CHAPTER II

NETWORKING BASED ON SHUFFLE NET AND ITS PERFORMANCE MEASURES
2.1 Introduction to Shuffle Net:

There exist many regular logical network topologies in literature like; Bus, Ring, Star which is simple topologies whereas Shuffle Net, De Bruijn Graph, Kautz Graph and Modified De Bruijn Graphs are advanced logical topologies. The basic purpose of this thesis is to Critically compare various advanced topologies with respect to different performance measures based on \((\Delta,d)\) Where \(\Delta\) is degree and \(d\) is the diameter of the Network. For the purpose of comparison in this thesis we have considered the values of \((\Delta,d)\) as 2, 3, 4 and 5 individually.

In this Chapter first we consider Shuffle Net logical topology which is denoted as \(S(\Delta,d)\) and calculated various performance measures of this topology. Before calculating these, preliminaries about Shuffle Net along with its Graphs are explained as follows:

Shuffle Net Topology was first proposed by Stone in the Year 1971 for parallel processing systems[74]. Later in 1987 the architecture of this network applicable in optical network topologies was first applied by Acampora[1] later Hulchyj,[33], Karol[39] and Karol & Shaikh[40] proved further properties and other interesting results of Shuffle Net.

A Shuffle Net is defined as follows:

Definition: The \(S(\Delta,d)\) Shuffle Net is a regular digraph of in-/out-degree \(\Delta\), \(N=\Delta^d\) nodes, and \(d\Delta^{d+1}\) arcs. The nodes are arranged in \(d\) columns, each column has \(\Delta^d\) nodes. The nodes in each column are connected to the next column via \(\Delta^{k+1}\) arcs in a generalization of the Perfect Shuffle pattern.
Constructions of the Shuffle Net Network is explained as follows:

A $S(\Delta,d)$ consists of $N = d\Delta^d$ ( $d = 1,2,3,\ldots$, $\Delta = 1,2,3,\ldots$) nodes which are arranged in $d$ columns of $\Delta^d$ nodes each, with $d^{th}$ column connected to the first i.e as if the topology is wrapped around a cylinder. The connectivity between successive columns is a $\Delta$-shuffle. In general, in a $(\Delta,d)$ Shuffle Net, node $(r,c)$ (i.e., at row $r$ and column $c$) is connected to nodes $(\Delta \cdot r \mod \Delta^d, (c+1) \mod d)$, $(\Delta \cdot r \mod \Delta^{d+1}, (c+1) \mod d)$ \ldots $(\Delta \cdot r \mod \Delta^d + \Delta, (c+1) \mod d)$.

Structural Properties:

Shuffle Net is a Symmetric and homogeneous topology, i.e the networks looks the same from any node. The structure of the Shuffle Net can be viewed as an ideal spanning tree rooted at each node. In Shuffle Net, due to its cylindrical connectivity pattern, routing is reliable and fault tolerant.

2.2 Algorithm for Calculation of various parameters of Shuffle Net.

In order to develop the algorithm for Shuffle Net first the structures of the Graphs are explained in the figure(2.2.1) and figure(2.2.2).

A $(\Delta,d)$ Shuffle Net consists of $N$ nodes which are arranged in $d$ columns of $\Delta^d$ nodes each. Where $(\Delta = 1,2,3,\ldots$, $d = 1,2,3,\ldots)$.
Following figures explains Shuffle Nets with ($\Delta = 2, d = 2$) and ($\Delta = 2, d = 3$)

Figure (2.2.1) The Shuffle Net $S(2, 2)$
Figure (2.2.2) The Shuffle Net S(2,3)
It is important to note from the above Figure (2.2.1) and Figure (2.2.2) that d\textsuperscript{th} column is connected to the first column as if the topology is wrapped around a Cylinder.

This Property can be explained in general as follows in a \( S(\Delta, d) \) Shuffle Net Node\((r,c)\) i.e node in ( \( r\)\textsuperscript{th} row belonging to \( C\)\textsuperscript{th} column) is connected to nodes \( \Delta.r \mod \Delta^d, (c+1) \mod d \), \( \Delta.r \mod \Delta^{d+1}, (c+1) \mod d \) \ldots \( \Delta.r \mod \Delta^p + \Delta, (c+1) \mod d \).

Thus a node at the \( r\)\textsuperscript{th} row and \( c\)\textsuperscript{th} column in \( (\Delta, d) \) Shuffle Net is assigned by an address \((r,c)\) where \( r = 0,1,2,\ldots,\Delta^d+1 \) and \( c = 0,1,\ldots,d+1 \). In the Year 1987 Sauer and Karol\cite{69} showed that in Shortest Path routing an intermediary node\((f,g)\) determines the column distance \( D \) between itself and the packets destination node \((c_{des}, r_{des})\). There are some variations present in literature about the definition of shortest path routing for instance shortest path with respect to distances in kms (or) Shortest path with respect to Time for transmitting a packet from one node to another node. The applicability of Shuffle Net is poor because the Number of nodes must be in the form \( d\Delta^d \) thus Shuffle Net exists for 8 nodes when \( (\Delta=2, d=2) \) and 24 nodes when \( (\Delta=2, d=3) \).

Procedure Shufflenet \((\Delta, d)\)
\[
\{ \\
\text{Shuffle\_nodes} = d \times \text{power}(\Delta, d); \\
\text{Shuffle\_numerator} = ((\text{Shuffle\_nodes}\times(\Delta -1)\times(3\times d -1))-(2\times d \times \text{power}(\Delta, d-1))); \\
\text{Shuffle\_denominator} = (2\times(\delta -1)\times(\text{Shuffle\_nodes}-1)); \\
\text{Shuffle\_average\_hop\_length} = (\text{float})\text{Shuffle\_numerator}/(\text{float})\text{Shuffle\_denominator}; \\
\}
\]
Shuffle_lmax = d*power(Δ, d);

In the present thesis the following parameters are considered for comparison purpose (wide chapter 1.3) in this section now we describe the formulae for these parameters with respect to Shuffle Net.

Number of nodes in the Network \( N = d\Delta^d \) \( \quad (2.2.1) \)

Average Hop Length

\[
\bar{h} = \frac{N(\Delta-1)(3d-1)-2d(\Delta^d-1)}{2(\Delta -1)(N-1)} \quad (2.2.2)
\]

Average Edge Loading

\[
\bar{L} = \frac{\text{Hsum}}{N^*\Delta^\Delta} \quad (2.2.3)
\]

LMAX = Maximum Edge loading

\( \bar{L} = \frac{\text{Hsum}}{N^*\Delta^\Delta} \quad (2.2.4) \)

Average Network Utilization

\( U_{\text{AVG}} = \frac{\bar{L}}{\text{LMAX}} \quad (2.2.5) \)

Throughput

\( \lambda = \frac{1}{\text{LMAX}} \quad (2.2.6) \)

Hsum: Sum of the shortest paths of all possible source and destination nodes.

2.3 Calculation of various parameters of Shuffle Net:

Using the algorithm given in sec(2.2) various parameters of Shuffle Net for \((\Delta, d = 2, 3, 4, 5)\) are given in the following table.
<table>
<thead>
<tr>
<th>( \Delta )</th>
<th>d</th>
<th>N</th>
<th>( \bar{H} )</th>
<th>( \bar{L} )</th>
<th>LMAX</th>
<th>( U_{AVG} )</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>2</td>
<td>8</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>24</td>
<td>3.26087</td>
<td>3</td>
<td>24</td>
<td>0.125</td>
<td>0.041667</td>
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<tr>
<td>2</td>
<td>4</td>
<td>64</td>
<td>4.634921</td>
<td>3.74194</td>
<td>64</td>
<td>0.058468</td>
<td>0.015625</td>
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<tr>
<td>2</td>
<td>5</td>
<td>160</td>
<td>6.069182</td>
<td>3.97959</td>
<td>160</td>
<td>0.024672</td>
<td>0.00625</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>18</td>
<td>2.176471</td>
<td>3.01449</td>
<td>18</td>
<td>0.167472</td>
<td>0.055556</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>81</td>
<td>3.5625</td>
<td>4.75</td>
<td>81</td>
<td>0.058642</td>
<td>0.012346</td>
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<tr>
<td>3</td>
<td>4</td>
<td>324</td>
<td>5.021672</td>
<td>6.2548</td>
<td>324</td>
<td>0.019305</td>
<td>0.003086</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1215</td>
<td>6.507413</td>
<td>7.20143</td>
<td>1215</td>
<td>0.005927</td>
<td>0.000823</td>
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<tr>
<td>4</td>
<td>2</td>
<td>32</td>
<td>2.258065</td>
<td>3.57143</td>
<td>32</td>
<td>0.111607</td>
<td>0.03125</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>192</td>
<td>3.691099</td>
<td>5.72801</td>
<td>192</td>
<td>0.029823</td>
<td>0.005208</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>1024</td>
<td>5.17302</td>
<td>7.75953</td>
<td>1024</td>
<td>0.007578</td>
<td>0.000977</td>
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<tr>
<td>4</td>
<td>5</td>
<td>5120</td>
<td>6.668294</td>
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<td>0.000195</td>
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<td>5</td>
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<td>50</td>
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<td>3.90444</td>
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<td>0.078088</td>
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<tr>
<td>5</td>
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<td>375</td>
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<td>375</td>
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<td>8.60668</td>
<td>2500</td>
<td>0.003443</td>
<td>0.0004</td>
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</tbody>
</table>

In order to calculate above parameters program was written in C- language.

Some interesting and relevant statistics were calculated parameter wise[57] and are given in the following table using the data given in Table (2.3.1).
<table>
<thead>
<tr>
<th>Parameters</th>
<th>( \bar{L} )</th>
<th>( \bar{H} )</th>
<th>( N )</th>
<th>LMAX</th>
<th>UAVG</th>
<th>( \lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>745.8</td>
<td>4.18627753</td>
<td>368.363514</td>
<td>359.363514</td>
<td>0.02140329</td>
<td>0.00857288</td>
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<tr>
<td>Standard Error</td>
<td>5.09139065</td>
<td>0.98179063</td>
<td>160</td>
<td>180</td>
<td>0.02928328</td>
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<td>Median</td>
<td>186.100356</td>
<td>0.06392203</td>
<td>0.01858906</td>
<td>0.02982306</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient of Variation</td>
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<td>38.498731</td>
<td>4.259351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>186.100356</td>
<td>38.498731</td>
<td>4.259351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sample Variance</td>
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<td>38.498731</td>
<td>4.259351</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>7.41633470</td>
<td>7.41633470</td>
<td>7.41633470</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Skewness</td>
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<td>2.688576</td>
<td>2.688576</td>
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<td>Range</td>
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<td>2</td>
<td>2</td>
<td></td>
<td></td>
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<tr>
<td>Maximum</td>
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<td>79273783</td>
<td>79273783</td>
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</tr>
<tr>
<td>Sum</td>
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<td>11187</td>
<td>11187</td>
<td></td>
<td></td>
<td></td>
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<td>Count</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table (2.3.2): Some interesting and relevant statistics for various parameters of Shuffle Net
2.4 Results and Discussions:

Critically comparing the calculated values of various parameters in Table(2.3.1) and the corresponding statistics of the parameters given in Table(2.3.2), One can draw the following conclusions:

❖ As 'Δ' (or) 'd' values are increasing the parameter representing the Total Number of Nodes(N) also increases. (or)

The Parameter representing the Total Number of Nodes (N) increases along with values of 'Δ' and 'd'. This implies that the Total Number of Nodes (N) has Positive relationship between the degree 'Δ' (or) diameter 'd' of the Network.

❖ The parameter Average Hop Length(H) also has similar relationship between the degree 'Δ' (or) diameter 'd'.

❖ Similar type of conclusions can be drawn with respect to Average Edge Loading(L) and Maximum Edge Loading (LMAX).

❖ When compared to Average Network Utilization ($U_{avg}$) which is decreasing as diameter 'd' increases and degree 'Δ' increases. Which implies that there exists a Negative relationship between $U_{avg}$ and 'Δ' (or)'d'.

❖ Similar type of relationship is exhibited between Throughput ($\lambda$) and degree 'Δ' and diameter 'd' of the Network.

❖ When comparing Coefficient of Variation the variation in parameter (H) is more consistent than other parameters, where as variation is more with respect to (LMAX and N).

Similar type of conclusions can also be drawn using other statistics like Median, Skewness, and Kurtosis and so on. To have the clarity among
various parameters with degree ‘Δ’ and diameter ‘d’, the data is presented in the form of bar diagrams in the following figures.

For Brevity Sake Diagrams are drawn for the parameters N and H only.

**Figure (2.4.1)** Total Number of Nodes (N) for Shuffle Net for various values of diameter ‘d’ when degree ‘Δ = 2’.
Figure (2.4.2) Total Number of Nodes (N) for Shuffle Net for various values of diameter ‘d’ when degree ‘Δ=3’.

Figure (2.4.3) Total Number of Nodes (N) for Shuffle Net for various values of diameter ‘d’ when degree ‘Δ=4’.

Figure (2.4.4) Total Number of Nodes (N) for Shuffle Net for various values of diameter ‘d’ when degree ‘Δ=5’.

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Figure (2.4.5) Average Hop Length($\overline{H}$) for Shuffle Net for various values of diameter ‘d’ when degree ‘$\Delta$ = 2’.

Figure (2.4.6) Average Hop Length($\overline{H}$) for Shuffle Net for various values of diameter ‘d’ when degree ‘$\Delta$ = 3’.
Figure (2.4.7) Average Hop Length($H$) for Shuffle Net for various values of diameter $d$ when degree $\Delta = 4$.

Figure (2.4.8) Average Hop Length($H$) for Shuffle Net for various values of diameter $d$ when degree $\Delta = 5$.
Remarks: Results of this Chapter are presented in the form of paper by Sarma, K.L.A.P., Satyanaraya, B and Praveen Kumar, P.T.V, “Stochastic approach on Multi hop light wave networks” in National seminar on Statistical Computing, held from 26th to 28th Nov, 2002 in the department of Statistics, T.M. Bhagalpur University, Bhagalpur, Bihar [64].