1. **FUNDAMENTAL ASPECTS**

The laminar flow of fluid bounded by one or two planes is of fundamental interest in fluid dynamics and it is useful to explore the interplay of various fluid forces and their interactions with external body forces. Apart from such practical interest, the study of these flows gained importance in fluid dynamics because such flows are ideal models of several flow situations occurring in nature. It is worth mentioning that Newtonian and non-Newtonian fluid dynamics, heat transfer, magnetohydrodynamics, hydrodynamic lubrication and flows through porous media are some of the important areas wherein the studies concerning the laminar flow between parallel planes or bounded by a single plane are important. We introduce a brief survey of the fundamental aspects concerning these areas of study which in turn will provide a foundation for formulating the problem of this dissertation.

(a) **Newtonian and non-Newtonian fluids**

The classical Navier - Stokes equation of motion is derived by assuming a linear relationship between stress tensor and the strain rate tensor in the fluid.
Fluids which obey this relationship are known as newtonian fluids. They possess a single rheological property called viscosity. Water, air, mercury, engine oil are some of the examples of newtonian fluids.

Many important industrial fluids are non-newtonian in their flow characteristics. These include paints, various suspensions, glues, printing inks, food materials, soap and detergent slurries, polymer solutions and many others. Because such fluids have more complicated equations that relate the stress to the velocity gradient than is the case with newtonian fluids, new branches in the fields of fluid mechanics and heat transfer are developed.

Another important characteristic of such fluids, because of their large apparent viscosities, is that they have a tendency towards low Reynolds and Grashof numbers and high Prandtl number. Thus laminar flow situations are encountered more often in practice than with newtonian fluids.

The fundamentals of non-newtonian laminar flow and heat transfer include an examination of the classification system for such fluids, the development of a method to predict the fully developed pressure drop in ducts both circular and non-circular in cross-sectional shape and a consideration of some aspects of the heat transfer processes.
The subject of thermophysical properties and their measurements is an important one when dealing with non-newtonian fluids and includes the consideration of both classical methods for making such measurements as well as several approaches which are unique to such fluids. It is unfortunate and time consuming that these property measurements must be made continuously when dealing with non-newtonian fluids because they are not pure substances and vary in their properties because of different preparation methods. The non-newtonian fluids can in turn be divided into purely viscous and viscoelastic fluids. The purely viscous time-independent fluids are defined as those whose shear stress depends only upon some function of the shear rate, and sometimes an initial yield stress. Viscoelastic fluids are those which possess properties of both viscosity and elasticity. We list below various non-newtonian fluids and some examples.

<table>
<thead>
<tr>
<th>Type</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>Water, air, mercury, engine oil</td>
</tr>
<tr>
<td>Pseudoplastic</td>
<td>paints, glues, blood, suspensions</td>
</tr>
<tr>
<td>Dilatant</td>
<td>wet sand, sugar and borax solutions</td>
</tr>
<tr>
<td>Bingham plastic</td>
<td>certain emulsions and paints</td>
</tr>
<tr>
<td>Thixotropic</td>
<td>printing inks, food materials, paints</td>
</tr>
<tr>
<td>Rheoplastic</td>
<td>clay suspensions</td>
</tr>
<tr>
<td>Viscoelastic</td>
<td>polymer solutions (e.g., polyox water)</td>
</tr>
</tbody>
</table>
The simplest type of non-newtonian fluids are the pseudoplastic and dilatant fluids whose relation between shear stress and shear rate can be expressed by an equation of the form \( \tau \propto \gamma^N \) where \( \gamma \) is the strain rate.

Because of this functional relation such fluids are called power law fluids and unlike newtonian fluids (which have only a single rheological property, i.e. viscosity) two independent properties are required to specify the relation between the shear stress and the shear rate. The term \( k \) is called the consistency and \( n \) is called the flow index. If \( n \) is less than one, the fluid is pseudoplastic and if greater than one, it is dilatant. The Couette flow problem helps researchers to investigate the interactions of various forces like viscous force, buoyancy force, electromagnetic force etc., in newtonian and non-newtonian fluids. The investigations on Couette flow with suction/injection are very important in boundary layer control.

(b) Heat transfer

We shall now describe heat transfer which is a phenomena associated with both newtonian and non-newtonian fluids. It is customary to categorise the various heat transfer processes into three basic types or modes, although,
as will become apparent as one studies the subject, it is certainly a rare instance when one encounters a problem of practical importance which does not involve at least two, and sometimes all three, of these modes occurring simultaneously. The three modes are conduction, convection and radiation.

Heat conduction is the term applied to the mechanism of internal energy exchange from one body to another, or from one part of a body to another body, by the exchange of the kinetic energy of the molecules by direct communication or by the drift of free electrons in the case of heat conduction in metals. This flow of energy or heat passes from the higher energy molecules to the lower energy ones (i.e., from a high temperature region to a low temperature region). The distinguishing feature of conduction is that it takes place within the boundaries of a body, or across the boundary of a body into another body placed in contact with the first, without an appreciable displacement of the matter comprising the body.

A metal bar heated on one end will, in time, become hot at its other end. This is the simplest illustration of conduction. The laws governing conduction can be expressed in concise mathematical terms, and the analysis of the heat flow can be treated analytically in many instances.
Convection is the term applied to the heat transfer mechanism which occurs in a fluid by the mixing of one portion of the fluid with another portion due to gross movements of the mass of fluid. The actual process of energy transfer from one fluid particle or molecule to another is still one of conduction, but the energy may be transported from one point in space to another by the displacement of the fluid itself.

The fluid motion may be caused by external mechanical means in which case the process is called forced convection. If the fluid motion is caused by density differences which are created by the temperature differences existing in the fluid mass, the process is termed free convection or natural convection. The circulation of the water in a pan heated on a stove is an example of free convection. The important heat transfer problems of condensing and boiling are also examples of convection-involving the additional complication of a latent heat exchange.

It is virtually impossible to observe pure heat conduction in a fluid because as soon as a temperature difference is imposed on a fluid, natural convection currents will occur as a result of density differences.
The basic laws of heat conduction must be coupled with those of fluid motion in order to describe, mathematically, the process of heat convection. The mathematical analysis of the resulting system of differential equations is perhaps one of the most complex fields of applied mathematics. Thus, for engineering applications, convection analysis will be seen to be a subtle combination of powerful mathematical techniques and the intelligent use of empiricism and experience.

Thermal radiation is the term used to describe the electromagnetic radiation which has been observed to be emitted at the surface of a body which has been thermally excited. This electromagnetic radiation is emitted in all directions; and when it strikes another body, part may be reflected, part may be transmitted, and part may be absorbed. If the incident radiation is thermal radiation (i.e., if it is of the proper wavelength) the absorbed radiation will appear as heat within the absorbing body.

Thus in a manner completely different from the two modes discussed above, heat may pass from one body to another without the need of a medium of transport between them. In some instances there may be a separating medium, such as air which is unaffected by this passage of energy.
The heat of the Sun is the most obvious example of thermal radiation.

We have described convection as the term applied to the heat transfer mechanism which takes place in a fluid because of a combination of conduction within the fluid and energy transport which is due to the fluid motion itself—the fluid motion being produced either by artificial means or by density currents.

Since fluid motion is the distinguishing feature of heat convection it is necessary to understand some of the principles of fluid dynamics in order to describe adequately the processes of convection. When any real fluid moves past a solid surface, it is observed that the fluid velocity varies from a zero value immediately adjacent to the wall to a finite value at a point some distance away.

Consider the case of a flow past a plane surface where the fluid velocity will vary from a uniform value at points away from the wall to zero at the wall. For fluids of low viscosity, such as air or water, the region near the surface, in which most of the velocity variation occurs, may be quite thin—depending on the free stream velocity of the fluid. In many applications—such as low speed
aerodynamics, hydraulics, etc. - it is possible to obtain satisfactory results by assuming that the fluid is inviscid. Hence, the flow may be treated as though it slips past the surface with no viscous retardation. However, since the process of convection of heat away from the wall (if the wall is at a temperature different from the free stream of the fluid) is intimately concerned with thermal conduction and energy transport due to motion in the fluid layers in the immediate vicinity of the wall, the simplification of assuming the fluid to be inviscid may not be made when analyses of heat convection are undertaken.

Since the region in which the retarding effect of the fluid viscosity plays a dominant role will often be a very thin layer near the wall, it is possible to simplify the description of the convection process by introducing the concept of the velocity boundary layer. The velocity boundary layer is defined as the thin layer near the wall in which one assumes that viscous effects are important. Within this region the effect of the wall on the motion of the fluid is significant. Outside the boundary layer it is assumed that the effect of the wall may be neglected. The exact limit of the boundary layer cannot be precisely defined because of the asymptotic nature of the velocity variation. The limit of the boundary layer is usually taken to be at
the distance from the wall at which the fluid velocity is equal to a predetermined percentage of the free stream value. This percentage depends on the accuracy desired, 99 or 95 percent being customary. Outside the boundary layer region the flow is assumed to be inviscid. Inside the boundary layer the viscous flow may be either laminar or turbulent. In the case of laminar boundary layer flow, adjacent fluid layers slide relative to one another but do not mix in the direction normal to the fluid streamlines. Thus, any heat that flows from the surface away from it does so mostly by conduction—although a transport of energy is also accomplished by virtue of the fact that the fluid has a velocity component normal to the surface.

In the presence of a temperature distribution in the fluid, the velocity field and the temperature field mutually interact, which means that the temperature and the velocity depend on one another. In the special case when buoyancy force $\beta g$ may be disregarded, and when the properties of the fluid may be assumed to be independent of temperature, mutual interaction ceases, and the velocity field no longer depends on the temperature field, although the converse dependence of the temperature field on the velocity field still persists.
This happens at large velocities (large Reynolds numbers) and small temperature differences, such flows being termed forced. The process of heat transfer in such flows is described as forced convection. Flows in which buoyancy forces are dominant are called free, the respective heat transfer being known as free convection. The density difference will produce a positive or negative buoyant force (depending on whether the surface is hotter or colder than the fluid) in the fluid near the surface. The buoyant force results in a fluid motion, substantially in the vertical direction, past the surface with consequent convective heat transfer taking place. The force of gravity is, then, the driving force which produces the fluid motion and maintains the convective process.

The heating of rooms and buildings by the use of 'radiators' is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens, etc., surrounded by cooler air are due to free convection, at least in part. The state of motion which accompanies free convection is evoked by buoyancy forces in the gravitational field of earth, the latter being due to density differences and gradients.

Venkatakrishna Murthy (1980, 1982, 1984), Leadon (1957) and Herts investigated the effects of heat transfer in a Couette flow.
(C) Hydrodynamic lubrication

The example of Couette flow indicates an essential method of approach to hydrodynamic and hydromagnetic aspects of lubrication. We describe the principle of hydrodynamic theory of film lubrication in brief. It is well known that, if there is movement between two machine parts pressed together, they will wear away rapidly unless a thin lubricating layer (oil) is placed between them. But the laws of friction between lubricated parts are quite different from those of dry friction. Coulomb has shown that dry friction depends primarily on pressure between the parts and the properties of the surfaces. Surface properties appear to have no effect on well-lubricated parts. The frictional force depends on the viscosity of the oil, the magnitude of the relative velocity, and the thickness of the lubricating layer. Since a viscous fluid adheres to the moving parts, their sliding velocity is transmitted to the fluid and the force depends on the internal friction within the lubricant provided there is no direct contact between the moving parts.

The distance between lubricated moving parts, that is, the thickness of the fluid film, is always very small and the viscosity of the lubricant high. For this reason the
Reynolds number is always small, and the inertia terms in the Navier-Stokes equation can be neglected in comparison with the friction terms; the creep flow exists.

A two-dimensional theory of lubrication was first developed by Reynolds. It showed that, if the lubricant layer is to transmit pressure between a shaft and a journal, the layer must have varying thickness, otherwise the stresses in the lubricant cannot balance the load of the shaft.

The flow system can be described most readily by referring to a slider on a flat surface. The width of both is assumed to be very large to achieve approximately plane flow. The mathematical formulation and solution is essentially that of Couette flow. The resultant load-carrying capacity for unit width of the slider is obtained by integrating the difference between the hydrodynamic pressure and the atmospheric pressure over the length of the slider.

(4) Magnetohydrodynamics

One dimensional laminar flow problem is one of the simplest problems in magnetohydrodynamics. The solution to this problem gives insight into the MHD generator, pump, flow-water, and bearings, and it forms the basic method for
treatment all viscous flow devices. Electromagnetic body forces act on the fluid and in turn the motion of the fluid in the presence of the electromagnetic field may generate an induced emf and alters the fields. The equations of motion are mathematically complex in original form but under certain approximations called MHD approximations, the electromagnetic body force can be reduced to a simple expression namely, \( \mathbf{J} \times \mathbf{B} \) where \( \mathbf{J} \) is the current density and \( \mathbf{B} \) is the magnetic flux density. If \( \mathbf{H} \) is the magnetic field we also have \( \mathbf{J} = \nabla \times \mathbf{H} \) and \( \mathbf{B} = \mu_0 \mathbf{H} \) where \( \mu_0 \) is the magnetic permeability. The term \( \mathbf{J} \times \mathbf{B} \) must be added to Navier-Stokes equation as an external force term, just as we add \( \rho \mathbf{g} \) as the buoyancy force term in density varying fluids. The electromagnetic force is usually non-conservative (rotational) and not derivable from a scalar potential function except in some rare cases. This is evident if we write \( \mathbf{J} \times \mathbf{B} = \frac{1}{\mu_0} (\mathbf{B}, \nabla) \mathbf{H} - \frac{1}{\mu_0} \nabla (\frac{\mathbf{B}^2}{2}) \).

The irrotational part is derived from a potential \( \frac{\mathbf{B}^2}{2 \mu_0} \) which may be combined with pressure. In MHD flows the fluid equations and the Maxwell's equations are coupled and must be solved simultaneously. Boundary conditions at ideal insulators and perfect conductors are derived using the principles of electrodynamics. The details are
described by Hughes and Young (1966).

(e) Flows through porous media

The Couette flow and Poiseuille flow through porous media are of considerable importance in geophysical problems. The movement of water, oil and natural gas through the ground, seepage underneath dams and large buildings, flow through packed towers in some chemical processes like filtrations etc., depend on this type of flow. A porous medium is a continuous solid phase with many void spaces, or pores, in it. Examples are sponges, cloths, wicks, paper, sand and gravel, filters, concrete, bricks, plaster walls, many naturally occurring rocks (e.g. sandstones and some limestones), and the packed beds used for distillation, absorption, etc. In many such porous solids the void spaces are not connected, so there is no possibility that fluid will flow through them. For example, foamed plastic hot-drink cups, and iceboxes have many pores, but because of the 'closed cell' structure of the plastic these pores are not interconnected. Thus these porous media form excellent barriers to fluid flow. On the other hand a pile of sand has fewer pores than a foamed polystyrene drinking cup, but its pores are all connected, so that fluids can easily flow through it. The porous media with no
interconnected pores are described as impermeable to fluid flow and those with interconnected pores as permeable. The velocity in a porous medium is usually not large and the flow passages so narrow that laminar flow may be assumed without hesitation. The permeability of a porous medium is its most useful fluid flow property. The permeability is a measure of the ease with which a fluid flows through the medium. If the permeability is higher, then the flow rate will be higher for a given pressure gradient. The porosity of a porous medium is another important property defined as the void volume, or volume of pore space divided by the total volume of the medium. The porosity of consolidated materials depends mainly on the degree of concentration. The porosity of unconsolidated materials depends on the packing of the grains, their shape, arrangement and size distribution. The specific surface of a porous material is defined as the total interstitial surface area of the pores per unit bulk volume of the porous medium.

The foundation on which laminar flow of fluids through porous media rests is Darcy's law. Darcy (1856), while experimenting with flow of water through sand filters, noted that the flow rate of water was proportional to the
difference in head of water across the filter. His basic equation was \( \frac{V}{m} = -k \frac{(h_2-h_1)}{L} \), where \( V \) is flow rate of water per unit cross-section area, \( k \) is a constant for the system, \( h_2-h_1 \) is the difference in fluid head across length \( L \). Since the work of Darcy, a number of experiments have studied the flow of various fluids through many types of porous solids. The basic relation of Darcy has been extended to cover flow of any fluid, and as with any law, its limitations and range of applicability have been defined. The use of Darcy's law is restricted to cases in which the flow is laminar or streamlined. The Darcy's law expresses that the seepage velocity is proportional to the pressure gradient and it does not have convective acceleration of the fluid. This law is therefore considered to be valid for low speed flows.

To study the flows through porous media we must introduce a simplified porous medium model that will be amenable to a mathematical treatment and that will incorporate the main features of a porous medium described so far.

One of the essential features of a porous medium, in connection with the flow of a fluid through it, is that it restricts the transport of the fluid to well defined channels. The porous medium is in fact a non-homogeneous medium. For the
sake of mathematical analysis it is replaced by a homogeneous fluid which has dynamical properties equal to those of non-homogeneous continuum. Thus one can study the flow of a hypothetical homogeneous fluid under the action of properly averaged external force (additional resistance $-\frac{\mu}{\tau} \nabla$ due to the medium). The complicated problem of the flow through a porous medium thus reduces to the flow problem of a homogeneous fluid with resistance of the medium taken into account. When the porosity of the medium is very close to unity, the fluid occupies almost all parts of the porous medium and a generalized Darcy's law must be used to represent the flow [Yamamoto and Iwamura (1976)].

The two simple concepts - Darcy's law and the law of conservation of mass - are used to describe the laminar flow of a fluid through porous medium.

2. INTRODUCTION

The Couette flow between two parallel porous planes is a basic configuration in fluid dynamics and its importance in several areas like boundary layer control, free convection, magnetohydrodynamics, lubrication theory and flows through
porous media is well known. Verma and Bansal (1966) considered the Couette flow of a viscous incompressible fluid with uniform suction at the stationary plane. For small values of the suction parameter, the series solutions are obtained for the exact non-linear equations. Venkatasiva-Murthy and Pennuraj (1982) considered the two dimensional Couette flow of a viscous incompressible temperature stratified fluid between two parallel planes which are maintained at different temperatures. The fluid is subjected to constant suction and injection at the boundaries which differ by a small quantity. This differential suction at the boundaries causes a vertical velocity in the fluid which depends on the vertical coordinate and consequently a horizontal velocity which depends linearly on the horizontal coordinate \( x \). The continuity equation gives similarity variables for velocity components. Though the buoyancy force term is taken into consideration, its effect is neglected to the first order in the course of development of the solution. The stratification affects only the temperature distribution. The solution of the linearised equations as well the non-linear effects are studied.

It is the purpose of the present dissertation to show that Couette flow situations in which the horizontal velocity depends on the horizontal coordinate \( x \) arise even in the
absence of differential suction/injection at the boundaries. Motivated by this aspect it will be shown that one such situation occurs when one of the planes is kept at a temperature which varies parabolically with the distance along the plane while the suction/injection velocities at the planes have the same constant value.

We consider the two dimensional flow of a viscous incompressible stratified fluid between two horizontal planes. The fluid is injected with a constant velocity at the upper plane and sucked with the same constant velocity at the lower plane. The lower plane is maintained at temperature varying parabolically along the plane. The upper plane is kept at a constant temperature. The upper plane also moves with a constant velocity. The buoyancy force is taken into consideration and this makes the velocity and temperature coupled in the differential equations. The varying temperature of the boundary induces a temperature distribution in the fluid which depends quadratically on the horizontal distance $x$. This in turn causes a horizontal velocity which depends on the horizontal distance. The solution is obtained by introducing similarity variables for velocity and temperature and solving the resulting coupled differential equations.