INTRODUCTION:

The magnetohydrodynamic heat transfer has gained significance in recent times owing to its applications in recent advancement of space technology. For example, the problem of controlling the skin friction and Aerodynamic heat transfer around the high speed vehicles has vital important with the advent of rocketary and supersonic flights. Because of the possibility of reducing the skin friction and heat transfer in liquid metals at low Prandtl numbers and convective gas flows at high Mach numbers by the application of magnetic field, several papers have appeared in the last two decades on MHD thermal convection.

The MHD heat transfer can be divided arbitrarily into two sections: one contains problems in which the heating is an incidental by-product of the electromagnetic fields (as in such MHD devices as generators, pumps etc.) and the second consists of problems in which the primary use of the electromagnetic fields is to control the heat transfer (as in the natural convection flows and aerodynamic heating).

These heat transfer problems are generally solved for two types of configurations: (1) Problems where the semi-infinite fluid is bounded by a non-porous (or porous) rigid boundary with a free-stream which moves with either a uniform or time dependent fluctuating velocity. (2) Problems where in the fluid is confined between non-porous (or porous) rigid boundaries. It is intutively evident that the temperature distribution around a hot boundary in a fluid stream gives rise to a thermal boundary layer across which the temperature gradient is large. Hence apart from the velocity distribution in the flow field the study of this thermal boundary layer and the influence of different forces (or kinematical factors) on this boundary layer is a major aspect of these heat transfer problems. When the buoyancy force is disregarded and when the properties of the fluid may be assumed to be independent of temperature, mutual interaction ceases and the velocity field no longer depends on the temperature field, although the converse dependence of the temperature field on the velocity field still persists. This happens at large Reynolds number and small temperature differences. Such flows being termed as forced flows and the process of heat transfer in such a flow is described as 'Forced convection'. Flows in which this buoyancy force is dominant is called free (natural) flow and the process of heat transfer is known as 'Free (natural) convection'. This free convection occurs at small Reynolds number and in the presence of large temperature differences.

The heat transfer in a conducting fluid under a uniform magnetic field neglecting the buoyancy force has been studied by several authors, viz. Lehnert (21), Blevis (5), Pai (33), Leaden (20), Schlichting (44), Sielgel (45), Geshuni and Zhukhovitskii (8), Poots (34), Sutton and Sherman (54), Osterle and Young (32), Soundalgekar (47), Rudraiah and Mariyappa (40). Siegel (45) has considered the effect of magnetic field on the fully developed convective heat transfer in a parallel plate channel for uniform wall heat flux. Geshuni and Zhukhovitskii (8) have considered one dimensional shear flow between infinite parallel walls maintained at different temperatures, the flow being wholly due to the temperature difference. This solution may be regarded as the convective analogue of the Hartmann flow maintained by a pressure gradient. Soundalgekar (47) has considered the short circuited generalised MHD couette flow when the walls are at equal and unequal temperature under MHD. He found that the percentage rate of heat transfer in general increases uniformly with the uniform increase in the value of both the pressure gradient and the Hartmann number. Unlike the cases of Blevis and Leadon, the rate of heat transfer at the lower stationary plate is affected by the magnetic field in case of pressure induced flow superposed on simple shear flow. Rudraiah and Mariyappa (40) have considered the steady laminar flow of an incompressible viscous, electrically conducting fluid between two parallel plates one in uniform motion and the other at rest with uniform suction under a uniform transverse magnetic field. Using similarity solutions he obtained the asympototic solutions of velocity and temperature distributions in cases of uniform wall temperature and uniform heat flux.

It was shown by sparrow and Cess (46), Gill and Casal (10) that the buoyancy force significantly affects the horizontal flows of low Prandtl number fluids which are more sensitive to gravitational field than the fluid with high Prandtl number. In fact the process of free convection as a mode of heat transfer has many applications. For example, cooling process of nuclear reactors where in liquid sodium and Mercury are used as coolants. As an another practical application in aerodynamics the possibility arises of altering the flow and heat transfer around high speed vehicles assuming that the air is sufficiently ionised. Hence the extent to which the thermal buoyancy force influences a forced flow is a topic of interest. Keeping this in view several authors (Gupta (14), Mishra and Muduli (27), Merric (26), Mohan (29), Rao et al (37)) have investigated the combined free and forced convection effects on the fully developed hydro-magnetic channel flows of a conducting fluid.

Gupta (14), has shown that the velocity distribution becomes asymmetric in the presence of buoyancy force. Also for Grasheff Number $G>0$ velocity increases in the lower half of the channel and decreases in the upper of the channel with increase in G and this situation gets reversed for $G<0$. Taking the linearly varying temperature dependent viscosity Soundalgekar (53) has extended the above mentioned problem. In the case of variable viscosity with more cooling of the plates there is a tendency of separation at the lower plate. This problem was studied in case of porous boundaries by Mishra and Muduli (27), using small perturbation technique. The flow will be governed by the suction Reynolds number R based on the constant suction and injection velocity, apart from the Grashoff number G and Hartmann number M. It was shown that an increase in R decreases the velocity and increases the induced magnetic field. Mohan (29) has formulated the combined free and forced convection flow in a rotating finitely conducting channel subjected to a uniform transverse magnetic field, Keeping in view its applications in the study of Geo-physical and Astro-physical phenomena of the coriolis force and its influence on the electro magnetic force, the buoyancy force and the various other body forces. Using the magnetic boundary conditions as mentioned in Gold (12) the velocity and magnetic field are obtained in general. The boundary layer thicknesses are calculated in various particular cases of interest and it was found that a strong interaction occurs between the Hartmann and Ekman layers because of the interplay of coriolis and the electro magnetic force. This problem has been extended to porous boundaries by Rao et al (37). The solution is obtained by using the small perturbation method with the suction Reynolds number S as the perturbation parameter. It is found that the suction and injection at the porous walls prevent the flow to reduce to two-dimentional in the limiting inviscid, slow motion case. This amounts to the violation of Taylor-Proudman theorem valid in the inviscid, rotating case in the presence of permeable boundaries. Problems where the fluid is semi-infinite in extent bounded by a rigid wall and the free-stream oscillates with time-dependent velocity are often encountered in engineering applications viz., aerodynamics of a helicopter rotor or in a fluttering aerofoil or in a variety of bio-engineering problems. The analysis of the hydromagnetic forced oscillatory (with free stream oscillation) flow has been dealt in detail by several authors notably Suryaprakash Rao (39), Soundalgekar (48), Lal (19), Prasada Rao and Krishna (36). Suryaprakash Rao (39) has discussed the hydromagnetic forced oscillatory flow with constant suction while Soundalgekar (49) extended the same for variable suction. They observed that the magnitudes of the skin friction, the wall temperature and the heat flux at the wall increases both with the increase in the magnetic field and the increase in the frequency of the fluctating stream. Taking the bueyancy force into consideration, this study has been extended to include the effect of free convection on the oscillatory flows by Soundalgekar (50) and K.N.V. Murthy (30). Most of the researches mentioned above are based on Lighthill method (24) which reduces the governing coupled non-linear partial differential equations to sets of ordinary differential equations which can be solved using the standard techniques (analytical or Numerical methods). The effects of free-stream oscillations on the upward flow of a viscous, incompressible fluid past a porous vertical plate was presented by Soundalgekar (50,51,52) for constant and variable suction respectively. Soundalgekar (50,51) has mentioned the possibility for non zero temperature differences $(T'_w - T'_\infty)$ to occur and made a comprehensive study of the free convective effects on the oscillatory flow past an infinite, vertical, porous plate with constant suction. He has also shown the significant contributions of the frictional heating to the flow and heat transfer characteristics. However, when the temperature differences $(T'_w - T'_\infty)$ are appreciably large, then the volumetric heat generation (absorption) term may exert strong influence on the heat transfer and as a consequence on the fluid flow as well. Recently heat transfer studies with internal heat sources have been proposed for earth's mantle (Gaskell (7), Runcorn (41), Tozer (55)) and for the outer region of star interiors (Bethe (3)). The volumetric rate of heat generation has been assumed to be either constant (Bird (4), Inman (17), Ostrach (31), Poppendiek (35), Sastri (43)) or a function of space variables (Chambre (6), Gill (11), Grosh and Cess (13), Helmann et al (15), Low (25)). Some authors have considered directly the viscous dissipation and the expansion effect (Gee (9), Modejski (28)).

In all the above investigations the boundaries are assumed to be flat. But there are many physical situations in which the surface of the solid boundaries are wavy in nature. For example, the surface formed by cleavage of mica contains irregularities of the order of 20 A in size, and the irregularities of the surface of an ideally smooth quartz crystal can be upto 100 A in height (Kragelskii (18)). Flow over a wavy wall has attracted the attention of authors in the recent times owing to its applications in different areas such as transpiration cooling of re-entry vehicles and rocket boosters, cross-hatching on ablative surfaces and film vapourisation in combusion chambers. Ackert (1) has treated compressible, inviscid fluid flows over wavy walls. Lighthill (23) extended the work of Ackert (1) by including the effects of viscosity in both the mean and distrubed flows. Inger (16) analysing compressible flow over wavy walls, has extended the analysis of Lighthill by including the effects of heat transfer. In all these investigations, the mean flow is assumed to a linearly within the distrubance sub-layer. Benzamin (2) has carried out an analysis for incompressible flows over flexible walls and pointed out that the mean flow (may) not be linear in the viscous sub-layer. Keeping this in view, Lekoudis, Nayfeh and Saric (22) have discussed the compressible viscous flow past wavy walls using linear analysis without restricting the mean flow in the distrubance layer. The effect of wavyness on a viscous flow past an infinite wall has been investigated by Sankar and Sinha (42). A transformation of the co-ordinate system was used which reduces the wavy wall to a flate wall in the transformed system. Using a perturbation technique similar to Lighthill's method, Vajravelu and Sastry (56) have extended the above analysis to free convection flow in a vertical wavy channel in the presence of a constant heat source. The hydromagnetic case of this problem has been studied by Rao, Krishna and Debnath (38).

In this dissertation an attempt has been made to study the flow and heat transfer aspect of an incompressible, viscous, electrically conducting fluid in a horizontal porous channel bounded by a wavy wall and a flat wall in the presence of a constant heat source. Using the long wave approximation the governing equations of flow and heat transfer have been solved. The influence of heat source, suction and wavyness of the boundaries on the flow has been brought out through the numerical analysis of the velocity distribution, temperature distribution, skin friction and Nusselt number.

2. FORMULATION AND SOLUTION :

Consider the flow of an incompressible, electrically conducting horizontal porous channel bounded below by a flat wall and above by a wavy wall. Choosing the Cartesian frame of reference $O(x',y')$ such that y'-axis is vertical. Let $y' = 0$ be the lower flat wall and let the wavy wall be represented by $y' = d + \epsilon^* \cos ('x')$, where ϵ^* the amplitude of the wave, is assumed to be small. A uniform magnetic field of strength H_0 is applied parallel to the y'-axis. The equations governing the steady two-dimensional flow and heat transfer in a viscous incompressible fluid occupying the channel are the momentum equations are

$$u' \frac{\partial u'}{\partial x'} + v' \frac{\partial u'}{\partial y'} = - \frac{\partial p'}{\partial x'} + \nu \nabla^2 u' - \left(\frac{\sigma \mu_e^2 H_0^2}{\rho} \right) u' \qquad (2.1)$$

$$u' \frac{\partial v'}{\partial x'} + v' \frac{\partial v'}{\partial y'} = - \frac{\partial p'}{\partial y'} + \nu \nabla^2 v' \quad (2.2)$$

the Continuity equation

$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \quad (2.3)$$

and the energy equation

$$\rho c_p \left(u \frac{\partial T}{\partial x'} + v \frac{\partial T}{\partial y'}\right) = k \nabla^2 T + Q \quad (2.4)$$

$$\left(\nabla^2 = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} \right)$$

where u' and v' are the velocity components, p is the pressure, σ is the electrical conductivity, μ_e the magnetic permeability, c_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid and Q is the constant heat addition/absorption.

The boundary conditions are

$$u' = 0, \quad v' = -v_o \quad (v_o > 0), \quad T = T_o \quad \text{at} \quad y' = 0$$

$$\left.\begin{aligned} v' \cos\phi - u' \sin\phi &= - v_o \cos\phi, \\ v' \sin\phi + u' \cos\phi &= v_o \sin\phi \\ T &= T_1 \end{aligned}\right| \quad \text{on} \quad y' = \eta(x') \quad (2.5)$$

where $\tan\phi = \frac{d\eta}{dx}$ is small.

Defining the non-dimensional variables (x, y, u, v, p, θ) as

$$(x,y) = (x',y')/d, \quad (u,v) = (u',v')d/\nu \; ; \; p = p'/\rho(\nu/d)^2 \; ;$$

$$\theta = \frac{T - T_o}{T_1 - T_o} \quad (2.6)$$

The governing equations and the boundary conditions in the non dimensional form reduce to

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \nabla^2 u - M^2 u \qquad (2.7)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \nabla^2 v \qquad (2.8)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (2.9)$$

$$P \left(u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} \right) = \nabla^2 \theta + \alpha \qquad (2.10)$$

$$\begin{aligned} &u = 0;\ v = -\phi;\ \theta = 0 && \text{on } y = 0 \\ &\left.\begin{aligned} v \cos\phi - u \sin\phi &= -\phi \cos\phi \\ v \sin\phi + u \cos\theta &= \phi \sin\phi \\ \theta &= 1 \end{aligned}\right\} && \text{on } y = \eta(x) \end{aligned} \qquad (2.11)$$

where

$M^2 = \dfrac{\sigma \mu_e^2 H_0^2 d^2}{\rho \nu}$, the Hartmann number

$P = \mu C_p / k$, the Prandtl number

$\alpha = Q\, d^2 / k(T_1 - T_0)$, the non-dimensional heat source parameter

$\epsilon = \dfrac{\epsilon^*}{d}$, the non-dimensional amplitude parameter

$\lambda = \lambda' d$, the non-dimensional frequency parameter

Using the perturbation method we write the total velocity and total temperature distributions as

$$u(x,y) = u_0(y) + \epsilon\, u_1(x,y) + \ldots\ldots\ldots\ldots$$
$$v(x,y) = -\phi + \epsilon\, v_1(x,y) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$p(x,y) = p_o(x) + \epsilon\, p_1(x,y) + \ldots\ldots\ldots\ldots$$

$$\theta(x,y) = \theta_o(y) + \epsilon\, \theta_1(x,y) + \ldots\ldots\ldots\ldots \qquad (2.12)$$

$(u_o(y), -s)$, $\theta_o(y)$ and $p_o(x)$ are the velocity distributions, temperature distribution and applied pressure of the mean flow. (u_1, v_1), θ_1, $p_1(x,y)$ are the perturbations over the velocity, temperature and pressure distributions respectively due to wavyness of the boundary substituting (2.12) in equations (2.7) - (2.10) and equating the like powers of ϵ we get

$$\frac{d^2u_o}{dy^2} + s\frac{du_o}{dy} - M^2 u_o = c \qquad (2.13)$$

$$\frac{d^2\theta_o}{dy^2} + (ps)\frac{d\theta_o}{dy} = -\alpha \qquad (2.14)$$

to the zeroth order and

$$u_o\frac{\partial u_1}{\partial x} + v_1\frac{du_o}{dy} - s\frac{\partial u_1}{\partial y} = -\frac{\partial p_1}{\partial x} + \nabla^2 u_1 - M^2 u_1 \qquad (2.15)$$

$$u_o\frac{\partial v_1}{\partial x} - s\frac{\partial v_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \nabla^2 v_1 \qquad (2.16)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad (2.17)$$

$$p\left(u_o\frac{\partial \theta_1}{\partial x} - s\frac{\partial \theta_1}{\partial y} + v_1\frac{\partial \theta_o}{\partial y}\right) = \nabla^2 \theta_1 \qquad (2.18)$$

to the first order, where $c = \frac{\partial p_o}{\partial x}$.

Assuming the slope of the wavy wall to be small with the help of (2.12) the boundary conditions (2.11) can be simplified to

$$\left.\begin{array}{llll} u_o = 0 \; ; & \theta_o = 0 & \text{on} & y = 0 \\ u_o = 0 \; ; & \theta_o = 1 & \text{on} & y = 1 \end{array}\right| \qquad (2.19)$$

$$\begin{array}{ll} u_1 = 0 \; ; \; v_1 = 0 \; ; \; \theta_1 = 0 & \text{on } y = 0 \\ u_1 = -\exp(i\lambda x)\, u_o' \; ; \; v_1 = 0 \; ; \; \theta_1 = -\theta_o' \exp(i\lambda x) & \text{on } y = 1 \end{array} \qquad (2.20)$$

where a prime denotes differentiation w.r.t. y. Introducing the stream function $\overline{\Psi}_1$ as

$$u_1 = \frac{\partial \overline{\Psi}_1}{\partial y} \quad ; \quad v_1 = \frac{\partial \overline{\Psi}_1}{\partial x}$$

into (2.15) and (2.16) and eliminating the non-dimensional pressure p_1, we get

$$u_o'' \overline{\Psi}_{1,x} - u_o (\overline{\Psi}_{1,xyy} + \overline{\Psi}_{1,xxx}) + \emptyset (\overline{\Psi}_{1,yyy} + \overline{\Psi}_{1,xxy}) + + 2\overline{\Psi}_{1,xxyy} + \overline{\Psi}_{1,xxxx} + \overline{\Psi}_{1,yyyy} - M^2 \overline{\Psi}_{1,yy} = 0 \qquad (2.21)$$

$$p(u_o \theta_{1,x} - \emptyset \theta_{1,y} + \theta_o' \overline{\Psi}_{1,x}) = \theta_{1,xx} + \theta_{1,yy} \qquad (2.22)$$

Keeping in view the conditions (2.20), we write the general solution for Ψ under the long-wave approximation ($\lambda << 1$) as

$$\overline{\Psi}(x,y) = \sum_i (\lambda^i \psi_i) \exp(i \lambda x) \qquad (2.23)$$

$$(i = 0, 1, 2, \ldots\ldots)$$

Substituting (2.23) in (2.21) and (2.22) and separating the terms of various orders in λ, we obtain the following differential equations to the order of λ^2,

$$\psi_o^{iv} + S\,\psi_o''' - M^2\psi_o'' = 0\ ;\quad t_o'' + S\,P\,t_o' = 0 \qquad (2.24)$$

$$\left.\begin{aligned} &\psi_1^{iv} + S\,\psi_1''' - M^2\,\psi_1'' = i\,(u_o\psi_o'' - u_o''\,\psi_o) \\ &t_1'' + S\,P\,t_1' = i\,P\,(u_o\,t_o + \theta_o'\,\psi_o) \end{aligned}\right\} \qquad (2.25)$$

$$\left.\begin{aligned} &\psi_2^{iv} + S\,\psi_2''' - M^2\,\psi_2'' = 2\,\psi_o'' + S\psi_o' + i(u_o\psi_1'' - u_o''\,\psi_1) \\ &t_2'' + P\,S\,t_2' = t_o + i\,P(\,u_o\,t_1 + \theta_o'\,\psi_1) \end{aligned}\right\} \qquad (2.26)$$

The boundary conditions (2.20) in terms of ψ_i are

$$\left.\begin{aligned} &\psi_o = 0\ ;\ \psi_o' = 0\ ;\ t_o = 0 \quad \text{on } y = 0 \\ &\psi_o = 0\ ;\ \psi_o' = u_o'\ ;\ t_1 = -\theta_o' \quad \text{on } y = 1 \end{aligned}\right\} \qquad (2.27)$$

$$\left.\begin{aligned} &\psi_i = 0\ ;\ \psi_i' = 0\ ;\ t_i = 0 \quad \text{on } y = 0 \\ &\psi_i = 0\ ;\ \psi_i' = 0\ ;\ t_i = 0 \quad \text{on } y = 1 \end{aligned}\right\} \text{ for } i \geq 1 \qquad (2.28)$$

Solving the equations (2.13) and (2.14) using the conditions (2.19), we obtain the mean flow solution as

$$u_o(y) = \frac{c}{M^2}\left[1 + \frac{(\text{Sinh}m(y-1) - \exp(S/2)\,\text{Sinh}(my))\exp(-S/2)}{\text{Sinh}\, m}\right]$$

$$\theta_o(y) = c_2\,(\exp(-PSy) - 1) - a_{40}\, y$$

where

$$m = \frac{(S^2 + 4M^2)^{1/2}}{4} \quad ; \quad a_{40} = \alpha / PS$$

$$c_2 = \frac{(1 + a_{40})}{(\exp(-PS) - 1)}$$

The perturbed flow solutions in terms of ψ and θ_1 are obtained by solving (2.15) - (2.18) using the conditions (2.20) which are given by

$$\Psi_0 = A_1 + A_2 y + [A_3 \operatorname{Cosh} (my) + A_4 \operatorname{Sinh} (my)] \exp(-\phi y/2)$$

$$\Psi_1 = i (B_1 + B_2(y) + [B_3 \operatorname{Cosh} (my) + B_4 \operatorname{Sinh} (my)]\exp(-\phi y/2) +$$
$$+ \phi_1 (y))$$

$$t_0 = c_4(\exp (-\phi p y) - 1)$$

$$t_1 = i (D_1 + D_2 \exp (-\phi p y) + \phi_2 (y)$$

where

$$a_1 = (m \operatorname{Sinh} m - \phi/2 \operatorname{Cosh} m) \exp (-\phi/2)$$

$$a_2 = (m \operatorname{Cosh} m - \phi/2 \operatorname{Sinh} m) \exp (-\phi/2)$$

$$a_3 = \exp (-\phi/2) \operatorname{Cosh} (m)$$

$$a_4 = \exp (-\phi/2) \operatorname{Sinh} (m)$$

$$a_{47} = (a_3 - 1 + \phi/2) (a_2 - m) - (a_1 + \phi/2) (a_4 - m)$$

$$A_1 = u_0' (1) (a_4 - m)/ a_{47}$$

$$A_3 = -A_1$$

$$A_4 = u_0' (1) (a_3 - 1 + \phi/2)/ a_{47}$$

$$A_2 = \phi/2 A_3 - m A_4$$

$$a_5 = (m^2 + \phi^2/4)$$

$$a_6 = m\phi$$

$$a_{42} = m^2 + \phi/2$$

$$a_7 = a_{42} \exp (\phi/2)$$

$$a_8 = - m (1 + \phi/2) \exp (\phi/2)$$

$a_9 = a_{42} A_3 - a_6 A_4$

$a_{10} = a_{42} A_4 - a_6 A_3$

$a_{11} = \frac{e}{M^2} \left(\frac{a_9}{\text{Sinh} m} - a_5 A_3 \right)$

$a_{12} = \frac{e}{M^2} \left(\frac{a_{10}}{\text{Sinh} m} - a_5 A_4 \right)$

$a_{43} = e\, a_5 / M^2$

$a_{44} = e\, a_6 / M^2$

$a_{45} = e\, a_7 / M^2$

$a_{46} = e\, a_8 / M^2$

$a_{13} = a_{43} A_1$

$a_{14} = a_{43} A_2$

$a_{15} = a_{44} A_1$

$a_{16} = a_{44} A_2$

$a_{17} = a_{44} A_3$

$a_{18} = a_{44} A_4$

$a_{19} = \frac{e}{M^2} \left(a_8 A_4 - a_7 A_3 - \frac{a_9 \exp(\mu/2)}{\text{Sinh } m} \right)$

$a_{20} = a_{45} A_4$

$a_{21} = a_{45} A_1$

$a_{22} = a_{45} A_2$

$a_{23} = a_{46}\ A_3$

$a_{24} = a_{46}\ A_1$

$a_{25} = a_{46}\ A_2$

$b_8 = m - \phi/2$

$b_9 = m + \phi/2$

$a_{26} = (m + 1)\phi/2$

$a_{27} = (m^2 \text{ Cosh } m - m\phi \text{ Sinh } m + \phi^2/4 \text{ Cosh } m) \exp(-\phi/2)$

$a_{28} = (m^2 \text{ Sinh } m - m\phi \text{ Cosh } m + \phi^2/4 \text{ Sinh } m) \exp(-\phi/2)$

$e_2 = (1 + a_{40})\ (\exp(-P\phi) - 1)$

$e_4 = \theta_o'(1)/(1 - \exp(-\phi P))$

$a_{41} = P\ \phi\ e_2$

$a_{31} = c\ e_4/m^2$

$a_{29} = a_{31}/\text{Sinh } m$

$a_{30} = a_{29} \exp(\phi/2)$

$a_{32} = a_{41}\ A_1$

$a_{33} = a_{40}\ A_1$

$a_{34} = a_{41}\ A_2$

$a_{35} = a_{40}\ A_2$

$a_{37} = a_{40}\ A_3$

$a_{36} = a_{41}\ A_3$

$a_{38} = a_{41} A_4$

$a_{39} = a_{40} A_4$

$b_1 = 2\, b_8^2\, (b_8^2 + \phi\, b_8 - M^2)$

$b_2 = 2\, b_9^2\, (b_9^2 - \phi\, b_9 - M^2)$

$b_3 = 16\, b_8^2\, (4\, b_8^2 + 2\, \phi\, b_8 - M^2)$

$b_4 = 16\, b_9^2\, (4\, b_9^2 - 2\, \phi\, b_9 - M^2)$

$$b_5 = \frac{4\, b_8^2 + 3\, \phi\, b_8 - 2\, M^2}{b_8\, (b_8^2 + \phi\, b_8 - M^2)}$$

$$b_6 = \frac{4\, b_9^2 - 3\, \phi\, b_9 - 2\, M^2}{b_9 (b_9^2 - \phi\, b_9 - M^2)}$$

$b_7 = 8\, \phi^2\, M^2$

$$d_1 = [\, \exp(-m)\, ((a_{14} - a_{16})\, b_5 + (a_{15} - a_{13})) - e(a_9 - a_{10})/M^2 + \\ + (a_{21} + a_{24}) - (a_{22} + a_{25})\, b_5\,] / b_1$$

$$d_2 = [\, \exp\, (-m)\, (a_{14} - a_{16}) - a_{25} - a_{22}] / b_1$$

$$d_3 = [\, \exp\, (m)\, (a_{13} + a_{15} + (a_{14} + a_{16})\, b_6) - e(a_9 + a_{10})/M^2 - \\ - a_{21} + a_{24} + (a_{25} - a_{22})\, b_5] / b_2$$

$$d_4 = [\, \exp\, (m)\, (a_{14} + a_{16}) + (a_{25} - a_{22})\,] / b_2$$

$$d_8 = (e\, a_{10}\, \exp\, (\phi/2) / M^2\, \text{Sinh}\, m + a_{20} + a_{23})$$

$$d_5 = [\, \exp\, (-m)\, (a_{11} + a_{12} + a_{17} + a_{18}) - a_{19} + d_8)] / b_3$$

$$d_6 = [\exp(m)(a_{11} - a_{12} - a_{17} + a_{18}) - a_{19} - d_8] / b_4$$

$$d_7 = [\mathrm{Cosh}(m)(a_{12} - a_{17}) + \mathrm{Sinh}\, m\,(a_{11} + a_{18}) - d_8] / b_7$$

$$\phi_1(y) = (d_1 - d_2 y)\exp(b_8 y) + (d_3 + d_4 y)\exp(-b_9 y) +$$
$$+ d_5 \exp(2b_8 y) - d_6 \exp(-2b_9 y) + d_7 \exp(-sy)$$

$$B_3 = e_5 / a_{47}$$

$$B_4 = (\phi_1'(0) - \phi_1'(1) - B_3(a_1 + s/2)) / (a_2 - m)$$

$$B_1 = -\phi(0) - B_3$$

$$B_2 = -a_1 B_3 - a_2 B_4 - \phi_1'(1)$$

$$e_5 = (\phi_1'(1) - \phi_1'(0))(a_4 - m) - (a_2 - m)(\phi_1(1) - \phi(0) - \phi'(0))$$

$$d_{11} = (a_{29}\exp(-m) - (a_{30} + a_{38} + a_{36}))/ b_{13}$$

$$d_{12} = (a_{29}\exp(m) - (a_{30} + a_{38} - a_{36}))/ b_{14}$$

$$d_{13} = (a_{29}\exp(-m) - (a_{30} - a_{39} - a_{37}))/ b_{15}$$

$$d_{14} = (a_{29}\exp(m) - (a_{30} - a_{39} + a_{37}))/ b_{16}$$

$$d_{15} = a_{31} + a_{33}$$

$$d_{16} = (a_{31} - a_{32}) P s - a_{34}/P^3 s^3$$

$$d_{17} = a_{34} / 2\, Ps$$

$$d_{18} = a_{34} / (Ps)^2$$

$$d_{19} = a_{35}$$

$$\phi_2(y) = P\,[d_{11}\exp((b_8 - P\phi)y) - d_{12}\exp(-(b_9 + P\phi)y) -$$
$$- d_{13}\exp(b_8 y) + (d_{17}y^2 + d_{18}y - d_{16})\exp(-Ps)y +$$
$$+ d_{14}\exp(-b_9 y) - d_{15} - a_{35}\,y]$$

$$D_2 = (\phi_2(0) - \phi_2(1)) \,/\, (\exp(-P\phi) - 1)$$
$$D_1 = - D_2 - \phi_2(0)$$

The perturbed velocity components u_1, v_1 and temperature θ_1 are given by

$$u_1 = - [\,\psi_r' \,\mathrm{Cos}\,(\lambda x) - \psi_i' \,\mathrm{Sin}\,(\lambda x)\,]$$
$$v_1 = - [\psi_r \,\mathrm{Sin}\,(\lambda x) + \psi_i \,\mathrm{Cos}\,(\lambda x)\,]$$

and $\theta_1 = t_r \,\mathrm{Cos}\,(\lambda x) - t_i \,\mathrm{Sin}\,(\lambda x)$

where

$$\psi_r + i\,\psi_i = \psi \quad ; \quad \psi_r' + i\,\psi_i' = \psi'$$
$$t_r + i\,t_i = t \quad ; \quad t_r' + i\,t_i' = t'$$

3. SKIN FRICTION AND HEAT TRANSFER CO-EFFICIENT (NUSSELT NUMBER) AT THE WALLS.

The shear stress τ_{xy} at any point in the fluid is given by $\tau'_{xy} = \mu(\frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'})$ which in the dimensionless form reduces to $\tau_{xy} = h^2\,\tau'_{xy}/\rho\nu^2 = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$. At the wavy wall

$(y = 1 + \epsilon \cos(\lambda x))$ and at the flat wall $(y = 0)$, τ_{xy} becomes

$$\left.\begin{aligned} \tau_w &= u_o'(1) + \epsilon\,[\,(u_o''(1) - \psi_o''(1)\cos(\lambda x) + \lambda \psi_1'' \sin(\lambda x)\,] \\ \tau_o &= u_o'(0) + \epsilon\,[\lambda \psi_1''(0) \sin(\lambda x) - \psi_o''(0) \cos(\lambda x)\,] \end{aligned}\right\} (3.1)$$

The heat transfer co-efficient N' is defined as

$$N' = -k \frac{\partial T}{\partial y}$$

which in the non-dimensional form reduces to

$$N = -\frac{dN'}{k(T_1 - T_o)} = \frac{\partial \theta}{\partial y}$$

At the wavy wall $(y = 1 + \epsilon \cos(\lambda x))$ and the flat wall $(y = 0)$, (3.1) takes the form

$$N_w = \theta_o'(1) + \epsilon\,[\,(\theta_o''(1) + t_o'(1) \cos(\lambda x) - \lambda t_1'(1) \sin(\lambda x)\,]$$

$$N_o = \theta_o'(0) + \epsilon\,[\,t_o'(0) \cos(\lambda x) - \lambda t_1'(0) \sin(\lambda x)\,]$$

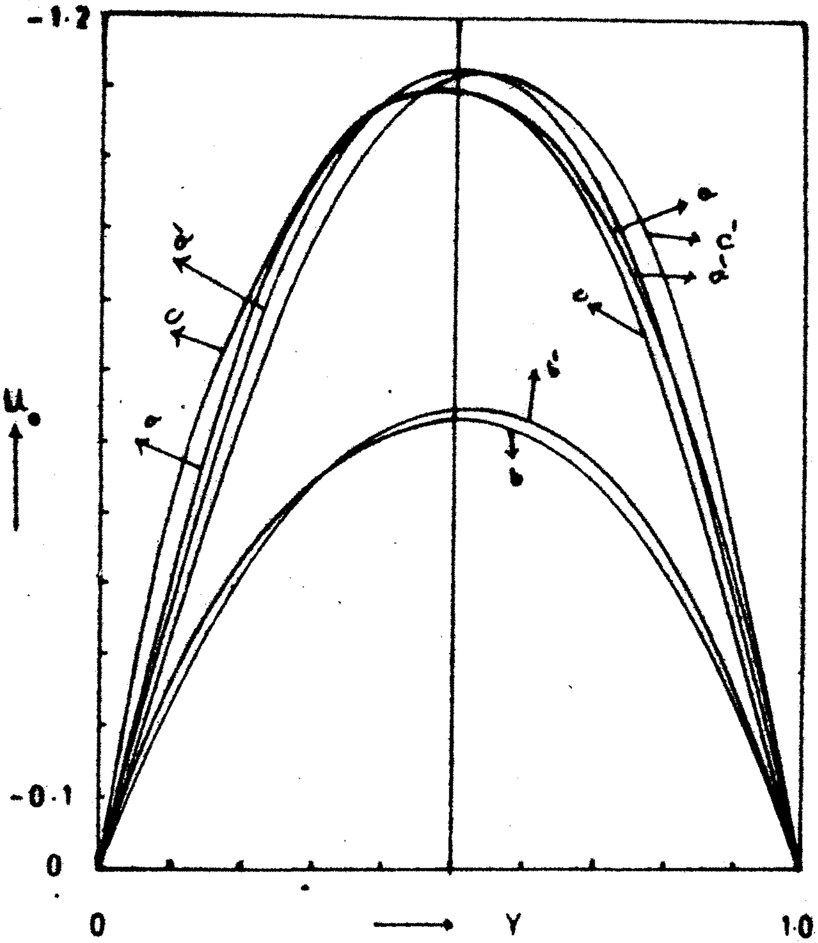

Fig. 1

Profiles for the zeroth order velocity (u_0)

	a	a'	b	b'	c	c'
M	1	1	3	3	1	1
β	0.2	-0.2	0.2	-0.2	0.4	-0.4

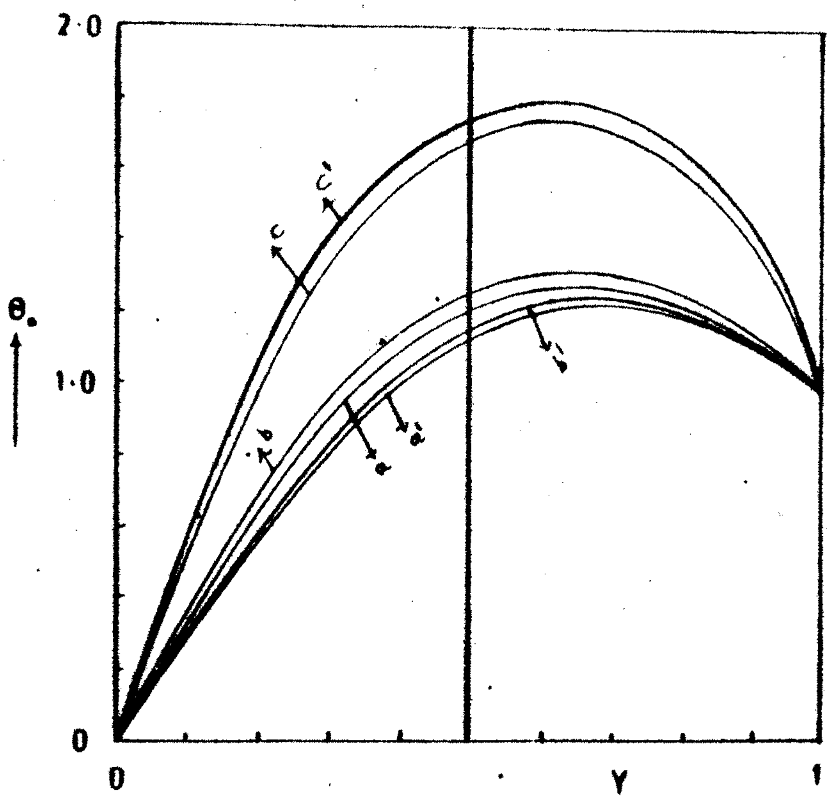

Fig. 2

Profiles for the zeroth order Temperature (θ_0)

with P = 0.71

	a	a'	b	b'	c	c'
M	1	1	1	1	1	1
β	0.2	-0.2	0.4	-0.4	0.2	-0.2
α	5	5	5	5	10	10

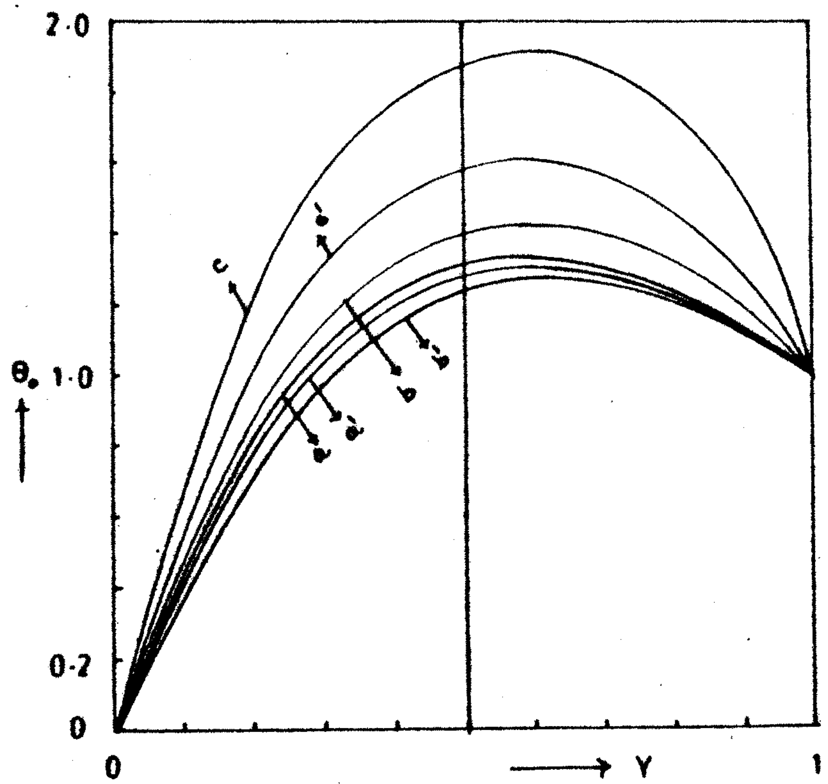

Fig. 3

Profiles for θ_o with P = 7

Curves as in Fig. 2