CHAPTER VI

EFFECTIVE MASS OF FREE CARRIERS: A NEW APPROACH
Chapter VI provides a means of studying effective mass of charge carriers in semiconductors from magneto optic dispersion measurements using new dispersions relation. The method is verified in case of CdS, InSb and GaAs.
6.1 INTRODUCTION:

Free carrier magneto optical experiments have usually been performed in the infrared, where \( \omega T = 1 \), and the results are fairly insensitive to the type of mechanism involved. In general the experiments are often performed on material in which the carrier distribution is degenerate and the resulting simplification of the transport integrals allows to describe the experiments with simple formulae derived from classical mechanics and Maxwell's equations [1]. However, in the case of semiconductors one deals with "effective mass" which may be small and anisotropic.

In the words of Adler & A.C. Smith [2] the definition for effective mass can be explained as follows:

"The electrons in any solid are acted upon not only by any applied electric or magnetic forces in question, but also by electrical forces from various atomic cores and other electrons in the periodic Crystal array. As indicated earlier, the size scale for computing the atomic and electronic forces is so small that quantum mechanics is essential for the determination of the motion produced by them. Accordingly, the additional accelerations computed by Newton's laws, based only on classical principles.
An applications of quantum mechanical principles is but necessary for visualization of electron motion due to the added forces.

The potential of the applied field forces must not vary much over a few interatomic distances of the Crystal structure. Under these circumstances it does not turn out to be possible to represent the effect of all the periodic Crystal forces in a parameter known as the 'effective mass', denoted by the symbol \( m^* \).

A classical picture of holes and conduction electrons is possible only because the major quantum features of the electronic motions in the solid can be buried in \( m^* \) 'effective mass' parameter \( m^* \).

The values for the effective masses of conduction electrons and holes in various materialise may lie in a range from less than \( 1/100 \) to greater than \( 1 \) times the normal mass of an isolated electron. The values larger than the normal mass may seem easy to imagine in terms of electrons which are relatively tightly bound. But conduction electron effective masses (less than the normal mass), and any particular values of the effective masses of holes, cannot be visualized in classical terms at all.
The fact, that small effective masses are very common, indicates the essentially quantum nature of the problem. As a matter of fact in many physics problems a careful treatment would require use of different values of effective mass in the three space directions, so the effective mass becomes not just a single number, but a matrix or tensor. For the cases of semiconductor devices to be considered it will be sufficient to take a single number for the effective mass of each species of carrier in each semiconductor material.

Evidently, the safest view is that the effective mass gives a quantum mechanical measure of the case with which an external force can accelerate the electrons and holes in the solid through the periodic hurdles supplied by the internal atomic potentials. In short the effective mass is therefore appropriate only to motions of the carriers with respect to the Crystal lattice.

At this point the consequences of the model of a pure semiconductor can be summarized as follows:

(1) In semiconductor such as germanium and silicon electrical conduction take place at ordinary temperatures via to distinct and independent quantum mechanical modes of
electron motion, which can most simply be described instead
as classical conduction by 'conduction' electrons with
charge \(-q\) and effective mass \(m_e\), and by holes with charge
\(+q\) and effective mass \(m_h\).

(2) In very pure germanium and silicon, conduction
electrons and holes always occur in equal numbers, since the
process of 'freeing' a bound electron must perforce leave a
vacancy (hole) behind. Thus two current carriers are made
available every time an electron is set "free", and the
material there by exhibits intrinsic conductivity."
6.2 MAGNETO-OPTICAL EFFECTS:

6.2.1 EFFECTS INVOLVED A CHANGE IN POLARIZATION STATE OF THE INCIDENT LIGHT:

Generally elliptically polarized light is encountered in the experiments that measure the rotation of the plane of polarization of the light. An elliptically polarized wave may be decomposed into two waves linearly polarized at right angles to each other. By specifying the relative amplitudes and phases of the two waves, these two waves can be given by

\[ E = a \cos (T + B) \]

and

\[ E = a \cos (T + B) \]

Now it is convenient to choose the parameters

\[ \chi = \tan \frac{a}{a} \]

\[ \beta = B - B \]

from this it is easy to show that

\[ \tan 2 \psi = \tan 2 \chi \cdot \cos \beta \]

and

\[ \sin 2 \phi = \sin 2 \chi \cdot \sin \beta \]
The angle $\theta$ measures the rotation of the major axis of the ellipse from axis 1, and $\tan \theta = b/a$ is the ratio of the minor to major axis of the ellipse (Fig. 6.1). This ratio is generally called as "ellipticity".

For $\sin \theta > 0$, the polarization is right handed.

For every polarization state of a light wave there corresponds one orthogonal polarization state. These two states can then be spanned over the two dimensional polarization space, so that any polarization state can be formed as a linear combination of the two orthogonal states. A device called an optical element may be characterized as this which in general, may transmit (or reflect) the orthogonal states differently and introduce a phase-shift between these two orthogonal states. Such an element is known as a dichroic retarder. The linear dichroic retarder introduces a phase shift $\Delta = \Delta_2 - \Delta_1$ between waves polarized linearly along axis 1 and axis 2, while transmitting them in a ratio

$$\frac{r_2}{r_1} = r$$
An incident light beam polarized at 45 between axis 1 and 2 (Fig. 6.1) will emerge from dichroic retarder with a polarization specified as

\[ \tan 2 \theta = \frac{2r}{1 - r} \cos \beta \]

\[ \sin 2 \theta = \frac{2r}{1 + r} \sin \beta \]

Ellipticity and the angle of inclination of major axis depend upon both the relative transmission and the relative phase shift of the two modes.

The free carriers in a magnetic field, traverse to the direction of propagation (VOIGT configuration), produce linear dichroic retardation and for propagation parallel to the magnetic field (FARADAY configuration) the carriers produce circular dichroic retardation.
Fig. 6.1 The parameters of elliptically polarized light. The arrowhead on the ellipse indicates the direction of circulation for a right-hand polarized wave propagating out of the plane of the paper.
The equations for circular dichroic retardation is given by

\[ \tan 2\theta = \tan \beta \]

\[ \sin 2\theta = \frac{2}{2} \frac{r - 1}{r + 1} \]

\[ \tan \beta = \left( \frac{r - r}{r + r} \right)^2 = \varepsilon^2 \]

where \( r^+ \) and \( r^- \) refer to the transmitted components of right, left circularly polarized components and \( r = (r^+/r^-) \).
6.2.2 FARADAY ROTATION:

When an isotropic dielectric placed in a magnetic field and a beam of linearly polarized light is sent through the dielectric in the direction of the field, a rotation of the plane of polarization of the emergent light is found to occur. In other words, the presence of the field causes the dielectric to become optically active. This phenomenon is known as "FARADAY EFFECT". The amount of rotation $\Theta$ of the plane of polarization of the light is proportion to the magnetic induction $B$ and $l$, the length of travel in the medium.

Thus the equation can be written as

$$\Theta = VBl$$

where $V$ is the constant of proportionality, and is known as "VERDET constant".

THEORY:

For the explanation of FARADAY effect, one can consider the equation of motion of the bounded electrons in presence of the static magnetic field $B$ and the oscillating electric field $E$ of the light wave.
The differential equation of motion can be written as

\[ \frac{d}{dt} \left( m \frac{\mathbf{r}}{2} \right) + k \mathbf{r} = -eE - e \left( \frac{\mathbf{dr}}{dt} \right) \mathbf{B} \]

\[ \ldots \quad (6.1) \]

where \( \mathbf{r} \) is the displacement of the electron from its equilibrium position and \( k \) is the elastic force constant. Here one can neglect the force due to the magnetic field of the optical wave as well as the damping effect, because these are not going to affect the understanding of the basic theory of FARADAY effect.

The optical field \( E \) has the harmonic dependence \( e^{-i\omega t} \).

Hence one can write

\[ -m \omega \mathbf{r} + kr = -\mathbf{E} + i\omega e \mathbf{r} \times \mathbf{B} \]

\[ \ldots \quad (6.2) \]

Multiplying equation (6.2) through out by \( Ne \) and by considering the polarization \( P \) of the medium to be equal to \( -Ne\mathbf{r} \), the equation changes to

\[ (-m \omega + K) P = -Ne \mathbf{E} + i\omega e \mathbf{P} \times \mathbf{B} \]

\[ \ldots \quad (6.3) \]
A solution of equation (6.3) will give an expression for \( P \) as

\[
\vec{P} = \epsilon \vec{X} \vec{E}
\]

where \( \vec{X} \) is the effective susceptibility tensor given by

\[
\vec{X} = \begin{bmatrix}
X_{11} + i X_{12} & 0 \\
-i X_{12} & X_{11} \\
0 & 0 & X_{33}
\end{bmatrix}
\]

where

\[
X_{11} = \frac{N_{e}}{m \epsilon_{0}}
\]

\[
X_{33} = \frac{N_{e}}{m \epsilon_{0}}
\]

\[
X_{12} = \frac{N_{e}}{m \epsilon_{0}}
\]

[Further expressions for \( X_{11}, X_{33}, X_{12} \) are given.]
In the derivation of above result it has been assumed that the magnetic field \( B \) is in the \( Z \) direction.

The above derivation for '\( P \)' gives scope to define two types of frequencies, '\( \omega_0 \)' and '\( \omega_c \)' as

\[
\begin{align*}
\omega_0 & = \sqrt{\frac{k}{m}} \quad \text{(resonance frequency)} \\
\omega_c & = \frac{eB}{mc} \quad \text{(cyclotron frequency)}
\end{align*}
\]

A substitution of value of \( \chi_{12} \) in equation for specific rotatory power \( \delta \), yields.

\[
\delta \approx \frac{\pi \text{Ne}}{\lambda m \epsilon_0} \left[ \begin{array}{c}
\omega \\
\omega - \omega_0
\end{array} \right] \left( \begin{array}{c}
\omega_0 \\
\omega - \omega_0
\end{array} \right)
\]

\[
\delta \approx \frac{\pi \text{Ne}}{\lambda m \epsilon_0} \left[ \begin{array}{c}
\omega B \\
\omega - \omega_0
\end{array} \right] \left( \begin{array}{c}
\omega B \\
\omega - \omega_0
\end{array} \right)
\]

..... (6.4)

in which an assumption that

\[
\omega \omega \ll \frac{\omega^2 - \omega_0^2}{\omega_0}
\]

is made.
6.3 MAGNETO OPTIC EXPERIMENTS AND EFFECTIVE MASS:

6.3.1 FARADAY EFFECT:

For the longitudinal configuration, the angle of rotation of the plane of polarization will be given by half the phase shift \( \beta \) introduced by the sample between the right and left circular polarization components of the incident light.

For a transmission through a sample of thickness \( d \), the rotation \( \Theta \) is given by

\[
\Theta = \frac{\omega d}{2c} (n^+ - n^-)
\]

Approximating \( (n^+ - n^-) \approx 2n (n^+ - n^-) \) and \( n \gg k \), and \( \omega \gg \omega_c \), one has an expression for \( \Theta \) as

\[
\Theta = \frac{-k \omega_p \omega_c d}{2n c (\omega - \omega_c)} \quad \ldots (6.5)
\]
But for \( w \gg w_c, \nu \), \( \Theta \) can be written as

\[
\Theta = \frac{3 - N c B \lambda d}{2 \beta c n F m^*} \quad \ldots \quad (6.6)
\]

From this it is clear that the rotation varies as the square of the incident wave length, linearly with magnetic field and it is proportional to the inverse square of the effective mass. Its sign depends on the sign of the charge carrier. The above equation can have a sign reversal when \( w \approx w_c \).

It was MICHELL [3] to suggest first that the FARADAY effect could be used to determine effective masses in semiconductors and the first measurements were made by BROWN [4]. The calculations by STEPHEN & LIDIARD [5], the showed that, just as in the case of free carrier susceptibility, the effective mass \( (m_f) \), can be interpreted by equation

\[
\frac{1}{m_f} = \left( \frac{1}{2} \right) \left( \begin{array}{c}
\frac{dE}{dk}
\end{array} \right)_F
\]

SMITH et.al [6,7] used this concept to determine the non-parabolicity of the InSb conduction band, by measuring \( m^* \) as a function of carrier concentration. Hence it is possible to satisfy the condition \( wT > 1 \) (rather than the harder condition \( w > 1 \)) with material of relatively low mobility, with the result that this method has been perhaps more used than most others for investigating new compounds and alloys.

Measurements of FARADAY rotation on Ga In As (DEMARS & WOOLEY 1973) [8] is one illustration of study of effective mass. The subject was has been completely reviewed by PILLER (1972) [9]. In the case of cubic Crystals with energy ellipsoids such as Ge (WALTON & MOSS 1961) [10] the FARADAY effect gives on average mass depending on the location and number of ellipsoids. SMITH et.al. (1959, 1962) [6,7], PALIK & WALLIS (1963) [11], KOLODZIEJCZAK et.al (1966) [12], VOROBLEV et.al. (1967) [13] have studied the effect due to hot electrons.
This effect was studied in the case of InP by AUSTIN (1960) [14], KESAMANLY et al. (1964) [15], MOSS & WALTON (1959) [16], for GaAs: CARDONA (1961) [17], UKHANOR & MALTSEV (1963) [18], DE MEIS & PAUL (1965) [19], PILLER (1966) [20], ALFAND & BAIRD (1968) [21] for GaP & AlSb: MOSS et al. (1962) [22], MOSS & ELLIS (1964) [23], for InAs: AUSTIN (1960), CARDONA (1961), PALIK & WALLIS (1961) [24], UKHANOV & MALTSEV (1963), SHULMAN & UKHANOV (1965) [25], SUMMERS & SMITH (1967) [26].
6.3.2 VOIGT EFFECT:

In this case for a transmission experiment transverse to the magnetic field through a sample of thickness \( d \), the phase shift between the waves with \( E_\parallel B \) and \( E_\perp B \) is given by

\[
B = \frac{\omega d}{c} (n_\perp - n_\parallel) \quad \cdots \quad (6.7)
\]

When one operates in the region where the amplitude of transmission is equal for the two polarization (\( r = 1 \)) there will be no rotation of the plane of polarization. This can occur when a linearly polarized is incident at an angle of \( 45^\circ \) to the external magnetic field. In this case an ellipticity of \( 2 \delta(\xi B) \) is introduced, which can be represented as

\[
2 \delta = \frac{-k d}{2 \pi c \omega} \frac{\omega_p^2 \omega_c^2}{\omega^2 - \omega_p^2 - \omega_c^2} \quad \cdots \quad (6.8)
\]
for \( w \gg w_p, w_c, \gamma \)

\[
2 \delta = \frac{4 \cdot 2 \cdot 3}{-N_e B \lambda d} \frac{3 \cdot 4}{16 \pi c n \epsilon_0 \mu_0^3} 
\]

\[2 \delta \] \( \text{Eq. (6.9)} \)

From the above equations it is clear that the ellipticity will change sign when

\[
\omega < \omega_p + \omega_c
\]

Because of the different dependence of VOIGT effect on magnetic field, mass and wave length, one may measure both FARADAY rotation and VOIGT effect on the sample to determine \( 2 \delta / \theta \) which is equal to

\[
\frac{2 \delta}{\theta} = \frac{\omega_c}{\omega}
\]
One can also determine the phase shift exhibited by free carriers in ellipsoidal valleys in VOIGT effect as

\[ \Delta \phi = \frac{q N A H I}{4 \pi c n (m^*)} \]  

where \( m^* \) is a VOIGT effective mass, which is a complicated \( v \) average of the effective masses in the ellipsoidal valley. The value of \( m^* \) exhibits a strong crystallographic dependence.
6.4 NEW RELATION:

From the equations of T.S. MOSS [27] the value of the FARADAY rotation $\theta$ by free carriers is given by

$$
\theta = \frac{q^2 N \lambda H_1}{4 \pi c n (m^*)^2}
$$

where $N$ is the free-carrier concentration, $\lambda$ is the wavelength of the radiation, $n$ is the index of refraction in the absence of magnetic field, and $m^*$ is the free-carrier effective mass. Here although uniform effective mass is assumed, corrections can be made for the ellipsoidal shape of the valleys and for the presence of two types of carriers.

To take care of both electrons and holes, $1/m^*$ has the effective value $1/m^* = 1/m_e + 1/m_h$, where $m_e$ and $m_h$ are the electron and hole effective masses respectively, but $m^*$ is mostly determined by $m_e$ since $m_h \approx 10 m_e$ [28].

The quadratic dependence of $\theta$ on $\lambda$ suggests that the free-carrier FARADAY rotation should be measured at long
wave lengths. However, angular frequency at which the electrons are driven must be large compared to the cyclotron frequency

\[ \omega = \frac{q H}{m c} \]

and further more the corresponding period must be short compared to the carrier-relaxation time.

If the carrier concentration \( N \) is known by other measurements like HALL effect, the effective mass can be calculated as follows:

From New dispersion formula the equation for can be represented as follows:

From equation of MORD one can have

\[ \delta = \frac{k \times m}{\rho \times \rho \times \lambda^2} \left[ \frac{2}{f (2 + f)} \frac{2}{(1 + f)} \right] \ldots \ldots \text{(6.12)} \]

where \( k = \frac{T \tau}{2 e N} \)

\( e = \) electronic charge

\( N = \) Avagadro's number

and \( f = \frac{\beta \lambda^2}{1 - \lambda^2} \)

where \( \beta = \frac{3V}{4 \pi N} \)
Now from MOSS RELATION one can have

\[ \Theta = \frac{3^2}{q N \lambda H 1} / 2 \pi c n (m^*) \]

or

\[ \Theta / H 1 = \frac{3^2}{q N \lambda} / 2 \pi c n (m^*) \]

\[ \delta = \frac{3^2}{q N \lambda} / 2 \pi c n (m^*) \]

\[ (m^*)^2 = \frac{3^2}{q N \lambda} / 2 \pi c n (\delta) \]

Now the value of calculated by MORD relation is substituted in the above equation to calculate the effective-mass.

\[ m^* = \left[ \frac{3}{q N \lambda} \right]^{1/2} \]

This method of evaluation of effective mass of the free carriers from an estimate of magneto optic rotation (from measurements in refractivity) has been applied to study the effective masses of CdS, InSb & GaAs. The details of the application of the method and the description of the results there on are presented in chapter VII.
REFERENCES:


