Preface

This Ph.D. thesis is the work done by the author and carried out under the guidance and supervision of Dr. J. Paulraj Joseph.

Graph theory has become a part of Discrete Mathematics which is indispensable in the field of Computer Science. It has applications to other sciences (physical, biological and social), Engineering and Commerce. It serves as a mathematical model for any system involving a binary relation. Graphs have an intuitive and aesthetic appeal because of their diagrammatic representation.

The theory of decomposition of graphs is a topic with fundamental theoretical interest. It also boasts a host of application to coding theory, design of experiments, crystallography, radioastronomy, radiolocation, all kinds of networks, serology and a variety of location problems. The origin of the study of graph decomposition and factorizations can be seen in various combinatorial problems, most of which emerged in the 19th century. Now a days, graph decomposition problems rank among the most prominent areas of research in Graph Theory and Combinatorics.

The thesis consists of 5 chapters. In Chapter 1, we collect the basic definitions and theorems on graphs which are needed for the subsequent chapters.

By a graph $G = (V, E)$, we mean a finite undirected graph without loops or multiple edges. Let $G = (V, E)$ be a connected simple graph of
order $p$ and size $q$. If $G_1, G_2, \ldots, G_n$ are edge disjoint subgraphs of $G$ such that $E(G) = E(G_1) \cup E(G_2) \cup \ldots \cup E(G_n)$, then $(G_1, G_2, \ldots, G_n)$ is said to be a decomposition of $G$. Different types of decomposition of $G$ have been studied in the literature.

In Chapter 2, we define a decomposition $(G_1, G_2, \ldots, G_n)$ of $G$ to be a continuous monotonic decomposition (CMD) if each $G_i$ is connected and $|E(G_i)| = i$ for each $i = 1, 2, \ldots, n$. A CMD in which each $G_i$ is a star is said to be a continuous monotonic star decomposition (CMSD) and a CMD in which each $G_i$ is a path is said to be a continuous monotonic path decomposition (CMPD). We obtain a necessary and sufficient condition for a graph to admit a CMD. We also characterize complete bipartite graphs, spiders, wheels, olive trees which admit a CMSD. While investigating the necessary condition for a CMPD, we obtain upper bounds for the maximum degree of a spider. We also find sufficient condition for a spider of diameter $n$ to have a CMPD.

The contents of this chapter have been published in [12].

In Chapter 3, we investigate graphs which admit a CMD $\{C_i/ i = 3, 4, \ldots, n\}$ where each $C_i$ is a cycle of length $i$ in $G$. Such a decomposition is called continuous monotonic cycle decomposition (CMCD). Two cycles are said to be incident with each other if they have exactly one vertex in common. We obtain a necessary condition for a graph to have a CMPD. We prove that $K_p$ does not admit a CMPD where $p \geq 4$. We find a necessary and sufficient condition for a graph with particular diameter to have a CMPD. ie $G$ with $n(2n+1)$ edges and diameter of $G = n^2 - 1$ admits a CMCD if and only if $G$ has $2n-2$ non-isomorphic blocks each of which is a cycle $C_i$ incident with at most two cycles at a distance $\lfloor i/2 \rfloor$. 
Caterpillar is a tree in which the removal of pendant vertices results in a path. Lobster is a tree in which the removal of pendant vertices results in a Caterpillar. Let \( L \) be a Lobster and \( P_c \) be the underlying path of length \( t \) obtained by the removal of pendant vertices two times successfully. Let \( N_2 \) denote the set of pendant vertices of \( L \) which are at a distance two from \( P_c \). Let \( N_1 \) denote the set of vertices at a distance one from \( P_c \). Let \( |N_2| = n_2 \) and \( |N_1| = n_1 \). A path which contains a pendant vertex is called a pendant path. A vertex which is adjacent to a pendant vertex is called a support. A vertex which is adjacent to two pendant vertices is called a 2-support.

In chapter 4, we investigate the decomposition of Lobster into continuous monotonic path decomposition. If \( L \) admits a CMPD \((P_1, P_2, \ldots, P_m)\), we prove that \( \frac{1}{2} (-1 + \sqrt{33} + 8\varepsilon) \leq m \leq \frac{1}{2} (7 + \sqrt{1 + 8\varepsilon}) \), with at most four distinct values. Then we characterize lobsters which admit a CMPD when \( m = \frac{1}{2} (-1 + \sqrt{33} + 8\varepsilon) \) and \( \frac{1}{2} (7 + \sqrt{1 + 8\varepsilon}) \) with reference to the number of vertices \( N_1 \) and \( N_2 \).

The vertices of \( P_c \) in a Lobster \( L \) with degree at least 3 is called a junction. An edge \( e = uv \) such that \( u \) is adjacent to a junction and \( v \) is adjacent to another junction is said to be a junction-neighbor. In chapter 5, we investigate continuous monotonic star decomposition of Lobster. We obtain a sufficient condition for a lobster to admit a CMSD. We also get a necessary condition which gives an upper bound for the diameter of \( L \). Then we characterize Lobsters with \( \text{diam}(L) = 2n - 1, 2n - 2, \) and \( 2n - 3 \) in terms of junction-neighbors.

Besides several new and interesting results in the field, the thesis contains a lot of open problems for further research.