CHAPTER –III

NETWORKING BASED ON DE BRUIJN GRAPHS AND ITS PERFORMANCE MEASURES

3.1 Introduction to De Bruijn Graphs:

The interconnection pattern of De Bruijn Graph was first proposed by De Bruijn in the year 1946[18] which is a logical topology applicable in optical networks a De Bruijn Graph with 'Δ' and 'd' as degree and diameter respectively is denoted by D(Δ,d) and is defined as follows :

Definition:

The De Bruijn Graph D(Δ,d) is a directed graph with the set of nodes $(0,1,2,\ldots\ldots\ \Delta)^d$ (i.e each node is represented with d digits, each of which can take values from 0,1,2,3,..... Δ). Each node has in-degree and out degree Δ, and have Δ nodes have self loops. The total number of nodes is given by $N = \Delta^d$. There is a directed edge from node(a_1, a_2, a_d) to node ($b_1, b_2, \ldots\ldots, b_d$) if and only if $b_i = a_{i+1}$ for $1 \leq i \leq d - 1$.

Topological Description:

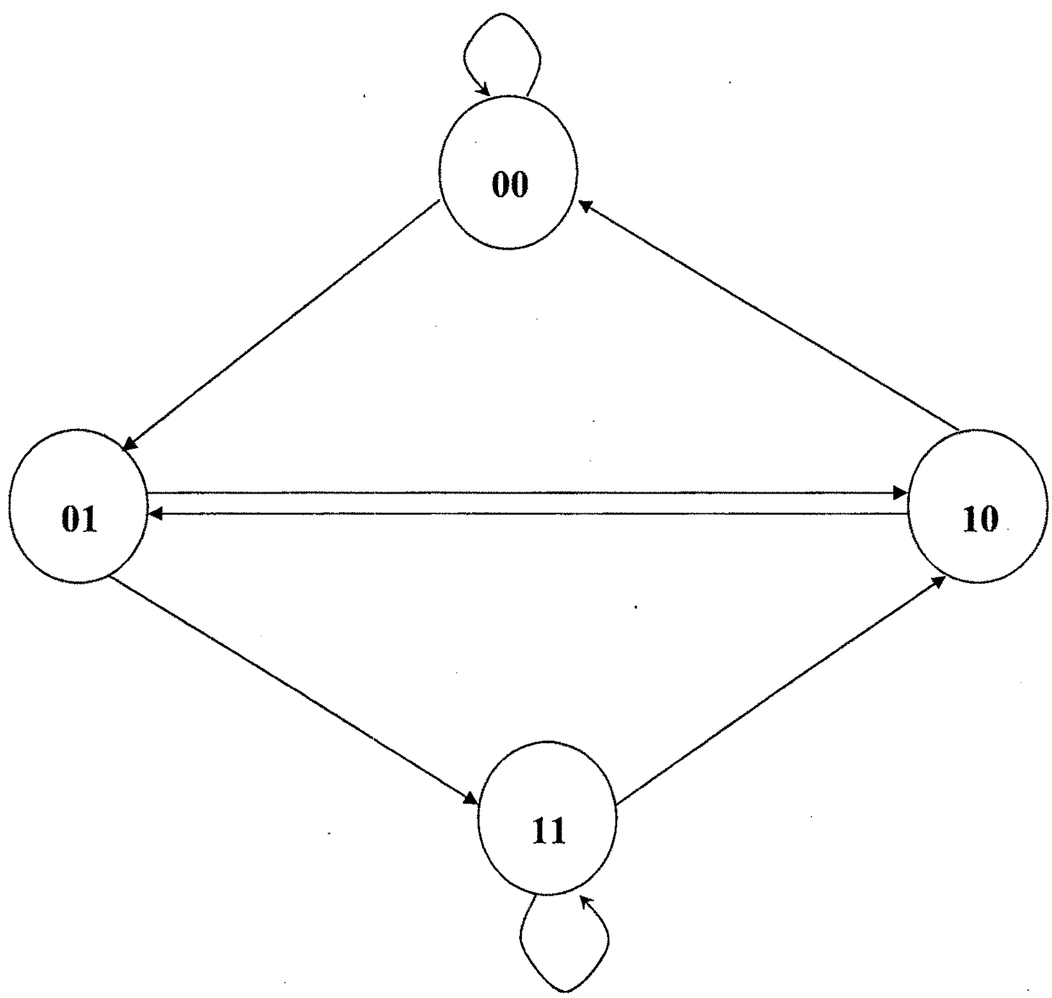
There is one to one correspondence between all the possible states of a Δ shift register of length d and the connectivity of the nodes in the De Bruijn Graph D(Δ,d). There is an edge from node i to node j if state j can be reached from state i with one shift operation in the shift register. Thus the connectivity pattern of a De Bruijn Graph is the state transition diagram of a Shift register.

Structural Properties:

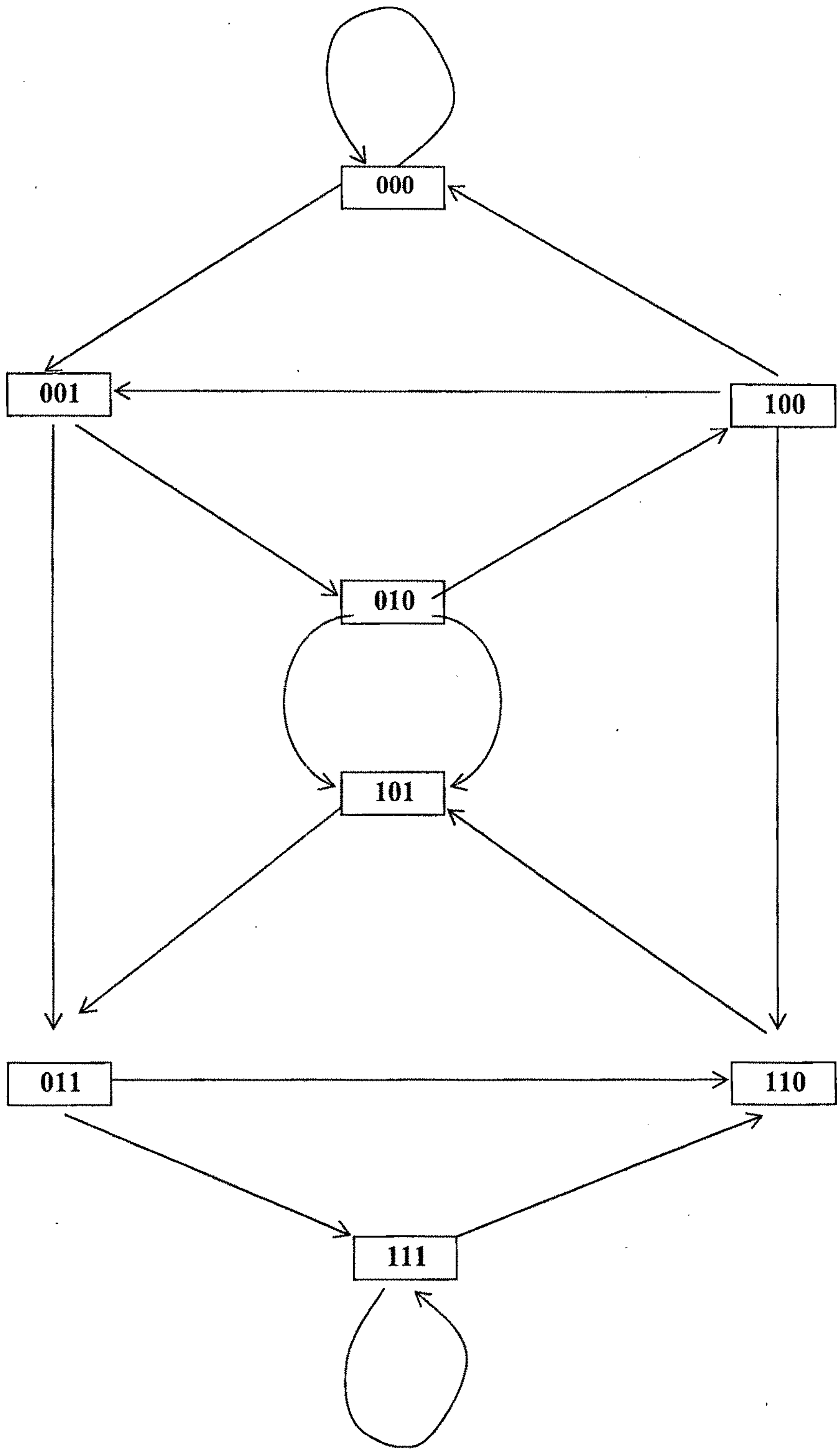
The De Bruijn Graph D(Δ,d) can be viewed as a two-column Graph, each column containing the same Δ^d nodes. The two columns are connected in a perfect Δ- shuffle pattern. The De Bruijn Graph structure is inherently asymmetric due to the node with self loops.

3.2 Algorithm for calculating various parameters of De Bruijn Graphs.

In order to develop the algorithm for De Bruijn Graphs first the structures of the Graphs are explained in the following figures for D(2,2) and D(2,3)



Figure(3.2.1) The De Bruijn Graph D(2,2)



Figure(3.2.2) The De Bruijn Graph D(2,3)

Following formulae are developed to measure various performance measures of De Bruijn Graphs .

Total Number of Nodes	N	=	Δ^{d}	----------(3.2.1)
Average Hop Length	$\overline{H}$	=	Hsum/N(N-1)	---------(3.2.2)
Average Edge Loading	$\overline{L}$	=	Hsum/(N* Δ - Δ)	----------(3.2.3)
LMAX (Maximum Edge load)				---------(3.2.4)
Average Network Utilization	U_{AVG}	=	$\overline{L}$ / LMAX	-----------(3.2.5)
Throughput	λ	=	1/LMAX	---------- (3.2.6)

Hsum: Sum of the shortest paths of all possible source and destination nodes.

An algorithm is developed for calculating above performance measures for De Bruijn Graph and is given as follows:

Schematic representation of De Bruijn Graphs D(2,2) and D(2,3) are shown in Figure(3.2.1) and Figure(3.2.2) respectively. From the Shift register analogy, a node in the De Bruijn Graph can be represented by a string (or sequence) of'd' digits. An edge from node 'x' to node 'y' can be represented by a string of (d+1) digits, the first d digits representing the node 'x' and the last 'd' digits representing node 'y'. Similarly, any path in the graph of length k hops can be represented by a string of (d+k) digits.

The description of routing algorithm for finding Shortest path from a source (say node A) to any destination (say node B) in a De Bruijn Graphs represented by Sivarajan.k and Ramaswami.R[43] is explained as follows:

Node A = ($a_1, a_2, \ldots\ldots a_d$) and Node B =($b_1, b_2, \ldots\ldots\ldots b_d$)

Define Shift-match(i ,A,B) , $0 \leq i \leq d$, Operation on two strings A and B to be true Iff

$(b_1,b_2,\ldots\ldots,b_{d-i}) = (a_{i+1},\ldots\ldots,a_d)$

and

false otherwise.

Define Merge (i,A,B), $0\leq i \leq d$ to be a string (or Sequence) of length d+i

given by

$(a_1\ldots\ldots,a_d, b_{d-i+1},\ldots\ldots b_d)$.

Using the above notations, the routing algorithm is defined as follows:

The Shortest path algorithm for De Bruijn Graphs, explained as follows:

Number of hops i =0

While(Shift_match(i ,A,B) is false)

i=i+1

end while

Shortest path = Merge(i,A,B)

For Example : Consider D(2,3) and let A =(001) and B=(111).

Here Shift_match(0,A,B) and

Shift_match(1,A,B) are False

And Shift_match(2,A,B) is True.

The Merge(2,A,B) yields (00111).

Hence the Shortest path is (001) →(011)→(111)

3.3: Calculation of various parameters of De Bruijn Graphs.

Using the above algorithm a program is written in c- language to calculate performance measures given in Section (3.2) and is given in the following table.

Table (3.3.1) Calculated values of performance measures of De Bruijn Graphs

Δ	d	N	$\overline{H}$	$\overline{L}$	LMAX	U_{AVG}	λ
2	2	4	1.5	3	3	1	0.33333
2	3	8	2.107143	8.428572	11	0.766234	0.090909
2	4	16	2.83333	22.66667	29	0.781609	0.034483
2	5	32	3.649194	58.3871	81	0.720828	0.012346
3	2	9	1.666667	5	7	0.714286	0.142857
3	3	27	2.478632	22.30769	31	0.719603	0.032258
3	4	81	3.386111	91.425	138	0.6625	0.007246
3	5	243	4.344047	351.8678	535	0.657697	0.001869
4	2	16	1.75	7	9	0.777778	0.111111
4	3	64	2.639881	42.23809	57	0.741019	0.017544
4	4	256	3.598529	230.3059	313	0.735802	0.003195
4	5	1024	4.584448	1173.619	1589	0.73859	0.000629
5	2	25	1.8	9	11	0.818182	0.090909
5	3	125	2.727742	68.19355	86	0.792948	0.011628
5	4	625	3.705897	463.2372	586	0.790507	0.001707

Some interesting and relevant statistics are calculated parameter wise[57] and are given in the following table using the data given Tab (3.3.1).

Table (3.3.2) some interesting and relevant statistics for various parameter of De Bruijn graphs

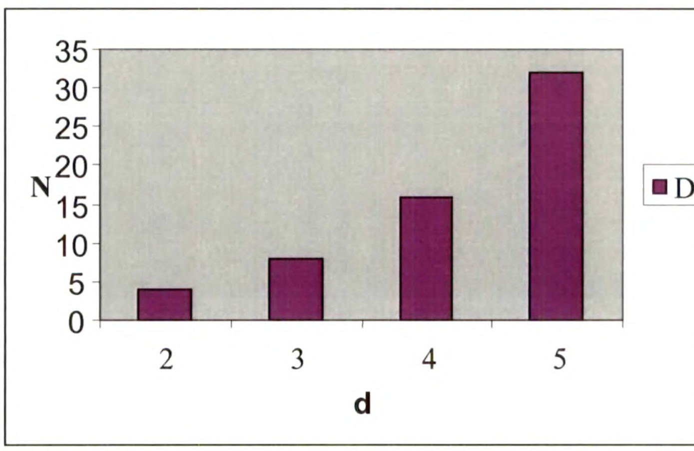
Parameters / Statistics	N	$\overline{H}$	$\overline{L}$	LMAX	U_{AVG}	λ
Mean	170.33333	2.851441	170.44509	232.4	0.76117213	0.05947
Standard Error	74.255586	0.255826	80.25249	108.6891067	0.02075196	0.02291
Median	32	2.727742	42.238094	57	0.7410192	0.01754
Coefficient of Variation	168.83991	34.74766	182.35583	181.1321428	10.5589792	149.218
Standard Deviation	287.59065	0.990809	310.81656	420.9511	0.08037201	0.08874
Sample Variance	82708.381	0.981703	96606.931	177199.8286	0.00645966	0.00787
Kurtosis	5.5253596	-1.073338	8.315548	8.231316591	5.41321803	6.4186
Skewness	2.376975	0.264872	2.7646295	2.743780434	1.83084271	2.36942
Range	1020	3.084448	1170.6188	1586	0.3423032	0.3327
Minimum	4	1.5	3	3	0.6576968	0.00063
Maximum	1024	4.584448	1173.6188	1589	1	0.33333
Sum	2555	42.77162	2556.6763	3486	11.417582	0.89202
Count	15	15	15	15	15	15

3.4 Results and Discussions.

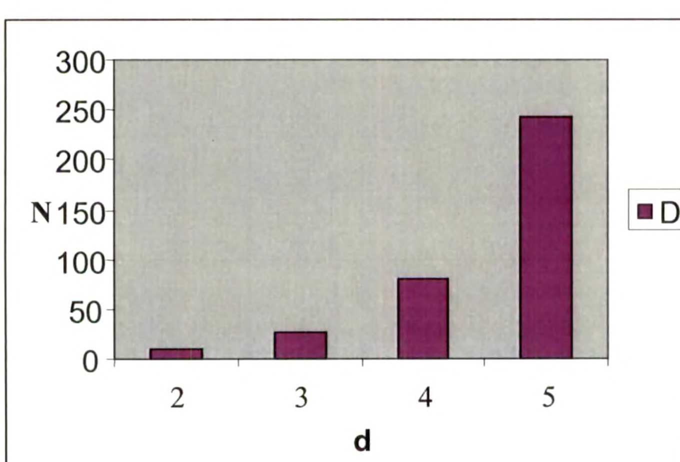
Critically comparing the calculated values of various parameters of Table (3.3.1) and the corresponding statistics of the parameters given in Table (3.3.2), one can draw the following conclusion:

- Conclusions drawn in Chapter 2 holds good for De Bruijn Graph also which means that both Shuffle Net and De Bruijn Graph have similar relationship exhibited between different parameters and the degree ' Δ ' and diameter 'd'. This means that there exist the relationship between N, $\overline{H}$, $\overline{L}$, LMAX with respect to degree ' Δ ' and diameter'd'.
- Similarly there exist negative relationship between the parameter Average Network Utilization (U_{AVG}) and Throughput (λ) with degree ' Δ ' and diameter'd'.
- It is interesting to note that both parameter $\overline{L}$ and LMAX have the approximately same C.V's . Larger variation is present with respect to $(\overline{L})$ and In De Bruijn Graph more consistency is exhibited in (U_{AVG}).

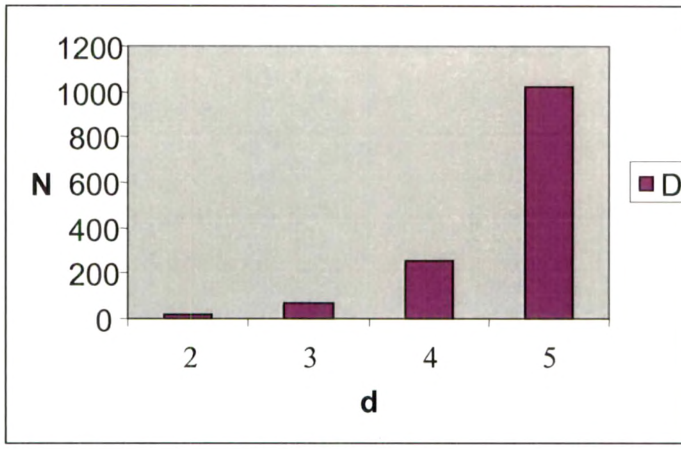
On similar lines of Chapter 2 the relation between N and $\overline{H}$ with respect to degree ' Δ ' and diameter 'd' are represented diagrammatically as follows:



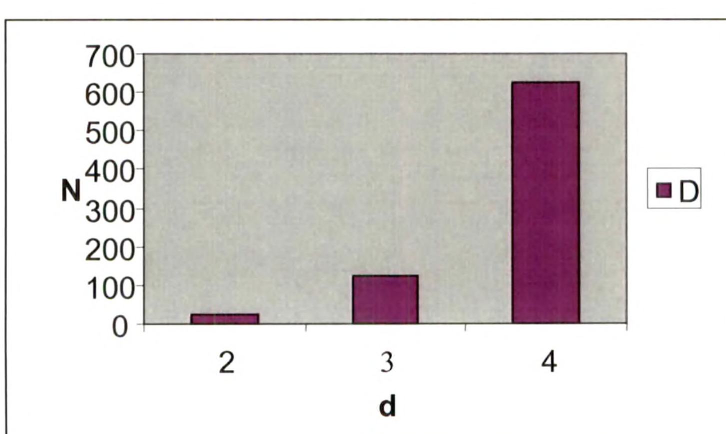
Figure(3.4.1) Total Number of nodes(N) for De Bruijn Graphs for various values of 'd' when ' Δ=2'.



Figure(3.4.2) Total Number of nodes(N) for De Bruijn Graphs for various values of 'd' when ' Δ=3'.



Figure(3.4.3)Total Number of nodes(N) for De Bruijn Graphs for various values of 'd' when 'Δ=4'



Figure(3.4.4)Total Number of nodes(N) for De Bruijn Graphs for various values of 'd' when 'Δ=5'

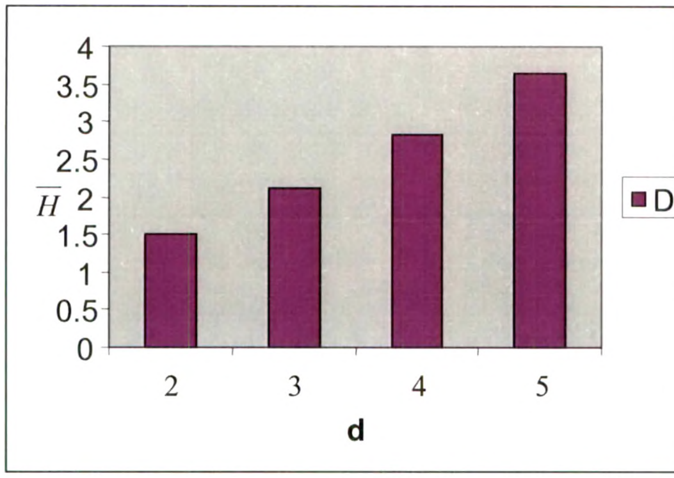


Figure (3.4.5)Average Hop Length($\overline{\mathrm{H}}$) for De Bruijn Graphs for various values of 'd' when 'Δ=2'

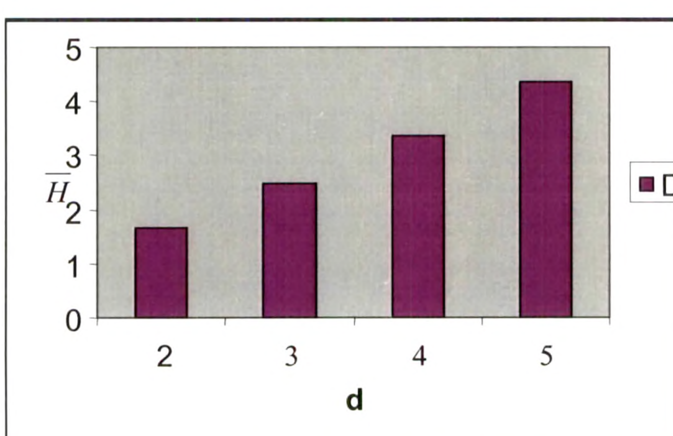


Figure (3.4.6) Average Hop Length($\overline{\mathrm{H}}$) for De Bruijn Graphs for various values of 'd' when 'Δ=3'

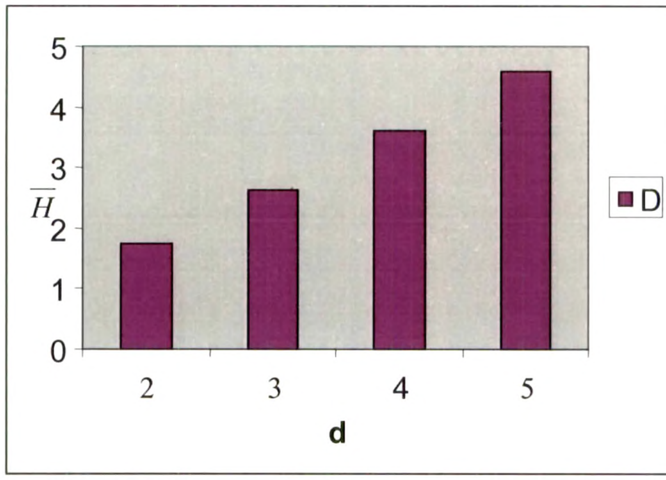


Figure (3.4.7) Average Hop Length ($\overline{H}$) for De Bruijn Graphs for various values of'd' when 'Δ=4'.

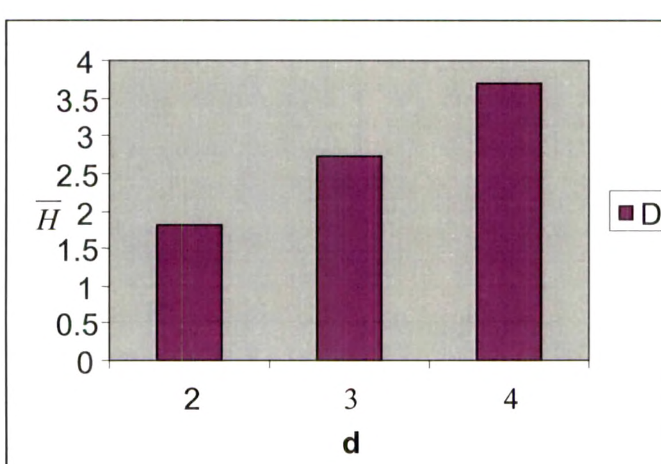


Figure (3.4.8) Average Hop Length($\overline{H}$) for De Bruijn Graphs for various values of 'd' when 'Δ=5'.

Remarks:

Sarma, K.L.A.P., Satyanarayana, B and Praveen Kumar, P.T.V," Shortest path routing algorithm based on Modified De BruijnGraphs" Presented a paper in International Conference on Recent Developments in Statistics and their Applications (I.C.R.D.A) held from 3rd to 4th Jan,2005 in the department of Statistics, S.V. University, Tirupathi [65].