CHAPTER 3

MODELING AND ANALYSIS OF EVENT DRIVEN SYSTEMS

3.1 OVERVIEW

Automatic control Systems characterized by a state space (Discrete or Continuous), where changes in a state are triggered by event occurrences are termed as Event driven systems. Event driven systems as discussed in Chapter 1 are of increasing importance in today’s world because they are growing in number, size and sophistication. It is therefore imperative to have systematic design methodologies in order to achieve desirable performance and to avoid catastrophic failures. These systems include automated manufacturing systems, process control, communication networks, computer operating systems, office information systems, etc. The systems may be asynchronous and sequential, exhibiting several characteristics like concurrency, conflict, mutual exclusion and non-determinism.

An event-driven system can be abstracted as a state machine in which the state changes when events occur. The Finite State Machines (FSM) or automaton are well established as a fundamental model for computation and computing machines. However, when they are used to model event systems in a straightforward manner, the exponential increase in the number of states makes it difficult to implement complex systems.
Thus, in this research work, the modeling power of Petri nets is utilized which provides a unified method for the design of event driven systems. Once effective models are developed, techniques to achieve FDI and FTC can be applied and analyzed.

### 3.2. MODELING AND ANALYSIS OF DISCRETE EVENT SYSTEMS

An important group of the man-made systems can be characterized as DEDSs. The basic property of DEDSs is that they are event-driven systems in comparison with time-driven systems. The event relations in DEDSs can be very complicated ones. DEDSs find its applications in modeling many complex processes. Batch chemical processes, according to Lyod [63] are regarded as DEDSs, which are systems characterized by activities such as start and end of discrete processing activities. In such systems, it is only the transitions between discrete processing states that matter, while between transitions, the overall system remains in a fixed state.

Batch processes take an important place in process industries. A batch process involves a sequence of phases that are carried out on a discrete quantity of material within a piece of operating equipment also termed as a resource. Even though plant resources work continuously, it is possible to classify their behavior into discrete states. A transition between states is caused by the occurrence of events signaling the beginning or end of continuous tasks on one hand, and a recipe demanding access to a resource on the other.

In batch processes, a recipe or product specification is a sequence of operations to be performed on certain quantities of raw materials and results in the product. Since the most accurate characterization of a product is by means of its recipe, it is natural to model each product by its recipe.
Recipes consist of five general kinds of elements, viz; sequence of operations, moving material, different ways to join material, adding material during an operation, and the splitting of material. The material can be chemical/chemicals with respect to any chemical processing system or any liquid/liquids with respect to liquid level system, etc. The elementary task models are formalized with respect to resource booking necessary synchronization of involved resources. Furthermore, general and reusable Petri net building blocks representing these elementary tasks are introduced. Using these building blocks, a recipe model can easily be put together.

To explain the modeling of a batch process a typical example shown in Figure 3.1 is considered. The process consists of two generic classes of resources (equipment devices), namely processors (units) and transporting devices. Processors are typically tanks, reactors, and other container-like units, fully equipped with control modules and other devices to manipulate a batch. Transporting devices, on the other hand, have as their main task to open and close connections between processors, causing and preventing material flow; typical examples are valves and pumps.

The Processors are modeled as set of places as mentioned above in the definition of Petri nets. The transporting devices are modeled as transitions. The connections between processors and transporting devices are modeled as directed arcs. The modeling of a recipe involves the following functions:

- **Operation**: - Different phases, e.g. heat, cool or react, applied to a part of the batch.
- **Move**: - Moving the batch from one unit to another.
- **Add**: - Adding material into a unit that already contains part of the batch (e.g. as part of an operation).
- Join: - Merging of two parts of a batch from two source units into a third target unit.

- Split: - Separating a batch into two disjunct parts.

An example for a batch process is shown in Figure 3.1.

![Figure 3.1 Example for a batch process](image1)

An example for a recipe modeled as a Petri net is given in Figure 3.2, where the corresponding resources indicated at the transitions Batch mixing are widely used in Pharmaceutical; Fine chemicals and Food Manufacturing and Consumer Goods (FMCG) industries. The principle is that the components are added to a vessel which is agitated or rotated in a manner designed to ensure that the ingredients are fully mixed. The ingredients are then heated in the vessel by passing steam for the development of the final product. Raw material from supply tanks, B₁ and B₂ are combined (as shown by J in Figure 3.2) in reactor P₁ where a reaction such as...
heating/cooling (operation as shown by O in Figure 3.2) takes place and the material is moved to reactor P_2. Finally, the batch is moved (as shown by M in Figure 3.2) to buffer tank P_3.

### 3.2.1 Batch mixing

![Diagram of batch mixing process](image)

**Figure 3.3** Example of a batch mixing process [39]

A typical batch mixing process is shown in Figure 3.3. Water and concentrates or essences are connected to ingredients valves which are designed as T-shaped valves with the effect that free-flushing process is not affected by any stagnant spaces. The mixing tank is linked to the outlet line of mixing system, which is completely filled with water, i.e., it is used as a so-called full-hose system to guarantee optimum accuracy. After the start, the pre-selected recipe is automatically processed, which means that concentrates and water are successively fed into the desired sequence. The concentrates are conveyed by a built-in pump. Water should bypass the concentrates pump at a higher capacity by means of an external pump. The
deaeating test vessel contain some level measurement probes for the functions (a) System stop in case of product deficiency, (b) Automatic deaeration, and (c) Accuracy test of the flow meter. When the batch has been passed through, the system is again filled with water, and the start of the same or a different product is possible.

### 3.2.2 Petri net based modeling of Batch mixing process

The Mixing Batch process as explained earlier has various stages of events which can be modeled easily by the use of Petri nets. For example, the presence of two chemicals A and B which has to be mixed thoroughly in the mixing tank are modeled by Places $P_1$ and $P_3$. The mixing tank itself can be modeled as Place $P_5$. The valves which control the flows are modeled as Transitions $T_1$-$T_4$. The Petri net structure developed as shown in Figure 3.4, helps in understanding typical structural properties like conflict, concurrency, avoidance of potential deadlock, etc [40].

![Petri net based model of a batch mixing process](image.png)
The meanings of each place and transition is given in Table 3.1.

**Table 3.1** Meanings of Places and Transitions for system model shown in Figure 3.4

<table>
<thead>
<tr>
<th>P/T</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>Chemical A available for mixing.</td>
</tr>
<tr>
<td>T₁</td>
<td>Start transfer of Chemical A to Tank 1. T₁ opens.</td>
</tr>
<tr>
<td>P₂</td>
<td>Chemical A is filled in Tank 1 represented by P₂.</td>
</tr>
<tr>
<td>T₂</td>
<td>Start of transfer of Chemical A to Mixing tank. T₂ opens to fill Chemical A in mixing tank.</td>
</tr>
<tr>
<td>P₃</td>
<td>Chemical B available for mixing.</td>
</tr>
<tr>
<td>T₃</td>
<td>Start transfer of Chemical B to Tank 2. T₃ opens to fill Tank 2.</td>
</tr>
<tr>
<td>P₄</td>
<td>Chemical B is filled in Tank 2 represented by P₃.</td>
</tr>
<tr>
<td>T₄</td>
<td>Start of transfer of Chemical B to Mixing Tank T₄ opens to fill Chemical B in mixing tank.</td>
</tr>
<tr>
<td>P₅</td>
<td>Mixing of both Chemicals in mixing tank in P₅.</td>
</tr>
<tr>
<td>T₅</td>
<td>Start of agitator operation.</td>
</tr>
<tr>
<td>P₆</td>
<td>Agitator runs to mix chemicals thoroughly.</td>
</tr>
<tr>
<td>T₆</td>
<td>End of mixing and Start of heating process.</td>
</tr>
<tr>
<td>P₈</td>
<td>Heating of the mixture (Chemical A + Chemical B) for producing Chemical C.</td>
</tr>
<tr>
<td>T₇</td>
<td>End of heating process and start of packaging of Final product.</td>
</tr>
<tr>
<td>P₉</td>
<td>Packaging process.</td>
</tr>
<tr>
<td>T₈</td>
<td>End of packaging process.</td>
</tr>
<tr>
<td>P₁₀</td>
<td>Final product out.</td>
</tr>
<tr>
<td>P₇</td>
<td>Control for sequential operation of valves 1-4.</td>
</tr>
</tbody>
</table>

The Input, output and incidence matrices as defined in Section 2.2.1 in Chapter 2 for the model shown in Figure 3.4 are as follows:

\[
\text{Input Matrix} = \begin{pmatrix}
    T₁ & T₂ & T₃ & T₄ & T₅ & T₆ & T₇ & T₈ \\
    P₁ & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    P₂ & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    P₃ & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    P₄ & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    P₅ & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    P₆ & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    P₇ & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
    P₈ & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    P₉ & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
    P₁₀ & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \quad (3.1)
\]
Output Matrix =
\[
\begin{pmatrix}
T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]  \hspace{1cm} (3.2)

and

Incidence Matrix =
\[
\begin{pmatrix}
T_1 & T_2 & T_3 & T_4 & T_5 & T_6 & T_7 & T_8 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
\end{pmatrix}
\]  \hspace{1cm} (3.3)

### 3.2.3 Petri net based analysis of Batch mixing process

To analyse the structure shown in Figure 3.4, the coverability tree graph is developed. For this purpose, the structure is modeled in MATLAB environment as shown in Figure 3.5 and the corresponding coverability graph developed and shown in Figure 3.6.
As discussed in Literature, with regard to properties of Petri nets in Chapter 2, reachability graph analysis is perhaps the simplest and the most straightforward approach to analyze the behavior of a Petri net. As the name suggests, such a technique relies on an exhaustive generation of all reachable markings from a given initial marking, in a hope to deduce the Petri net properties by examining the structure of the reachability graph. In spite of its simplicity, the applicability of the technique of reachability graph analysis is rather limited; it can only be applied to bounded (i.e., finite) Petri nets with small reachability sets. Even for bounded Petri nets which exhibit finite reachability graphs, the technique is “expensive” in the sense that it suffers from state explosion phenomenon, as the sizes of the

**Figure 3.5** Modeled structure in MATLAB environment
reachability sets grow beyond any primitive recursive function even for bounded Petri nets in the worst case.

The coverability graph analysis in turn presents an alternative to the technique of reachability graph analysis by abstracting out certain details to make the graph finite. A coverability tree actually relates each node of a marking with a generalized marking as shown in Figure 3.6. Thus, it can be found that a reachability graph (possibly of infinite size) captures exact information about the set of reachable markings of a Petri net, whereas a coverability graph (always of finite size) provides an over-approximation of the reachability set, should it be infinite.

![Coverability Tree graph of Petri net model as shown in Figure 3.4](image)

**Figure 3.6** Coverability Tree graph of Petri net model as shown in Figure 3.4
3.2 MODELING AND ANALYSIS OF CONTINUOUS EVENT SYSTEMS

A continuous process can be considered as Continuous Event Systems (CESs) characterized by a continuous state space where the changes in the state are triggered by event occurrences. In order to model continuous processes, the concept of continuous Petri nets has been developed.

To understand the modeling of CESs using continuous Petri nets, system of tanks as shown in Figure 3.7 can be considered. It comprises two tanks which are emptied permanently (except if they are empty) with a flow of 5 and 7 litres/second respectively. The tanks are also supplied in turn, with a valve whose flow is 12 litres/second. The latter has two positions; when it is in position A, it feeds tank 1, and it supplies tank 2 if it is in position B. To commutate between positions A and B, the valve needs 0.5 seconds, during which, the valve behaves as if it is in its precedent position.

![Figure 3.7 Example for Continuous process [43]](image-url)
The equivalent representation using continuous Petri net is shown in Figure 3.8.

![Figure 3.8 Equivalent Petri net representation [43]](image)

The continuous transitions, T₁, T₂, and T₃ represent only a positive flow for the three valves. Refer Figure 3.8, Places in continuous Petri nets are represented by a double line to distinguish them from places of a discrete Petri net. Transitions in continuous Petri nets are shown by white rectangular boxes. The firing of transitions, T₁, T₂ and T₃ represent material flow through valves. The marking of places, P₁ and P₂ represent quantities of liquid in tank 1 and tank 2, respectively.

### 3.3.1 Benchmark systems

In this research work, for clear understanding of modeling and analysis of continuous event systems using continuous Petri nets, a system of three connected tanks as shown in Figure 3.9 is considered. The system considered is a typical benchmark system, as the operation of such systems can be related to any real-time working process. The main reasons to consider the three tank system in this Research work is that it resembles a typical mixing process which is used in most industries.
Figure 3.9 Three tank system model [42]

As observed in Figure 3.9, the liquid present in the reservoir tanks 1 and 2 are pumped continuously through pump 1 and 2 to main tanks, MT1 and MT2, through valves V1 and V2. Tanks MT1 and MT3 are connected by means of valve V3. Similarly, tanks MT2 and MT3 are connected by valve V4. After considerable amount of time, the collected liquid in tank MT3 is drained out through valve V5. It is supposed that tanks MT1, MT2 and MT3 contain 5, 10 and 5 cm$^3$ (initial values), respectively. In order to ensure constant flow of liquid to tanks MT1, MT2 and MT3, the valves V1, V2, V3, and V4 are operated with flow rates, 1 cm$^3$/sec, 1 cm$^3$/sec, 0.5 cm$^3$/sec, and 0.5 cm$^3$/sec, respectively. Liquid from tank MT3 is drained after 140 sec by using valve 5 operating at 0.25 cm$^3$/sec. Pumps 1 and 2 are discharging liquid to tanks MT1 and MT2 from reservoir tanks having a capacity of 5 cm$^3$ and 10 cm$^3$, respectively.

The levels in the corresponding process tanks are sensed by suitable level sensors for automatic control. The reasons as stated earlier to consider the three tank system as shown in Figure 3.9 are to analyze a system which resembles a typical mixing
process present in most industries. Since most mixing systems are characterized under continuous process, the attempt to consider a continuous system for modeling, analysis and FDI is done.

3.3.2 Petri net based modeling of continuous event systems

The continuous Petri net shown in Figure 3.10 describes the behavior and working of the system of tanks. Places of the continuous Petri nets are represented by circles with double lines to distinguish them from places and transitions of discrete Petri nets (as denoted by circles with single line). The places are actually represented as equivalent for tanks and pumps as shown in Figure 3.9, whereas the transitions represent the valves. The rectangular boxes as shown in Figure 3.10 represent the transitions. The firing of transitions, T_1 to T_5, represents material (water is considered here) flow through valves V1 to V5, and pumps 1 and 2 (represented by places P_1 and P_3), respectively. The marking of places, P_2, P_4 and P_5, represent quantities of liquid in main tanks, MT1, MT2 and MT3, respectively.

The working of the continuous Petri net model is as follows: Initially transitions T_1 (valve V1) and T_3 (valve V2) fires to remove tokens in places P_1 (pump 1) and P_3 (pump 2) and put them in places P_2 (main tank MT1) and P_4 (main tank MT2) respectively. Once the places P_2 and P_4 reach maximal value (maximum level), the transitions T_3 (valve V3) and T_4 (valve V4) are fired to remove tokens from P_2 and P_4 and put in place P_5 (main tank MT3) where combination of the tokens (mixing operation) take place. After a considerable amount of time, when place P_5 reaches maximal value (maximum level), transition T_5 (valve V5) is fired to remove tokens from P_5 until it reaches zero. The representation is shown in Figure 3.10.
Here, the considerations made in the system are: flow through the valves is constant, valves used are of linear types and pumps are operating at constant speed. Hence, the usage of constant continuous Petri net (CCPN) model [64] is sufficient enough to represent the working of system shown in Figure 3.10.

3.3.3 Petri net based analysis of continuous event systems

To analyze the system modeled by continuous Petri net, places $P_1$ and $P_3$ in Figure 3.10 are considered marked and all instantaneous firing speeds are equal to their maximal value. The pre-incidence ($W^-$), post-incidence ($W^+$) and incidence matrix ($W=W^+-W^-$) for the model shown in Figure 3.10 are given as follows:

\[
W^- = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{pmatrix}, \quad (3.4)
\]
In order to calculate the marking evolution, let at initial time $t = 0$, the values of transitions be considered as $v_1=1$, $v_2=0.5$, $v_3=1$, $v_4=0.5$, and $v_5=0.25$, respectively. Thus, the Initial value of transitions, $v(0)$, is given by

$$v(0) = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ 1 \\ 0.5 \\ 0.25 \end{bmatrix}.$$  

Similarly, initial marking, $m(0)$, is given by

$$m(0) = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 10 \\ 10 \\ 5 \end{bmatrix}.$$
At time $t = 5$ sec, the new marking $m(5)$ is obtained using Equation (2.7) as discussed in section 2.2 in Chapter 2. The final value obtained is shown in Equation (3.9).

Let

$$m(5) = \begin{bmatrix} 5 \\ 5 \\ 10 \\ 10 \\ 5 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 0.5 \\ 1 \\ 0.5 \\ 0.25 \end{bmatrix} dt = \begin{bmatrix} 75.8 \\ 5.12 \\ 5.0 \\ 75.0 \\ 5.0 \end{bmatrix}.$$  

Similarly, at time $t = 10$ sec, $t = 20$ sec, $t = 40$ sec and $t = 140$ sec, the new markings $m(10)$, $m(20)$, $m(40)$ and $m(140)$ are obtained and represented by a graph for easy understanding with the magnitude of place markings on the y-axis with respect to time on x-axis and shown in Figure 3.11.

![Figure 3.11 Plot of Place markings with $P_1=5$, $P_2=10$, $P_3=10$, $P_4=5$, $P_5=5$](image)

Figure 3.11 shows that the places are marking dependent and are subjected to changes as the input values of markings for the corresponding places are changed. For example, the magnitude of input places or in process terms, the pump speed can
be changed by keeping the valve flow rates fixed, and accordingly, the plots can be obtained. Figure 3.12 shows the plot of evolution of places for $P_1 = 10$, $P_3 = 5$, $P_2 = 0$, $P_4 = 0$ and $P_5 = 0$.

**Figure 3.12** Plot of Place markings with $P_1=10$, $P_3=5$, $P_2=0$, $P_4=0$, $P_5=0$  

It is thus clear from Figure 3.12 that by changing the pump speeds, i.e., values of $P_1$ and $P_2$, the desired values for the tanks represented by $P_2$, $P_4$ and $P_5$ are changed to 6, 2.5 and 10, respectively (as compared to 2.5, 5 and 12.5 for earlier as shown in Figure 3.11). Similar graphs can be obtained by maintaining $P_1 = 10$, $P_3 = 10$, $P_2 = 0$, $P_4 = 0$ and $P_5 = 0$ as shown in Figure 3.13.

**Figure 3.13** Plot of Place markings with $P_1=10$, $P_3=10$, $P_2=0$, $P_4=0$, $P_5=0$
For structural based analysis as done in section 3.2.3 for DEDSs, here, firstly, the reachability graph is developed for the system modeled as Petri net as shown by Figure 3.10 and is shown in Figure 3.14. The reachability graph comprises macro markings denoted by \( m^* \) which are totally 8 in number, i.e., \( m_1^* \) to \( m_8^* \) as per Definition 7 in section 2.2.

**Figure 3.14** Reachability graph of Petri net model shown in Figure 3.10

### 3.3 MODELING AND ANALYSIS OF HYBRID EVENT SYSTEMS

HDSs as discussed in Literature are defined as a dynamic system that integrates explicitly continuous systems and discrete event systems. The modelling, analysis and control issues of HDSs are based on the determination of marking evolutions, which are nothing but the values of places (place markings) at every instant. Based on the observability (measurable) of the markings, the evolution graph is developed [65], which is later used for various analysis purposes. Hybrid Petri nets as a modeling tool are useful for the study of HDSs [66] because they combine discrete and structural aspects with continuous evolution.
3.4.1 Hybrid event systems

Similar to the discussions made in the earlier sections, in order to understand the modeling and analysis of hybrid event systems, two tank system as shown in Figure 3.15 is considered.

This system is made up of two tanks (numbers 1 and 2), four valves (numbers 5, 6, 7 and 9, corresponding to the index of speeds associated with them), which can be opened and closed and a pump. There is a permanent supply of tank 1, 1 liter/s, when valve 5 is open. Liquid flows from tank 1 to tank 2 via valve 6 (2 liter/s, assumed to be always open). Liquid in tank 2 can leave it in three ways with their priorities obtained by gravity. The highest priority corresponds to valve 7 (1 m$^3$/s, if it is open). The second priority is the pipe leading to the pump (1 m$^3$/s). The third priority corresponds to valve 9 (1 m$^3$/s, always open). At initial time, it is assumed that both tanks are empty and valves, 5 and 7 are closed. Valve 5 will be open periodically from $t = 5$ sec to $t = 10$ sec, then from $t = 15$ sec to $t = 20$ sec, etc. Valve 7 will be open periodically from $t = 10$ sec to $t = 15$ sec, then from $t = 25$ sec to $t = 30$ sec, etc.

![Figure 3.15 Representation of two-tank system](image-url)
The behavior of the system shown in Figure 3.15 is modeled by the timed hybrid Petri net as shown in Figure 3.16 in which places $P_8$ and $P_9$ model the liquid quantities in tanks 1 and 2, and transitions $T_5$ to $T_9$ model the valves and the pump.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{figure3.16}
\caption{Timed Hybrid Petri net model of two tank system}
\end{figure}

The behavior of a system modeled as hybrid Petri net can be analyzed by an evolution graph represented in time. This graph is represented in the form of a Petri net. Each place corresponds to an IB-state (where IB means Invariant Behavior), i.e., a constant instantaneous speed state of the continuous part represented by $v = (v_1, v_2, ..., v_n)$ and the corresponding marking of the discrete part $m = (m_1, m_2, ..., m_n)$. From the graphs shown in Figure 3.17, it can be observed that the system is reachable as each new marking is reachable from the earlier marking. From the evolution graph shown in Figure 3.18, it can also be concluded that the system is observable.
As seen in Figure 3.18, during the first IB-state, $v_8, v_9 = 0$ since $m_1$ and $m_4 = 0$. At $t = 5$, $P_1$ becomes marked (valve 5 is open). From this time, $v_5 = 1$. Because of the priority $T_8 < T_9$ ($T_7$ is not enabled), $v_8 = 1$ and the speed vector $v = (v_5, v_6, v_7, v_8, v_9) = (1, 1, 0, 0, 0)$.

Likewise, as valve 5 is closed at $t = 10$, the flow from $T_5$ to $T_9$ disappears, whereas the flow in the loop continues. In this case, the speed vector, $v = (0, 1, 1, 1, 1)$ which marks the completion of third IB state. Similarly the subsequent states can be analyzed.
3.4.2 Petri net modeling of Hybrid event systems

In this section, a three tank benchmark system is considered and modeled in a similar manner as explained in section 3.2.1, wherein the system was modeled using continuous Petri nets. But here, the semantics of timed Hybrid Petri nets are applied (instead of D-elementary Petri nets considered in [67]) and improved results are obtained.

Figure 3.18 Evolution graph of system shown in Figure 3.15
The system of three connected tanks as shown in Figure 3.19 comprises tanks T1 and T2, which are supplied by liquid flowing continuously through pumps 1 and 2 connected to valves V1 and V2. Tanks T1 and T3 are connected by means of valve V3. Similarly, tanks T2 and T3 are connected by valve V4. After considerable amount of time, the collected liquid in tank T3 is drained out through valve V5. The details of specifications for the system are listed out in Table 3.2.

**Figure 3.19** Structure of three tank system

<table>
<thead>
<tr>
<th>Item No.</th>
<th>Contents</th>
<th>Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Tank T1</td>
<td>Maximum height= 10cm (initial height=5cm)</td>
</tr>
<tr>
<td>2.</td>
<td>Tank T2</td>
<td>Maximum height= 20 cm (initial height=10cm)</td>
</tr>
<tr>
<td>3.</td>
<td>Tank T3</td>
<td>Maximum height= 35 cm (initial height=5cm)</td>
</tr>
<tr>
<td>4.</td>
<td>Pump 1</td>
<td>Continuous type, flow rate=5 cm$^2$/sec</td>
</tr>
<tr>
<td>5.</td>
<td>Pump 2</td>
<td>Continuous type, flow rate=10 cm$^2$/sec</td>
</tr>
<tr>
<td>6.</td>
<td>Valve V1</td>
<td>Constant speed, flow rate=0.33 cm$^2$/sec</td>
</tr>
<tr>
<td>7.</td>
<td>Valve V3</td>
<td>Constant speed, flow rate=0.25 cm$^2$/sec</td>
</tr>
<tr>
<td>8.</td>
<td>Valve V2</td>
<td>Constant speed, flow rate=0.1667 cm$^2$/sec</td>
</tr>
<tr>
<td>9.</td>
<td>Valve V4</td>
<td>Constant speed, flow rate=0.125 cm$^2$/sec</td>
</tr>
<tr>
<td>10.</td>
<td>Valve V5</td>
<td>Constant speed, flow rate=0.033 cm$^2$/sec</td>
</tr>
</tbody>
</table>
The reason for choosing different heights for tanks is to ensure that valves V1-V5 operate at different flow rates, and hence the need for continuous monitoring and controlling is considered since improper maintenance of flow rates eventually results in faulty behavior of the system [68]. Improper supply of liquid from the pumps and wrong measurement of level by the level sensors (not shown in the model) would cause the system to fail. Hence, these faulty conditions must be diagnosed and detected at an appropriate time so that they can be either isolated or tolerated depending on their nature and operating conditions [69]. The reason for considering the three tank system as shown in Figure 3.19 is that it resembles a typical mixing process [70], where mass flow is considered as a major parameter.

As discussed in Chapter 1, FDI using Petri nets are classified under model based approach and qualitative based subclass. Thus, the specifications listed in Table 3.2 are sufficient to perform qualitative analysis to achieve FDI, even though parameters like tank diameters; valve $C_V$, cross section of pipes, pump curves are considered normally for quantitative analysis, and to achieve state models based FDI as discussed in [71].

The equivalent timed hybrid Petri net shown in Figure 3.20 describes the behavior of the system of tanks. The places and transitions of the continuous part (valves V1-V5 and pumps 1 and 2) are represented with empty boxes and double lined circles to distinguish them from places and transitions of discrete part (Control signals) as shown by darkened boxes and circles. The firing of transitions, $T_1$, $T_2$, $T_7$, $T_8$ and $T_{13}$, represent material flow through valves V1 to V5 and pumps 1 and 2, respectively. The marking of places, $P_2$, $P_9$ and $P_3$, represent quantities of liquid in tanks, $T_1$, $T_2$ and $T_3$, respectively. The details are listed out in Table 3.3.
Table 3.3 Description of Places/Transitions (P/T) for system model shown in Figure 3.20

<table>
<thead>
<tr>
<th>Names of P/T</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>P₁</td>
<td>Equivalent of pump 1</td>
</tr>
<tr>
<td>P₈</td>
<td>Equivalent of pump 2</td>
</tr>
<tr>
<td>P₂</td>
<td>Equivalent of main tank MT1</td>
</tr>
<tr>
<td>P₉</td>
<td>Equivalent main tank MT2</td>
</tr>
<tr>
<td>P₃</td>
<td>Equivalent of main tank MT3</td>
</tr>
<tr>
<td>T₁, T₂, T₇, T₈, T₁₃</td>
<td>Equivalent of valves V₁-V₅</td>
</tr>
<tr>
<td>P₄, P₅</td>
<td>Discrete places for controlling operation of T₁</td>
</tr>
<tr>
<td>T₃, T₄</td>
<td>Discrete transitions for controlling operation of T₁</td>
</tr>
<tr>
<td>P₆, P₇</td>
<td>Discrete places for controlling operation of T₂</td>
</tr>
<tr>
<td>T₅, T₆</td>
<td>Discrete transitions for controlling operation of T₂</td>
</tr>
<tr>
<td>P₁₀, P₁₁</td>
<td>Discrete places for controlling operation of T₇</td>
</tr>
<tr>
<td>T₉, T₁₀</td>
<td>Discrete transitions for controlling operation of T₇</td>
</tr>
<tr>
<td>P₁₂, P₁₃</td>
<td>Discrete places for controlling operation of T₈</td>
</tr>
<tr>
<td>T₁₁, T₁₂</td>
<td>Discrete transitions for controlling operation of T₈</td>
</tr>
</tbody>
</table>

3.4.3 Petri net based analysis of hybrid event systems

As analyzed in previous sections, the analysis of the system and its model considered in earlier section is performed by finding the pre-incidence, post incidence and incidence matrices, which are as follows:
Pre-incidence matrix =

\[
\begin{pmatrix}
T_3 & T_4 & T_5 & T_6 & T_9 & T_{10} & T_{12} & T_{14} & T_{15} & T_1 & T_2 & T_7 & T_8 & T_{13} \\
\text{P}_4 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\text{P}_5 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_6 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\text{P}_7 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{10} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{P}_{11} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{12} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{P}_{13} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\text{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\text{P}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\]

(3.10)

Post-incidence matrix =

\[
\begin{pmatrix}
T_3 & T_4 & T_5 & T_6 & T_9 & T_{10} & T_{12} & T_{14} & T_{15} & T_1 & T_2 & T_7 & T_8 & T_{13} \\
\text{P}_4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\text{P}_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\text{P}_7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{10} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\text{P}_{11} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_{12} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
\text{P}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\text{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
\text{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\text{P}_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\text{P}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
\end{pmatrix}
\]

(3.11)

and

Incidence matrix =

\[
\begin{pmatrix}
T_3 & T_4 & T_5 & T_6 & T_9 & T_{10} & T_{12} & T_{14} & T_{15} & T_1 & T_2 & T_7 & T_8 & T_{13} \\
\text{P}_4 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_5 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_6 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_7 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{10} & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{11} & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{12} & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{13} & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\text{P}_{14} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
\text{P}_{15} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 \\
\text{P}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
\text{P}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
\text{P}_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\
\text{P}_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\
\text{P}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
\end{pmatrix}
\]

(3.12)
Figure 3.21  Evolution graph for system model shown in Figure 3.20
As observed in matrices from Equation (3.11) to Equation (3.13), the places $P_4$-$P_7$ and $P_{10}$-$P_{15}$ are discrete places, whereas places, $P_1$, $P_2$, $P_3$, $P_8$ and $P_9$ constitute the continuous places. Likewise, transitions, $T_3$-$T_6$, $T_9$-$T_{12}$ and $T_{14}$-$T_{15}$ are discrete transitions and $T_1$, $T_2$, $T_7$, $T_8$ and $T_{13}$ are continuous transitions, respectively.

To analyze the model, the details given by Equation (2.10) as discussed in section 2.3 in Chapter 3 are evaluated to develop the evolution graph [72] as shown in Figure 3.21.

The continuous part and discrete part of the model are analyzed individually by developing the reachability and observability graphs [73] as shown in Figure 3.22 and Figure 3.23, respectively.

Figure 3.22 Reachability graph of continuous part of the model shown in Figure 3.20
The graphical results obtained for the system model considering the marking evolutions of the continuous place, $P_3$, (tank 3) are shown in Figure 3.24 and Figure 3.25, respectively. It can be observed from Figure 3.24 that marking of $P_2$ (tank 1) reaches its maximum value, i.e., in time $t = 4$ sec initially. At the same time, a rise in marking of $P_3$ (tank 3) can be observed which eventually reaches the maximum value at time $t = 6.5$ sec. Finally, fall in $P_3$ marking from maximum value to zero occurs at time $t = 11$ sec. The graphical plots of the control signals given by discrete places, $P_4$ and $P_6$, are shown in Figure 3.24, whereas the graphs of the marking values with respect to place $P_8$ (tank 2) along with the control signals denoted by discrete places, $P_{10}$ and $P_{12}$, are shown in Figure 3.25.
Thus, in this chapter, detailed description of modeling and analysis of typical event driven systems using Petri nets are presented and discussed. In the next Chapter, complete description of the proposed method to achieve estimation based FDI and FTC is presented along with simulation results.