CHAPTER V

NETWORKING BASED ON MODIFIED DE BRUIJN GRAPHS AND ITS PERFORMANCE MEASURES
5.1 Introduction to Modified De Bruijn Graphs:

In the year 1999 B. Satyanarayana proposed a new graph known as Modified De Bruijn Graph based on De Bruijn Graph explained in chapter 3 [67]. A Modified De Bruijn Graph with respect to degree ‘Δ’ and diameter ‘d’ is denoted by M(Δ, d) whose construction is explained as follows.

Construction of Modified De Bruijn Graphs:

The construction of Modified De Bruijn Graphs M(Δ, d) is mainly based on De Bruijn Graphs D(Δ, d). The network for Modified De Bruijn Graph is defined for total number of nodes (N), where N = Δ^d (Δ ≥ 2, d ≥ 2). The basic difference between M(Δ, d) and D(Δ, d) is that presence of self loops in D(Δ, d) are removed because of the fact that the necessity does not arise to transmit an information/packet to the same node where it is generated.

For example:

One can talk on phone to the other phone but not to the same phone from where he is calling. Similarly, messages are to be sent to one computer (Source) to another computer (Destination) but not to the same computer (Source) itself. Thus self loops are removed in D(Δ, d) to obtain M(Δ, d). In M(Δ, d) self loops are removed and the nodes (0_0, 0_1, ..., 0_d), (1_1, 1_2, ..., 1_d), ..., ((Δ - 1)_1, (Δ - 1)_2, ..., (Δ - 1)_d) are connected to one another in a cyclic manner.

Structural properties of M(Δ, d):

In M(Δ, d) ‘Δ’ self edges that are physically added to D(Δ, d) thus M(Δ, d) network distance between nodes for significant number of node pairs are reduced the reduction of distance between the nodes in M(Δ, d) makes the graph more efficient than D(Δ, d).
5.2 Algorithm for calculation of various parameters of Modified De Bruijn Graphs.

In order to develop the algorithm, schematic representation of connectivity of $M(\Delta,d)$ is given in the following figures for $M(2,2)$ and $M(2,3)$.

Figure (5.2.1) The Modified De Bruijn Graph $M(2,2)$. 
Figure (5.2.2) The Modified De Bruijn Graph $M(2,3)$. 
In order to calculate various performance measurers for $M(\Delta, d)$ formulae are developed and are given as follows:

\begin{align*}
\text{Total Number of nodes } & \quad N = \Delta^d \quad \text{----------(5.2.1)} \\
\text{Average Hop Length } & \quad \bar{H} = \frac{H_{\text{sum}}}{N(N-1)} \quad \text{----------(5.2.2)} \\
\text{Average Edge Loading } & \quad \bar{L} = \frac{H_{\text{sum}}}{N^* \Delta} \quad \text{----------(5.2.3)} \\
\text{LMAX (Maximum Edge load)} & \quad \text{----------(5.2.4)} \\
\text{Average Network Utilization } & \quad U_{\text{avg}} = \frac{\bar{L}}{\text{LMAX}} \quad \text{----------(5.2.5)} \\
\text{Throughput } & \quad \lambda = \frac{1}{\text{LMAX}} \quad \text{----------(5.2.6)}
\end{align*}

$H_{\text{sum}}$: Sum of the shortest paths of all possible source and destination nodes.

An algorithm is developed for calculation of various performance measures of Modified De Bruijn Graph is given as follows:

Modified De Bruijn Graphs have a self-routing property, namely, packets can be routed from a source(src) to a destination (dest) using distributed routing algorithm in which the packets are forwarded to intermediate node, basing on a routing decision given only on the label of the destination node of the packet. The basic difference in De Bruijn and Modified De Bruijn Graph is, in De Bruijn Graphs self loops are present where as in Modified De Bruijn Graphs self loops are removed wide Fig(5.2.1) and Fig(5.2.2).

In this section, we explain the notations and definitions used in the development of shortest path algorithm.

A Shortest path routing algorithm for Modified De Bruijn Graphs (networks) from a source node $\text{Src} = (a_1, a_2, \ldots, a_d)$ to a destination node $\text{Dest} = (b_1, b_2, \ldots, b_d)$. 

\[ \text{83} \]
Δ: Degree of the network

d: Diameter of the network (Which denotes the maximum length between any nodes)

N: Total number of nodes in the Network.

The nodes of the Modified De Bruijn Graphs (network) are classified into two types.

(i) Self-nodes, in which all Symbols in the label are same.

(ii) Non-self nodes, in which all symbols in its label are not same.

The source-destination pairs are categorized into four types

(i) Self-node to Self-node

(ii) Non-Self node to Non-Self node.

(iii) Self-node to Non-Self node

(iv) Non-Self node to Self-node.

Is Self(Node) -> Finds whether it is a Self-node or not.

Both Self -> Finds the Shortest distance in number of hops between a Self-node(source) to a Self-node(destination) using self-nodes alone.

PreselftoDest-> refers to the preceding self-node to the destination node.

DistPreselfToDest-> Finds the preceding self-node to a Non-self node(destination) and also returns the shortest distance between the PeselftoDest(self-node) and destination.

AdjselftoSrc -> refers to the adjacent self-node to the source node.

DistofAdjselfToSrc-> Finds the adjacent self node to source(Non-self node)

And returns the shortest distance between the source and AdjselftoSrc.

DistSrcToAdjself-> finds the distance between source node and the immediate adjacent self node.

Shiftmatch( i,Src,Dest), 0 ≤ i ≤ d → An operation on the two strings

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Src and Dest to be TRUE iff src = \( (b_1, b_2, \ldots, b_{d-i}) = (a_{i+1}, a_{i+2}, \ldots, a_d) \)

And false otherwise.

Merge \((i, \text{Src}, \text{Dest})\), \(0 \leq i \leq d \) → is a String(or Sequence) of length \(d+i\)

Given by \( (a_1, a_2, \ldots, a_d, b_{i}, b_{i+1}, \ldots, b_d) \).

Find NormalLength Find the distance between source node and the destination node using the De Bruijn routing.

Transmit Next node refers to the adjacent node from source node to which the packet has to be transmitted.

Algorithm for Shortest path routing in Modified De Bruijn graph.

Procecurce Modified _de Bruijn route (Src, Dest, A, d)

Begin

If(Src = Dest) consume packet;

Return;

If (\((\text{IsSelf(Src)} = \text{FALSE}) \) and (\text{IsSelf(Dest)} = \text{FALSE})) then

Len = both-nonself(Src, Dest);

Else if (\((\text{IsSelf(Src)} = \text{TRUE}) \) and (\text{IsSelf(Dest)} = \text{TRUE}))

Len = bothself(Src, Dest);

Else if (\((\text{IsSelf(Src)} = \text{TRUE}) \) and \text{IsSelf(Dest)} = \text{FALSE}) then

Len = Source_self(Src, Dest);

Else if (\((\text{IsSelf(Src)} = \text{FALSE}) \) and IsSelf(Dest = TRUE))

Len = dest_self(Src, Dest);

Transmitnext_node;

Return len;

end.

Procedure findNormalLength(Src, Dest, A, d)
Begin
For(i = 0 to d)
Begin
If(Shift_match(i, Src, Dest) = FALSE)
Break;
End;
Merge(Src, Dest, i, sPath);
Return i ; (i is the path length)
End;
Procedure both_self(Src, Dest, Δ, d)
Begin
Length = findNormalLength(Src, Dest, Δ, d);
If (Length < d) return Length;
Count = DistBothSelf(Src, Dest);
If (Length < count) return Length;
Return count;
End;
Procedure both_nonself(Src, Dest, Δ, d)
Begin
Length = findNormalLength(Src, Dest, Δ, d);
If Length < d return Length;
Length 1 = DistOfSrcToAdjSelf(Src, AdjSelfToSrc) +
DistOfPreSelfToSrc(PreSelfToDest, Dest) +
DistBothSelf(AdjSelfToSrc, PreSelfToDest);
If Length > Length 1
Return Length;
End;

Procedure SourceSelf(Src, Dest, A, d)
Begin
Length = FindNormalLength(Src, Dest, A, d);
If(Length < d) return length;
Length1 = DistBothSelf(Src, PreSelfToDist) + DistPreselfToDest(PreSelfToDest, Dest);
If(Length < Length1) return Length;
Return Length1;
End;

Procedure DestSelf(Src, Dest, A, d)
Begin
Length = FindNormalLength(Src, Dest, A, d);
If(Length < d) return Length;
Length = DistOfAdjselfToSrc(Src, AdjSelftoSrc) + DistBothself(AdjSelfToSrc, Dest);
If(Length < Length1) return Length;
Return Length1;
End;

For Example:

A=2 and d=3 and A =(001) and B(101)

Here A and B are recognized as Nonself to Nonself nodes respectively and it pertains to both nonself routing procedure.

First, Find normallength(i,A,B) which yields = 2 hops

Which is less than diameter ‘d’ which is 3 in this case
Hence 2 is the shortest path length for A and B.

5.3 Calculation of various parameters of $M(\Delta, d)$:

Using the above algorithm a program is written in c-language. Various parameters of the network under consideration are calculated and are given in the following table.

Table (5.3.1) Calculated values of Performance measures of Modified De Bruijn Graphs

<table>
<thead>
<tr>
<th>$\Delta$</th>
<th>d</th>
<th>N</th>
<th>$\bar{H}$</th>
<th>$\bar{L}$</th>
<th>LMAX</th>
<th>$U_{AVG}$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>4</td>
<td>1.333333</td>
<td>2</td>
<td>3</td>
<td>0.66667</td>
<td>0.33333</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>8</td>
<td>1.964286</td>
<td>6.875</td>
<td>8</td>
<td>0.85937</td>
<td>0.125</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>16</td>
<td>2.733333</td>
<td>20.5</td>
<td>26</td>
<td>0.788462</td>
<td>0.038462</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>32</td>
<td>3.584677</td>
<td>55.5625</td>
<td>80</td>
<td>0.694531</td>
<td>0.0125</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>9</td>
<td>1.625</td>
<td>4.33333</td>
<td>7</td>
<td>0.619048</td>
<td>0.14257</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>27</td>
<td>2.448718</td>
<td>21.2222</td>
<td>31</td>
<td>0.684588</td>
<td>0.032258</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>81</td>
<td>3.370633</td>
<td>89.8889</td>
<td>138</td>
<td>0.651369</td>
<td>0.007246</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>243</td>
<td>4.33716</td>
<td>349.8642</td>
<td>535</td>
<td>0.653952</td>
<td>0.001869</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>16</td>
<td>1.733333</td>
<td>6.5</td>
<td>9</td>
<td>0.722222</td>
<td>0.111111</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>64</td>
<td>2.630952</td>
<td>41.4375</td>
<td>57</td>
<td>0.726974</td>
<td>0.017544</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>256</td>
<td>3.595037</td>
<td>229.1838</td>
<td>313</td>
<td>0.732216</td>
<td>0.003195</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1024</td>
<td>4.583257</td>
<td>1172.168</td>
<td>1589</td>
<td>0.737677</td>
<td>0.000629</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>25</td>
<td>1.791667</td>
<td>8.6</td>
<td>11</td>
<td>0.781818</td>
<td>0.090909</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>125</td>
<td>2.724194</td>
<td>67.56</td>
<td>86</td>
<td>0.785581</td>
<td>0.011628</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>625</td>
<td>3.704795</td>
<td>482.3584</td>
<td>586</td>
<td>0.789008</td>
<td>0.001707</td>
</tr>
</tbody>
</table>
Some interesting and relevant statistics are calculated parameter wise [57] and are given in the following table using the data given in Table (5.3.1).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>N</th>
<th>$\bar{H}$</th>
<th>$\bar{L}$</th>
<th>LMAX</th>
<th>$U_{AVG}$</th>
<th>$\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>170.333</td>
<td>2.810705</td>
<td>169.2036</td>
<td>231.9333</td>
<td>0.726232</td>
<td>0.062017</td>
</tr>
<tr>
<td>Standard Error</td>
<td>74.2559</td>
<td>0.26259048</td>
<td>80.23005</td>
<td>108.7519</td>
<td>0.017035</td>
<td>0.023223</td>
</tr>
<tr>
<td>Median</td>
<td>32</td>
<td>2.724194</td>
<td>41.4375</td>
<td>57</td>
<td>0.726974</td>
<td>0.017544</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>168.8399</td>
<td>36.1833974</td>
<td>183.6425</td>
<td>181.6015</td>
<td>9.084764</td>
<td>145.028</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>287.5906</td>
<td>1.01700856</td>
<td>310.7297</td>
<td>421.1943</td>
<td>0.065976</td>
<td>0.089941</td>
</tr>
<tr>
<td>Sample Variance</td>
<td>82708.38</td>
<td>1.03430641</td>
<td>96552.92</td>
<td>177404.6</td>
<td>0.004353</td>
<td>0.008099</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.52536</td>
<td>-1.0498533</td>
<td>8.319056</td>
<td>8.220688</td>
<td>-0.43998</td>
<td>5.57213</td>
</tr>
<tr>
<td>Skewness</td>
<td>2.376975</td>
<td>0.24301218</td>
<td>2.765519</td>
<td>2.741879</td>
<td>0.254546</td>
<td>2.206653</td>
</tr>
<tr>
<td>Range</td>
<td>1020</td>
<td>3.249924</td>
<td>1170.168</td>
<td>1586</td>
<td>0.240327</td>
<td>0.332704</td>
</tr>
<tr>
<td>Minimum</td>
<td>4</td>
<td>1.333333</td>
<td>2</td>
<td>3</td>
<td>0.619048</td>
<td>0.000629</td>
</tr>
<tr>
<td>Maximum</td>
<td>1024</td>
<td>4.583257</td>
<td>1172.168</td>
<td>1589</td>
<td>0.859375</td>
<td>0.333333</td>
</tr>
<tr>
<td>Sum</td>
<td>2555</td>
<td>42.160575</td>
<td>2538.054</td>
<td>3479</td>
<td>10.89349</td>
<td>0.930248</td>
</tr>
<tr>
<td>Count</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>
5.4 : Results and Discussions:

Critically comparing the calculated values of various parameters Table (5.3.1) and the corresponding statistics of the parameter given in Table (5.3.2), one can draw the following conclusions:

❖ Modified De Bruijn Graphs also exhibits similar type of relation as exhibited by Shuffle Net (or) De Bruijn Graphs (or) Kautz Graphs with respect to all parameter for different values of degree ‘Δ’ and diameter ‘d’.

❖ With respect to Coefficient of Variation the parameter $(U_{AVG})$ is showing more consistency than other parameters.

❖ Maximum Variation is observed in Modified De Bruijn Graph with respect to the parameter $(L)$.

❖ More Consistency is observed with respect to $U_{AVG}$ in Modified De Bruijn Graph.
On similar lines of previous chapters parameter $N$ and $H$ are represented diagrammatically for various values of degree $\Delta$ and diameter $d$.

Figure (5.4.1) Total Number of nodes ($N$) for Modified De Bruijn Graph for various values of $d$ when $\Delta=2$

Figure (5.4.2) Total Number of nodes ($N$) for Modified De Bruijn Graph for various values of $d$ when $\Delta=3$
Figure (5.4.5) Average Hop Length ($\overline{H}$) for Modified De Bruijn Graphs for various values of $d$ when $\Delta = 2$

Figure (5.4.6) Average Hop Length ($\overline{H}$) for Modified De Bruijn Graphs for various values of $d$ when $\Delta = 3$
Figure (5.4.7) Average Hop Length ($\bar{H}$) for Modified De Bruijn Graphs for various values of ‘d’ when ‘$\Delta = 4$’

Figure (5.4.8) Average Hop Length ($\bar{H}$) for Modified De Bruijn Graphs for various values of ‘d’ when ‘$\Delta = 5$’
Remarks:

Results of this Chapter was communicated for publication in the research Journal – (ANIJMIT) by Sarma, K.L.A.P., Satyanaraya, B and Praveen Kumar, P.T.V, entitled “Shortest Path routing algorithm based on Modified De Bruijn Graphs”.

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