being physically disallowed, the part of the modal field dissociates itself from the fibre and get lost through radiation from its sides [24].

The curvature of the optical fibre waveguide has two effects: Firstly, the stretching of the waveguide at the outside of the bend leads to a velocity increase in the outer wing of the mode and hence to an apparent decrease of the propagation constant β. As a result, β to be replaced by ${}^{\beta R'}/_{r\,+\,R'}$ the mode field now decreases as

$$exp\left[-\int_{a_0}^{r}\gamma(r)d\,r\right] \qquad (2.16)$$

Second, a loss mechanism arises at $r = r_0$, where the mode velocity reaches the velocity of light in the cladding material $[\gamma(r_0) = 0]$. The mode energy passing this radius is radiated off. The resulting loss leads to the exponential term in equation (2.13)

The approximate form of $\gamma^2(r)$ is written in the form-

$$\gamma^2(r) = \gamma^2(0) - {}^{2\beta^2 r}/_{R'} \qquad (2.17)$$

Again if β is replaced by $n_1 k_0$, then inserting equation (2.13) in equation (2.17), we get

$$\alpha_B = \frac{2\gamma^2(0)}{n_1 k_0} exp\left[-\frac{2}{3} n_1 k_0 R' \left(\frac{\gamma^2(0)}{n_1^2 k_0^2} - \frac{2a_0}{R'}\right)^{3/2}\right] \tag{2.18}$$

where, $$\gamma(0) = \sqrt{\beta^2 - k_0^2 n_{cl}^2} \tag{2.19}$$

Where, R' represents the radius of curvature, a_0 is the core radius, n_1 and n_{cl} are the RI of core and cladding of the fibre respectively, β is the propagation constant and $k_0 = \omega/c$. Thus the expression of loss coefficient due to bending is a function of the core and cladding refractive index and the radius of bending of the fibre.

2.4 PRINCIPLE OF BARE AND TAPERED OPTICAL FIBRE REFRACTOMETER

Optical fibres can be used to measure the refractive index of liquids through intensity modulation of light by the measurand [24]. Kumar, A., *et al.* [27] had constructed a refractometer using bare tapered multimode fibre whose geometry has been shown in Figure-2.5. The principle and working of the refractometer given in this section is the foundation of the refractometer described in Chapter 4. The light (Gaussian) from a source entering into a cladded multimode fibre of core radius a_i (fibre 1) is coupled into an uncladded fibre (fibre 2) of smaller radius a_0 through an intermediate taper which is also uncladded.

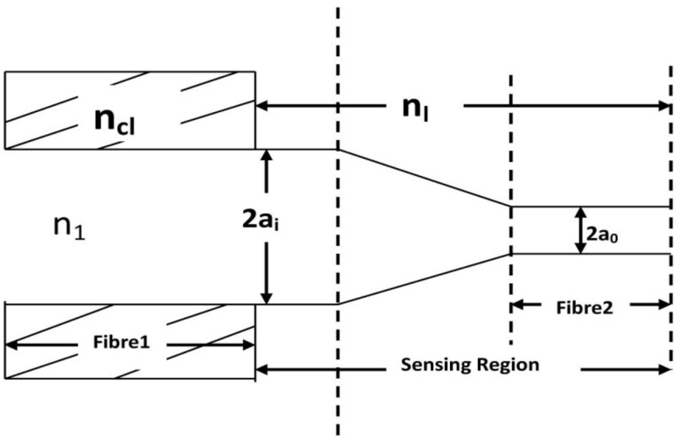

Fig. 2.5: Geometry of a tapered multimode fibre refractometer

The tapered portion can be thought as of interconnection of two fibres; fibre 1 of core diameter $2a_i$ and fibre 2 of core diameter $2a_0$ ($a_0 < a_i$). The RI of the core in fibre 1, 2 and taper all have the same core refractive indices ($= n_1$) and the RI of the cladding for the initial section of fibre 1 is n_{cl} while the RI of cladding for the later section of the fibre 1, the taper portion and fibre 2 is the same ($= n_l$). A guided mode of effective index $\widetilde{\beta_1}(= n_1 \cos\theta_1, \theta_1$ being the characteristic propagation angle) in fibre 1 get transformed to a corresponding characteristic propagation angle $\theta(Z)$ as it propagates down the taper.

$$a(Z)\sin\theta(Z) = a_i \sin\theta_i \tag{2.20}$$

where $\theta(Z)$ denotes the angle that the ray makes with the axis of the taper at a distance Z from the input end of the taper and $a(Z)$represent the radius of the taper at a distance Z from its thick end. Accordingly the effective mode $\widetilde{\beta_1}$ in fibre 1 will get transformed through the taper to a mode $\widetilde{\beta_1} = \widetilde{\beta_2}$ in fibre 2 with $\widetilde{\beta_2}$ as

$$\widetilde{\beta_2} = n_1 \cos\theta_2 = n_1 \left[1 - R^2 \frac{n_1^2 - \widetilde{\beta_1^2}}{n_1^2}\right]^{\frac{1}{2}}$$

$$= \left[n_1^2 - R^2\left(n_1^2 - \widetilde{\beta_1^2}\right)\right]^{\frac{1}{2}} \tag{2.21}$$

where, $R = a_i/a_0$ represents taper ratio. For a mode to be guided in fibre 2, one must have

$$\widetilde{\beta_1} \geq \left[n_1^2 - \frac{n_1^2 - n_l^2}{R^2}\right]^{\frac{1}{2}} \cong \widetilde{\beta_{min}} \tag{2.22}$$

If P_0 represents the total power injected into the guided mode of fibre 1 then the power in the modes with $\overline{\beta_1} > \overline{\beta_{min}}$ will be

$$P_b = P_0 \left[\frac{n_1^2 - \widetilde{\beta_{min}^2}}{n_1^2 - n_{cl}^2}\right] \tag{2.23}$$

which on substitution of $\widetilde{\beta_{min}}$

$$P_b = P_0\left[\frac{n_1^2 - n_l^2}{R^2(n_1^2 - n_{cl}^2)}\right] \tag{2.24}$$

where, $R = a_i/a_0$

It is evident from equation (2.24) that power coupled to fibre 2 through the taper increases linearly with the decrease in n_l^2 and it become zero when n_l becomes equal to n_1. Therefore, the maximum value of RI of a liquid that can be measured by this refractometer is $n_l^{max} = n_1$. Similarly, for minimum value of RI that can be measured by this refractometer is given by the condition $P_b = P_0$. Thus, the minimum value of RI that can be measured by this refractometer can be expressed as $n_l^{min} = \sqrt{n_1^2 - R^2(n_1^2 - n_{cl}^2)}$. The lower limit of n_l give the lower limit for the working of the refractometer, which can be extended right upto $n_l = 1$ by selecting an appropriate value of R. As example, for a value of R=4, n_{cl}=1.46 and n_1= 1.48, the value of n_l^{min} = 1.11785. Therefore, if bare tapered multimode fibre is exposed to air (n_{air} =1.0003), transmitted power from fibre 1 to fibre 2 will be P_0.

This type of refractometer is constructed from a plastic clad silica core fibre by uncladding a small portion of the fibre and then the uncladded portion is made into a taper by heating and pulling the uncladded portion. The tapered portion of the fibre is immersed in a liquid of refractive index $n_l(< n_1)$. By changing the liquid of different refractive index, aand monitoring the power reaching fibre 2, a calibration curve is drawn. This calibration curve is used to find the refractive index of unknown liquids.