2.1 Introduction

The basic principles of the electrical theory of resistivity and the SP methods have been discussed in this chapter. The methods used to measure the properties of geologic materials can be divided into two types: methods using applied currents and those using naturally occurring currents. The method that uses applied currents is electrical resistivity and the method that uses naturally occurring current flow is self potential. The combination of surface level electrical resistivity and self potential methods has been employed in this study as well. These methods are used generally for planning efficient and economical test drilling programmes and to have a general preliminary idea of the lithological conditions.

2.2 Resistivity Method

One of the most widely used methods of geoelectric exploration is known as the resistivity method. In this method, current is introduced into the ground by two or more current electrodes, and the potential difference is measured between two points suitably placed with respect to the current electrodes. The potential difference per unit current sent through the ground is a measure of the electrical resistance of the ground between the probes. The measured resistance is a function of the geometrical configuration of the electrodes and the electrical
parameters. There are two types of resistivity measurements; one is geoelectric profiling and the other is geoelectric sounding.

2.2.1 Geoelectric Profiling

In profiling, the electrodes and probes are shifted without changing their relative positions. This gives us an idea of the surface variation of resistance values within a certain depth. The depth is related to the current electrode separation. Resistivity profiling with a fixed depth of investigation is mainly used in locating lateral variation and thus is useful in mineral exploration.

2.2.2 Geoelectric Sounding

In sounding, the positions of the electrodes are changed with respect to a fixed point and the measured resistance values at the surface reflect the vertical distribution of resistivity values in a geological section. This method is popularly known as VES (vertical electrical sounding). The principle of VES was established in the 1920s (Gish and Rooney, 1925). Electrical resistivity sounding with a varied depth of penetration is useful in delineating extensive aquifers constituting horizontally stratified earth.

2.2.3 Electrode Configuration

In profiling or in sounding, there is scope for electrode arrangement to be made in different ways in the field and such arrangements are called configurations. The different electrode configurations commonly differ in terms of 1) number of electrodes used, 2) mutual interval of electrodes, 3) mode of
arrangement of electrodes, and 4) whether the electrodes are arranged symmetrically or not with reference to the point of investigation.

2.2.4 Symmetrical Electrode Configuration

In this setup four electrodes are placed on a line on the surface, two for energizing the earth and the other two for measuring the resultant voltage. These are arranged symmetrically on either side of the point of investigation. The Wenner and Schlumberger configuration belongs to this category.

2.2.4.1 The Wenner Configuration

This was developed by Wenner in 1915. In this configuration, the outer electrodes A and B are used to send current into the ground and the inner electrodes M and N are used to measure the potential. The current and potential electrode pairs have a common midpoint and in this setup the distances between adjacent electrodes are equal i.e. AM = MN = NB = a in Fig. 2.1.

![Figure 2.1 Wenner Electrode Configuration](image)

The apparent resistivity measured with the Wenner array (electrode set up is called array) is given by $\rho = (2\pi a)(\Delta V/I)$; Where $a$ = electrode separation, $V$ = potential difference measured, and $I$ = current sent into the ground.
2.2.4.2 The Schlumberger Configuration

This was developed by Schlumberger in 1916. This method measures the potential gradient rather than the potential difference. For this purpose, potential electrodes are kept at smaller separations compared to the current electrodes. But the current and potential pairs of electrodes have a common midpoint. The separation of current electrodes \( AB = L \) and potential electrodes \( MN = a \) (Fig. 2.2).

![Figure 2.2 Schlumberger Configuration](image)

In general \( MN \leq \frac{1}{5} AB \) relation is maintained in this investigation. Here A and B are the current electrodes and M and N are the potential electrodes. The resistivity measured with this array is given by the formula

\[
\rho_a = \frac{\pi}{4} \frac{V}{I} \left( \frac{L^2 - a^2}{a} \right)
\]  

(2.1)

2.2.5 Asymmetrical Electrode Configuration

The dipole-dipole, three electrodes and the central gradient configuration belong to this group.
2.2.5.1 Dipole-Dipole Type

In this, two dipoles are used, each consisting of two similar electrodes. One pair is used for energization and the other pair for measuring potential difference. In this set up, the distance between the electrodes of a pair should be less than 5 or 6 times the distance between the centres of two dipoles. The spacing of the electrodes in each pair is $AB = MN = a$, while the distance between their midpoints is $OO' = L$ (Fig. 2.3).

![Figure 2.3 Dipole-Dipole Configuration](image)

The measured resistivity is, $\rho = \pi \frac{V}{I} \left( \frac{L(L^2 - a^2)}{a^2} \right)$ \hspace{1cm} (2.2)

2.2.5.2 Three Electrodes Type

In this system (Fig. 2.4), either one current electrode or one potential electrode is kept at infinitely (i.e. at a far off distance) and the other electrodes remain in their positions as in the symmetrical set up. In profiling, this set up of three electrodes is moved and $\rho$ readings are obtained.

![Figure 2.4 Three Electrode Configuration](image)
2.2.5.3 Central Gradient Type

In this system, the current electrodes remain fixed, a large distance apart, and the potential electrodes are moved in the central regions (Fig. 2.5)

![Central Gradient Configuration](image)

**Figure 2.5 Central Gradient Configuration**

In asymmetrical electrode configurations, the midpoint of potential electrodes is the point of investigation. These configurations are particularly suited to investigate thin (narrow) veins or dike which are vertical or steeply inclined.

Of all these various electrode configurations the most commonly employed configurations are the Wenner and the Schlumberger arrays to measure the earth resistivity. The work presented in this thesis uses the Wenner configuration. The Wenner array holds several advantages over the Schlumberger array including the simplicity of the apparent resistivity formula, requirement of small current values to produce measurable potential differences and availability of a large album of theoretical master curves for two, three, and four layer earth models (Mooney and Wetzel, 1956).
2.2.6 Measurement of Earth Resistivity

Surface resistivity measures the electrical resistivity of the subsurface materials, which include soil and groundwater characteristics, from survey stations at the ground surface. Surface resistivity can be used to study lateral changes and vertical cross sections of the natural hydrogeologic setting. Surface resistivity can also be used to study contamination of soil and groundwater and to locate buried objects.

Electrical resistivity is a fundamental property of rocks and is closely associated with their lithology. The direct current method is commonly used for determining the resistivities of the subsurface rocks by passing a known current through the ground and measuring the strength of the current flow at the surface. These resistivities are then used to predict the lithology of the rocks in the subsurface and delineate the subsurface rock layers and any inconsistencies associated with them (Dobrin and Savit, 1988). The resistance offered to the current flow is dependent on the mineralogy, particle arrangement, water content and salinity of the underlying earth layers (Woods, 1994). In soils or rocks, the flow of an electrical current occurs through

1) the pore water which acts as a conductor, and

2) the soil or rock which may act as a conductor or an insulator/resistor depending upon the nature of the mineral.

Hence, the nature of the pore water as well as the mineral will affect the resistivity. Resistivity can provide information about soil type. For example, clay will conduct electricity through both the pore water and the clay itself, whereas sand will transmit current primarily through its pore water.
Resistivity is governed by Ohm's Law which states that the change in potential across a resistor is proportional to both the current and the resistance at constant temperature. It is a fundamental property of the material and is independent of geometry. The flow of current in a medium is based on the principle of conservation of charge and is expressed by the relation,

\[ \text{div} \mathbf{J} = \frac{\partial \rho^'}{\partial t} \]  

where \( \mathbf{J} \) is the current density (A/m\(^2\)) and \( \rho^' \) is the charge density (C/m\(^2\)). This relation is also known as the equation of continuity. For stationary current this relation is reduced to,

\[ \text{div} \mathbf{J} = 0 \]  

(2.4)

The current density \( \mathbf{J} \) is related to the electric field intensity \( \mathbf{E} \) (V/m) as

\[ \mathbf{J} = \frac{1}{\rho} \mathbf{E} = -\frac{1}{\rho} \text{grad} V \]  

(2.5)

where \( V \) is the electric potential (volts) and \( \rho \) is the resistivity (\( \Omega \)-m) of the medium.

For an isotropic medium, \( \text{div} \left( \frac{1}{\rho} \text{grad} V \right) = 0 \)

(2.6)

i.e.

\[ \text{grad} \left( \frac{1}{\rho} \right) \cdot \text{grad} V + \frac{1}{\rho} \text{div} \text{grad} V = 0 \]  

(2.7)

This is the fundamental equation of electrical prospecting with direct current. If the medium is homogeneous, \( \rho \) is independent of the coordinate axes and the relation will be

\[ \nabla^2 V = 0 \]  

(2.8)
This is Laplace's equation. The determination of the potential at any point $P$ at
the surface of an infinite homogeneous medium of resistivity $\rho$ due to a point
source of current $I$ is the basis for the evaluation of potential distribution in a
layered earth. Laplace's equation in spherical polar coordinates is reduced to,

$$\frac{d}{dr} \left( r^2 \frac{dV}{dr} \right) = 0$$

(2.9)

On integrating we get,

$$V = C_1 + \frac{C_2}{r}$$

(2.10)

As the potential is taken to be zero at a large distance from the source, $C_1 = 0$. The
current density at a distance of $r$ will be,

$$J = -\frac{1}{\rho} \frac{\partial V}{\partial r} = -\frac{1}{\rho} \frac{\partial}{\partial r} \left( \frac{C_2}{r} \right) = \frac{1}{\rho} \frac{C_2}{r^2}$$

(2.11)

As the equipotential surfaces are spherical and the electric field lines as well as
the current lines are radial, the total current flowing out of a spherical surface of
radius $r$ is,

$$4\pi r^2 J = (4\pi/\rho)C_2 = I$$

(2.12)

$$C_2 = I\rho/4\pi$$

(2.13)

Therefore the potential at any point due to a current source at the surface of the
homogeneous earth is

$$V = \frac{I\rho}{2\pi r}$$

(2.14)

In practice, the current is introduced into the ground by means of two electrodes,
i.e., a source and a sink; and the potential at any point due to this bipolar
arrangement is:
where \( r_1 \) and \( r_2 \) are the distances of the point P from the source and the sink respectively.

**Apparent Resistivity:** In actual conditions, though the medium around the electrode array is assumed to be homogenous and isotropic, because of the mud, flushed zone, invaded zone and adjacent formations we have to substitute instead of resistivity, the apparent resistivity. The apparent resistivity is the weighted average of all these effects that depend upon the spacing of the electrodes.

In Wenner configuration, the four electrodes A, M, N and B are placed at the surface of the ground along a straight line so that AM = MN = NB = 'a' and AB=L is always equal to 3MN. A and B are the outer electrodes, and are used for sending current into the ground. M and N are the inner electrodes and are used to measure potentials.

For Wenner array, \( \rho = (2\pi a)(\Delta V/I) \) \hspace{1cm} (2.16)

\[ \rho = 2\pi a R \] \hspace{1cm} (2.17)

Thus the resistivity is a function of the single distance variable 'a'. In Wenner array, the potential electrodes and current electrodes are moved after each measurement.
A direct current 'I' is introduced into the homogeneous and isotropic earth by means of two point electrodes A and B. The potential difference between the two points M and N on the surface is measured (Fig. 2.6).

\[ V_M = \frac{I \rho}{2\pi} \left( \frac{1}{AM} - \frac{1}{BM} \right) \]  (2.18)

\[ V_N = \frac{I \rho}{2\pi} \left( \frac{1}{AN} - \frac{1}{BN} \right) \]  (2.19)

The Potential difference between the points M and N on the surface is

\[ \Delta V = V_M - V_N = \frac{I \rho}{2\pi} \left[\left( \frac{1}{AM} - \frac{1}{BM} \right) - \left( \frac{1}{AN} - \frac{1}{BN} \right) \right] \]  (2.20)

where \( \rho \) is the resistivity of the ground. Thus the resistivity of the homogeneous earth can be determined from the measurements on the surface. In the symmetrical arrangement , the points A, M, N, B are taken on a straight line so that the points M and N are symmetrically placed about the centre O of the spread AB (Fig. 2.6a).
Substituting \[ AN = BM = \frac{L + a}{2}, \quad AM = BN = \frac{L - a}{2}, \] (2.21)

\[ \Delta V = \frac{4I}{\pi} \left( \frac{l}{L^2 - l^2} \right) \rho \] (2.22)

The resistivity is

\[ \rho = \frac{\pi}{4} \left( \frac{L^2 - l^2}{l} \right) \frac{\Delta V}{I} \] (2.23)

In the Wenner arrangement \( L \) is equal to 3a and \( a \) is known as spacing or separation of the adjacent electrodes.

\[ \rho = 2\pi a \frac{\Delta V}{I} \] (2.24)

### 2.3 Self Potential Method

Self potential method is one of the oldest geoelectric methods and it is still used in many fields of applied Geophysics. The SP method was used by Fox as early as 1830 on the sulphide veins in a Cornish mine but the systematic use of the SP and the resistivity methods dates from about 1920. This method is also known as the spontaneous polarization method or simply as the SP method. The self potential method, as its name implies, is based upon measuring the natural potential differences which generally exist between any two points on the ground. The potential differences, partly constant and partly fluctuating are associated
with electric currents in the ground. The constant and unidirectional potential differences between any two points can arise due to various electrochemical mechanisms (Parasnis, 1997). Sato Mooney (1960) suggested that SP fields result from electric currents that are produced when a conducting body connects regions of different electrochemical potentials. The SP values ranging from a fraction of a millivolt to a few tens of millivolts, sometimes attain values of the order of a few hundred millivolts and much more. Such large potentials are observed over sulphide and graphite ore bodies and are also found in coal and manganese deposits (Parasnis). The measurement of self potential is quite easy. Any voltmeter with sufficiently high input impedance may be connected to a pair of non polarizing electrodes driven some 10-15cm into the ground and lead off.

Simple metal stakes are inadequate as electrodes. Electrochemical reactions take place between the metal and moisture in the ground, causing the build up of spurious charges on the electrodes, which can falsify or obscure the small natural self potentials. To avoid or minimize this effect nonpolarizing electrodes are used.

Potentials generated by subsurface environmental sources are lower than those induced by mineral and geothermal anomalies and often associated with high noise polarization level (Corwin, 1990). Therefore, the usage of non polarizing electrodes is mandatory when the SP method is applied in soil and environmental studies. The nonpolarizing electrode consists of a metal element immersed in a solution of salt of the same metal with a porous membrane between the solution and the soil (Corwin and Butler, 1989). Because of easy breakage of the membrane and leakage of the electrode solution we adopted firm
nonpolarizing electrodes (carbon cores from the exhausted electrical cells) (Pozdnyakov et al., 1996a).

2.3.1 Origin of Self Potential

Nourbehecht (1963) identified the source of self potential anomalies in the coupling between irreversible processes. He outlines the generation of self potential anomalies by geologically common mechanism as streaming potentials, thermoelectric potentials and diffusion potentials.

Streaming potential or electrofiltration is an electric potential difference that is developed between the ends of a capillary tube through which an electrolyte is flowing. In geophysical work it can be found associated with the flow of water through sand, porous rocks, moraine etc. Electrofiltration potentials have been used to detect leakage spots on submerged slopes of earth dam water reservoirs (William Lowrie, 1997), to detect concealed karstic cavities, springs etc. The voltage can be positive or negative and may amount to some hundreds of millivolts.

Differences in concentration of electrolytes in the ground from place to place may be expected to produce electric potential differences and is called diffusion potential. Kilty (1984) made an extensive study of diffusion potentials on peat and bogs, who found the SP values down to about -120 mV caused by concentration differences. If the concentration of electrolytes in the ground varies locally, potential differences will be set up in accordance with the equation

\[ U = \frac{2vRT}{u + vnF} \ln\left(\frac{c_1}{c_2}\right) \]  

(2.25)
where \( u \) and \( v \) are the velocities of positive and negative ions in the electrolyte, \( R \) is gas constant, \( T \) is absolute temperature, \( F \) is Faraday’s constant, and \( n \) is the valence of the ions. And this may cause the SP anomalies of the order of fractions of a millivolt to tens of millivolts.

The potential difference due to the temperature difference in the ground from place to place is known as thermoelectric potential. Self potential measurements were used in geothermal exploration by Corwin (1982). There are four distinct potential differences in the self potential mechanism of ore bodies (kilty, 1984). These are:

1) The voltage drop within the ore body \( (V_{ore}) \)

2) A voltage across the ore body electrolyte interface at the site of electronation

3) A voltage across the ore body electrolyte interfaces at the site of de-electronation and

4) The potential difference within the electrolyte.

The sum of these voltages forms an equation of the form

\[-V_{ore} + (\Delta \phi_e - \Delta \phi_d) = IR,\]  

(2.26)

The voltage drop within the ore body is larger in magnitude at equilibrium. As the current (and consequently the voltage \( IR \)) in the electrolyte increases, the voltage across the ore body decreases. This is consistent with the observation that the potential across a spontaneously running cell decreases with the increasing current. The potential at the electrode-electrolyte interface also changes with the increasing current in a way that decreases the
quantity \((\Delta \phi_z - \Delta \phi_d)\). Near equilibrium the deviation of the interface potentials from equilibrium values, or over potentials (Bockis and Reddy 1970) are linearly related to the current. Therefore, near equilibrium the voltage \(IR\) or self potential is linearly related to over potential.

The generation of self potentials takes place under nonequilibrium conditions (Kilty 1984). Equilibrium for an electrode-electrolyte combination would require zero net current flow across the interface; but, without a net current flow there is no voltage in the electrolyte and no SP anomaly. In an ore body the separate sites of electronation and deelectronation are short circuited. So there is no way to equilibrium. The net current \(i\) flowing across the ore electrolyte interface is related to the equilibrium current \(i_o\) and the over potential at the interface through the Butler-Rolmer equation (Bokris and Reddy 1970)

\[
i = i_o \left[ \exp((1 - \beta) F \eta / RT) - \exp(-\beta F \eta / RT) \right], \tag{2.27}
\]

where \(\beta\) is the factor that depends on the symmetry of \(i\) versus \(\eta\) relationship for either direction of the charge transfer reaction. It depends on the composition of the electrolyte and electrode.

Because the net current flowing through the electrolyte’s resistance is the cause of an SP anomaly, the factors that determine the current (the over potential, resistance of the electrolyte and ore body and the equilibrium potential) are more important in determining the expected size of an SP anomaly than is the equilibrium potential alone.
SP anomaly by surface traversing and mapping may sometimes give distorted effects due to overburden. Skianis et al (1991, 1993) made a mathematical approach to describe the influence of overburden on the formation of self potential and deduce a relationship between the resistivity of upper layer and SP anomalies on ground surface. Telford et al (1988) explained two examples of SP anomalies of sulphide minerals, one at a depth of 90 m and another at a depth of 120 m respectively. The first one gave good surface anomaly and the second one did not show up any on the surface. Both have been drilled. It was found that the first bore has the top soil as sand and the second one has top soil with clay cover. From these examples it is clearly seen that the overburden has a pronounced effect on surface SP. To overcome these problems, self potentials are measured with increments of potential electrodes spacing with a fixed center. This technique is akin to Vertical Electrical Sounding (VES). As suggested by Schlumberger and Doll (in Rider, 1986), the SP arises from the electrical disequilibrium created by connecting formations vertically (in the electrical sense) when in nature they are isolated. This electrical disequilibrium created by connecting formations can be studied vertically by electric coring. By self potential and Vertical Electrical Sounding in Wenner configuration, electrical coring across the formations can be obtained vertically. SP data are collected in the field as potential drop across a passive dipole, normally consisting of a pair of nonpolarizable electrodes (Ives and Janz 1961).

Many source mechanisms have been proposed to explain the genesis and time and space patterns of the SP field, both in Applied Geophysics (Sato and Mooney 1960; Orellana 1982; Kilty 1984; Parasnis 1986; Furness 1992) and in Tectonophysics (Burton 1985, Bernard 1991, Johnston 1989). From the physical point of view, the common aspect of the many source models is that an electric charge polarization is set up which is responsible for electric current circulation in
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The study of the SP field caused by any configuration of buried stationary electric charge accumulations inside any complex resistivity geometry of the geological structure can be considered in the framework of the classical geoelectrical problem of finding the special variation of the electric potential due to point current sources and (or) sinks (Patella, 1997).

Consider any geological structure, characterized by a flat air earth boundary, where a generic point on the origin of a Cartesian coordinate reference system, defined by the unit vectors $K_x$, $K_y$ and $K_z$ pointing downwards into the earth. For simplicity, an observation point $P$ moves along a straight line in the air earth boundary, which is assumed to be the $x$-axis in order to simulate a simple self potential profiling technique. In the half space problem, the geoelectrical potential $U$ at $P$ is given by (Alfano 1959; Keller and Frischknecht 1966).

$$U(P) = \frac{1}{2\pi} \left[ \int V \frac{\rho}{r} \, dV + \int \frac{E \cdot \nabla \rho}{\rho} \, dV \right],$$

(2.28)

where $J$ is the current density and $E$ is the electric field vectors.

The first integral in Eq. (2.28) gives the primary contribution to the potential at $P$, caused by the point sources and sinks (Dey and Morrison 1979), given by
\[ U_1(P) = \frac{1}{2\pi} \int \frac{\rho \text{div} J}{r} dV = \sum_{n=1}^{N} \frac{\rho_n I_n}{2\pi r_n} , \]  
(2.29)

where \( \rho_n \) is the resistivity about the \( n^{th} \) point source or sink.

The second integral in Eq. (2.28) gives the secondary contribution to the total potential at \( P \), due to resistivity variations inside the volume \( V \).

The secondary potential \( U_2 \), for a total of \( M \) buried discontinuity surfaces,

\[ U_2(P) = \frac{1}{2\pi} \int \frac{E \cdot \text{grad} \rho}{\rho} r dV = \frac{1}{2 \pi \varepsilon_0} \sum_{m=1}^{M} \sum_{t=1}^{T_m} \frac{\sigma_{m,t}}{r_{b,m,t}} \Delta S_{m,t} , \]  
(2.30)

The total potential \( U \) of \( P \) can be written as

\[ U(P) = \sum_{n=1}^{N} \frac{\alpha_n}{r_n} + \sum_{m=1}^{M} \sum_{t=1}^{T_m} \frac{\beta_{m,t}}{r_{b,m,t}} , \]  
(2.31)

\( U(P) \) can be expressed as a sum of \( N + \sum_{m=1}^{M} T_m \) terms of similar type, i.e. all varying as \( 1/r \). The first group represents the primary potential, i.e. the potential caused by the discrete set of point sources and sinks. The second group represents the secondary potential namely the contribution due to the discrete set of continuous aggregates of induced electric charges along the buried resistivity boundary surfaces.

The following notation will be used for the total potential \( U(P) \),

\[ U(P) = \sum_{q=1}^{Q} \frac{\Gamma_q}{r_q} \]  
(2.32)

where, \( \Gamma_q \) is the intensity of the \( q^{th} \) charge accumulation, either primary or induced and \( r_q \) is the corresponding distance from an observation site \( P \).
As a flat air-earth boundary has been assumed with the origin of the \((x, y, z)\) coordinate reference system the \(x\)-axis has been taken as the SP profiling line along which the observation point \(P\) can move, Eq. (2.32) becomes

\[
U(x) = \sum_{q=1}^{Q} \frac{\Gamma_q}{[(x-x_q)^2 + h_q^2]^{1/2}}
\]

and the electric field surface component \(E_x\) for different position \(x\) of the observation site \(P\) is,

\[
E_x(x) = \sum_{q=1}^{Q} \frac{\Gamma_q (x-x_q)}{[(x-x_q)^2 + h_q^2]^{3/2}}
\]

2.3.3 The Self Potential power spectral density

Any electric field surface component can be viewed as a continuous superposition of stationary harmonic waves with wave numbers distributed in the interval \(0 \leq p \leq \infty\).

The Fourier transform of \(E_x(x-x_q,h_q)\) is

\[
\Phi_x(p;x_q,h_q) = \int_{-\infty}^{\infty} E_x(x-x_q,h_q)e^{-px} dx
\]

And the inverse Fourier transform of \(\Phi_x(p;x_q,h_q)\) exists and equals the electric field component \(E_x(x-x_q,h_q)\)

\[
E_x(x-x_q,h_q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_x(p;x_q,h_q)e^{px} dp
\]

The SP power spectral density is defined by the real and even function,

\[
\Psi_x(p) = \Phi_x(p) \cdot \Phi_x^*(p) = |\Phi_x(p)|^2
\]

It is the spectral density of the electric power per unit resistance and unit interval on the wave number scale. It gives the energy per unit time of any distribution of
electric charges in the lower half-space. The additional electric power per unit resistance which is necessary to change the steady state of the SP field from wave number \( p \) to \( p + dp \), is given by \( \Psi'(p)dp \). According to Parseval's theorem (Bath 1974), the total power associated with the electric field component \( E_x(x) \) over the whole selected x-axis is given by

\[
\int_{-\infty}^{\infty} E_x^2(x)dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi'(p)dp
\]

(2.38)

2.3.4 Space-Domain (SD) Analysis

The charge occurrence probability (COP) function: Generally this analysis consists of the determination of the distribution of the sources in the subsoil, knowing their effects on the free surface. This corresponds to finding the location of the electric charge accumulations from the known SP drops along a selected straight line profile. To solve this problem, the electric field component of total power modified as

\[
\int_{-\infty}^{\infty} E_x^2(x)dx = \sum_{q=1}^{Q} \int_{-\infty}^{\infty} E_x(x)E_x(x-x_q,h_q)dx
\]

(2.39)

\[
\sum_{q=1}^{Q} \mathcal{R}(x_q,h_q) = \frac{\sum_{q=1}^{Q} \int_{-\infty}^{\infty} E_x(x)E_x(x-x_q,h_q)dx}{\int_{-\infty}^{\infty} E_x^2(x)dx} = 1
\]

(2.40)

The Coefficient \( \mathcal{R}(x_q,h_q) \) for each \( q = 1, 2, 3, \ldots, Q \) could be computed, then the fraction of the total power associated with the electric field component \( E_x(x) \) over the entire selected profiling x-axis contributed by each \( q^{th} \) charge source or sink could be calculated. The total electric field component
along any chosen profiling line due to the existing charge distribution can be measured.

By repeated iterations fairly complete charge distribution geometry can be observed. Then using the known cross correlation bounding inequality (Bendat and Piersol, 1986), the observed electric field is modified as

\[ E_x(x - x_q, h_q) = \Gamma_q \mathfrak{I}_x(x - x_q, h_q), \quad (2.41) \]

where

\[ \mathfrak{I}_x(x - x_q, h_q) = \frac{(x - x_q)}{\left[(x - x_q)^2 + h_q^2\right]^{3/2}}, \quad (2.42) \]

The function \( \mathfrak{I}_x(x, h) \) is the space domain scanning function (SDS). \( E_x(x) \) is known as field data function. Space domain function combines with the continuously recorded electric field component; the following new basic inequality is derived.

\[ -1 \leq \eta(x_q, h_q) \leq +1, \quad (2.43) \]

where

\[ \eta(x_q, h_q) = C h_q^{3/2} \int_{-\infty}^{\infty} E_x(x) \mathfrak{I}_x(x - x_q, h_q) \, dx, \text{ for } h_q > 0 \quad (2.44) \]

The function of \( \eta(x_q, h_q) \) will be denoted the charge occurrence probability (COP) functions. Positive values of \( \eta(x_q, h_q) \) are the result of a major influence from positive charge accumulations. Negative values of \( \eta(x_q, h_q) \) are the result from negative charge concentrations. The numerical value of the \( \eta(x, h) \) function for any arbitrarily assigned pair of coordinates \( (x, h) \) in the tomoplane, will give the probability that a positive \( (\eta > 0) \) or negative \( (\eta < 0) \) charge, located there is responsible for the observed SP surface field.
2.3.5 Schematic Diagram Explaining the SP Measurements at Different depths in the Borehole and on the Surface with Increasing of Electrode Spacing

Figure 2.6b Schematic diagram for recording SP log in the borehole  
Figure 2.6c Schematic diagram for recording SP log on the surface of the ground.

The SP or spontaneous polarization is caused by the migration of ions in the solution, by chemical reaction at the mineral surfaces and by ionic segregation by the flow of electrolytic fluid through the pores in rock with differential pressure acting together or individually.

In the conventional SP well logging technique, the potential differences are measured between a reference electrode fixed at the surface and a moving electrode in the borehole (Fig. 2.6b). The measured potential is akin to the potential difference between the negative and positive terminals of the source battery (Fig. 2.6d)
In the present study, a new approach is adopted to measure the subsurface SP through the two non polarizing electrodes planted at the surface separated by a distance ‘a’. The electromotive forces generated at different levels of the horizontal formations with increasing depths are received vertically by increasing electrode spacing on both sides of the fixed centre of observation with equal distances (Fig. 2.6c). Thus the potential gradient which is perpendicular to the bed boundaries are measured by increasing electrode spacing. It is found that the SP measurements obey the Ohms law, \( V = I \cdot R \) where \( I \) is the current flow within the formations, \( R \) is the resistance value of the ground between the two electrodes (Fig. 2.6e).

\[ V = I \cdot R \] (Ohm’s Law)

The progressive increase of electrode spacing laterally in steps of ‘a’ records the SP values generated at different layers of the beds is tabulated. Graphs are plotted for SP Vs electrode spacing. The series of measurements of the natural potential developed in various levels of the layers are made by increasing the electrode spacing in successive steps about a fixed point is called vertical exploration or depth probing.
2.4 Instrumentation

Aquameter – CRM (Computer Resistivity meter) is a modern version of earth resistivity meter, makes use of the advanced technology of microprocessors. The aquameter CRM 500 consists of a generator and a receiver housed in a single cabinet. It possesses a single averaging facility of averaging 4, 6, 16 and 64 signals and an option of using 5 to 500 mA current settings in 5 ranges. Resistivity could be read in mΩ, Ω, kΩ levels. Self potential cancellation is done automatically. It takes only 6 to 8 seconds to measure the resistance after canceling the SP and display the values. Another advantage of this equipment is the error diagnosis cropping due to poor contacts, unwanted current settings, resistance settings, low battery power and many more. The accuracy of the measurement is ± 8% upto 0.003 Ω.

Figure 2.7  Panel View of the Resistivity meter
2.5 Field Measurements

In the present study vertical electrical sounding (VES) method is adopted. Vertical sounding is a means by which the apparent resistivity variations in a vertical direction are investigated. This method consists of expanding the electrode separation about a common point and making a series of measurements with assumption that the larger the electrode spacing the more is the depth of investigation (Fig. 2.8). This means that as we go on increasing the inner electrode spacing we get information of deeper and deeper layers.

![Figure 2.8 Depth of current penetration increases with increasing electrode spacing](image)

The interpretation of sounding data primarily aims at understanding the subsurface conditions, such as the number of layers present, their thickness, the resistivity of individual layers and a few other geoelectrical parameters. From the knowledge of local geology and the resistivities of layers, it is often possible to identify the lithological units in a given area which can be made use of in hydrogeological studies. The soundings curves are interpreted either by
theoretical or empirical methods. The theoretical methods involve the computation of apparent resistivity values for various electrode separations for a given set of subsurface conditions.

For the electrical survey, Wenner configuration was adopted. It is best adapted to lateral profiling. When we use the Wenner configuration, the resistivity and self potential can be measured in the same observing point simultaneously. That is, the resistivity has been measured with the Wenner configuration, and the self potential is also measured between the inner electrodes of the same configuration but disconnecting the current electrodes.

The greatest depth of detection is achieved by Wenner array when the array spread is broad. And it improves considerably when compared to other arrays (Apparao et al, 1992). The depth of the investigation of the resistivity prospecting and self potential prospecting is synchronizing with each other in the Wenner Configuration. The Wenner array is the most efficient array in terms of the received voltage per unit of transmitted current (Roy and Apparao 1971). The Wenner array has the lowest number of measurements compared with the other arrays employed in geoelectrical imaging surveys. (Xu and Noel, 1993)

In the present survey aquameter CRM is used. It displays the resistivity values directly in terms of ohm, Ω (or) milli ohms, mΩ (or) kilo ohms, kΩ. There is no need of calculating the resistivity value from the voltmeter and ammeter readings. To measure the self potential, the current electrodes are disconnected and the self potential is measured between the two inner electrodes MN. The
potential value is displayed in the same aquameter in the range of millivolts to volts and the scale is +mV or -mV.

Keeping the place of observation constant, the electrode spacing has been gradually increased to a maximum range of 425 meters. For every electrode spacing self potential as well as apparent resistivity values were recorded. The self potential was first measured; at that time no current electrodes were connected. The nonpolarizing electrodes driven some 10-15 cm into the ground and the leads of potential electrodes were connected to the potential terminals of the aquameter P1, P2. The potential value was displayed on the LED screen.

After taking the self potential readings, the current electrodes were connected to the current passing terminals of the aquameter CRM. While DC current was being sent to the earth through the current electrodes the resistance value was measured by the meter. The resistance values are displayed in the LED screen. The measured resistance values were calculated into apparent resistivity using the relation \( \rho = 2\pi a R \). The same procedure is repeated for different increasing electrode spacing and the measured values of the self potential and the resistivity with their respective electrode spacing are tabulated for the detailed studies.