INTRODUCTION
Basics of vibration and vibration of plates are provided in this section. An informative literature survey is mentioned. Aim and objective of the thesis with thesis outline is also given in this chapter.
INTRODUCTION

1. Basics of Vibration

Any motion that repeats itself after a regular interval of time is called vibration or oscillation. In other words, vibration is mechanical oscillation of a particle, a group of particles or a body from its position of equilibrium. Most human activities involve vibration in one form or other. For example, we hear because our ear drums vibrate. Breathing is associated with the vibration of lungs and walking involves (periodic) oscillatory motion of legs and hands. The whole of our body shivers when we feel cold. We cannot speak and express ourselves without the use of vibration. Not only within us, the universe around us also governed by periodic motion. Example- Day and night cycle on the earth.

Vibration occurs in every rotating and reciprocating machinery as a consequence of forces which are generated in such machines. These may be due to the following reasons such as electromagnetic forces, misalignment, faulty bearings or gears, eddy currents, rotor imbalance etc. These forces act through the bearings to move the stationary parts of the machine. The design of string instruments, such as guitars, is based on the phenomenon strings vibrating at a certain frequency. The guitar has all of the components of a vibration system including a vibration damper, and a place to store potential and kinetic energy.

In fact, many significant failures of machines and structures in the past are attributed to serve vibrations to which they are subjected. It is always possible to anticipate vibration problem at the design stage itself. The subject is thus important not only to mechanical and civil engineers but also to aeronautical engineers and practitioners who are always interested to know about the vibrational behaviour of structure or machine before finalize their designs.

Vibrations can be found in our daily life in so many ways i.e. music, cars, trains, planes, mobiles, electric motors, engines etc. all are directly related with vibrations. Sometimes, vibrations affect our daily life indirectly. For example, noise pollution has become intolerable problem for the society in these days. Everyone wants to control noise or sound. But it is
Interesting to know when we control the sound (frequency of sound) then actually, we control vibrations.

Actually, human ear is a very sensitive and highly accurate receiver of vibrations and ear drum (a part of human ear) receives the sound in the form of energy from the subject and then it converts the energy into sound waves and make it audible (transmit messages to brain).

Vibrations can be classified into two main categories:

- Controlled Vibrations
- Uncontrolled Vibrations.

Controlled Vibrations are those vibrations which lie in a finite region or one can say that those vibrations in which frequencies can be controlled by using scientific technique. Vibration in electric motors, vehicles, machines, musical instruments and various mechanical structures are the examples of controlled vibrations. Controlled vibration effects are necessary in health industry, paper industry, designing of structures, building construction and many more areas. On the other hand, uncontrolled vibrations lie in an infinite region or those vibrations which cannot be controlled by scientific devices and techniques. Vibrations generate during earthquake, nuclear plants shut down due to failure of turbines and volcanoes etc. are uncontrolled vibrations. Suddenly failure of machines or plants due to uncontrolled vibrations may be occurred sometimes and also affects economy and time management system of a country.

Recently, Northern Electric Power Grid failed to work suddenly in India during 30-31st July, 2012 and nearly 18-20 states faced a lot of troubles. Due to that, more than 300 million people (about 25% of the population of India) were suffered with no-electricity, shut downs of trains (including Delhi Metro System), airports and water delivery systems during this tenure of nearly 18-20 hours. Approximately 32 crore rupees were spent to restart the grid and to get normal supply of electricity. Actually, failure of power plants directly affects the economy, safety and time management of the nation.
The main reason of the failure of grid was over loading of electricity i.e. extra or bulk consumptions of electricity by states. Actually, each state has its own generating station with a limited electric power which is directly connected with the grid. Generating stations of states are also interconnected with one another so that they can transfer (use) the power of one another in case of emergency. If a state requires more electricity than its limit, it can be taken from the other generating station which directly affects the balance of electricity in other states. Due to overloading of electricity, voltage downs slowly. To maintain the voltage, feedback control system of every generating station tries to increase the speed of turbine. Now what happens: overloading of electricity slows the speed of the turbine and feedback control system continuously tries to increase the speed of turbine, here this disturbance produce vibrations which cannot be controlled after a saturated stage. In other words, turbines are stopped to rotate (work) and generating stations becomes unable to produce electricity and hence shut down occurs. In this condition, we said that grid is fail.

There are different classes of vibration: free (undisturbed) and forced (disturbed), damped and un-damped (has negligible or no damping), nonlinear and linear and random (unpredictable) and deterministic etc. Free vibrations are those in which energy is neither added nor removed from the vibrating system. It will just keep vibrating forever at the same magnitude. Simple pendulum is an example of free vibrations. Clockwork mechanism is example of forced vibrations. In a clockwork mechanism, energy stored in a spring transferred a bit at a time to the vibrating element. Other examples of forced vibrations are machine tools, electric bells etc.

Linear and non-linear vibrations are depending on the behaviour of the system. If in a vibratory system; mass, spring and dashpot behave in linear manner thus obtained vibrations are known as linear in nature. On the other hand, if any one of vibratory system; mass, spring and dashpot behave in non-linear manner, the vibrations caused are known as non-linear in nature.

Damped vibration is one in which there is an energy loss from the vibrating system. This loss may be in form of mechanical friction or in form of turbulence as the vibrating system
disturbs its surroundings. The amplitude of a damped vibration will eventually decay to zero. Actually, there are no free vibrations in nature. They are all damped to some extent. While undamped vibration suffers no energy loss. Lightly damped vibrations have slight energy loss which may or may not be negligible because energy loss depends on the nature of the observation of the vibrator.

2. **Vibration of Plates**

Structural components are generally subjected to dynamic loadings in their working life. Very often these components may have to perform in severe dynamic environment wherein the maximum damage results from the resonant vibration. Susceptibility to fracture of materials due to vibration is determined from stress and frequency. Maximum amplitude of the vibration must be limited for the safety of the structure. Hence vibration analysis has become very important in designing a structure to know in advance its response and to take necessary steps to control the structural vibrations and its amplitudes.

With the upsurge of technology, researchers and scientist are always trying to control the vibrations of structures or machines to enhance their efficiency and life actively by using many theoretical, experimental and computer methods. Whenever we say about active vibration control in any machines or structures, we need active vibration control for each and every part of that structures or machines which may be a shaft, cylinder, plates etc. Therefore, area of active vibration control in structures or designs is too wide to discuss at once.

Vibration control in plates depends on various factors i.e.

a) **Shape and size of the plate**: Plates can be made up of different shapes such as Rectangular, Circular, Parallelogram, Elliptic, Triangular etc. having different sizes i.e. dimensions of their different edges.

b) **Material used for plates**: Different types of materials are generally used in making plates by the requirement of design engineers i.e. homogeneous, non-homogeneous, isotropic, orthotropic, anisotropic etc. These materials can be explained as follows:
Homogeneous materials are those materials in which elastic constants (young’s modulus, Poisson ratio, modulus of rigidity) of the material do not vary (remain constant) from point to point.

Non-homogeneous materials are simply those materials which are not homogeneous i.e. which do not show identical properties of elastic constants at all points. In other words, elastic constants (young’s modulus, Poisson ratio, modulus of rigidity) of non-homogeneous materials vary (not constant) from point to point.

Isotropic materials are those materials whose physical properties (density, strain, stress) at a point are same in all directions.

Orthotropic materials have two or three mutually orthogonal two fold axes of rotational symmetry so that its mechanical properties are in general, different along each axis. One common example of an orthotropic material with two axis of symmetry would be a polymer reinforced by parallel glass or graphite fibers.

Anisotropy is the property of being directionally dependent, as opposed to isotropy, which implies identical properties in all directions. It can be defined as a difference, when measured along different axes, in a material’s physical or mechanical properties (absorbance, refractive index, conductivity, tensile strength etc.). An example of anisotropy is the light coming through a polarizer.

c) Thickness of the plates: - Plates can also be categorized as per their thickness. If thickness of any plate remains same throughout the middle plane of the plate, it is called uniform thickness but when the thickness varies point to point of the plate, it is called non uniform thickness.

Plates having non-uniform thickness are also called tapered plates. This tapering can be of different types such as linear, parabolic, exponential etc. Plates of variable thickness are commonly used in many engineering applications like spacecrafts, submarine, nuclear reactor ships etc. So it is the need of the hour to get deep knowledge of the plate’s behaviour and their characteristics, which would in turn help to perceive their potential in many fields. Plates of
variable thickness are frequently used in order to economize the plate material or to lighten the plates, especially when it is used in the wings of high-speed and high performance aircrafts. Since a lot of factors directly affect the vibrational properties of materials or plates, they must be analyzed from engineering perspective with easy and fast computational technique for the betterment of structures.

Also, non-homogeneous plates of variable thickness are vigorously used in engineering and architectural practice i.e. in the designs of space vehicle, marine, automobile etc. Non-homogeneity along with variable thickness not only reduces the cost of the material but also provide more strength, lighten the loads, increase reliability etc. of the structures. The consideration of non-homogeneous plate’s material together with non-uniform thickness of the structural components not only ensures the reduction in weight and size but also meet the desirability of high strength in various technological situation of aerospace industry, ocean engineering and optical equipments.

In the field of mechanical engineering, new discoveries can’t be possible without considering the effect of vibration as almost all machines and engineering structures experience. In the modern technology, an interest towards the effect of high temperatures on non-homogeneous rectangular plates of variable thickness is developed due to applications in various engineering branches where metals and their alloys exhibits visco-elastic behaviour (a property of material that exhibit both elastic and viscous characteristics when undergoes deformation). The reason for these is that during heating up periods, structures are exposed to high intensity heat fluxes and material properties undergo significant changes; in particular thermal effect cannot be negligible.

In recent years, the dynamic behaviour of thin isotropic rectangular tapered plates (plates with variable thickness) has received considerable attention due to its wide technical importance in high speed atmospheric flights, nuclear energy applications, space technology etc. Tapered plates possesses a number of attractive features such as material saving, weight reduction, stiffness enhancing and also meet the desirability of economy. Therefore, the demand for this
type of structure has rapidly increased due to industrial stringency, especially in aerospace vehicles in which light weight is essential. However, this type of structure can lead to unwanted instances of high vibration. Overtime, vibrational effects can have long-term as well as short-term damaging effects on the structure. Such phenomena are potentially dangerous as they can create a complete unbalance of the structure which can then ultimately fail.

In modern technology, scientists and researchers are continuously working to develop new materials which are necessary for the betterment of machines or systems of structures. In this connection, visco-elastic* materials are vigorously used for their characteristics of less weight, high strength, high boiling point, more reliability etc.

When a structure works under the influence of elevated or high temperature field, its properties undergo significant changes. Also, non homogeneity may be occurred in material due to temperature variation, which cannot be neglected.

3. Literature Survey

Literature in form of Journals/Books/Monographs and other authentic databases is cited as follows:

Gupta et. al. [7] evaluated thermal gradient effect on vibration of non-homogeneous orthotropic rectangular plate having bi-linear thickness variations. Free vibrations of a sectorial plate had been discussed by Bhattacharya and Bhowmic [12]. Gupta and Goyal [13] worked on eigenfunction method to analyze the asymmetric response of linearly tapered circular plates subjected to transverse loads, uniformly distributed over an annular sectorial area of the plate. Gupta et. al. [14] presented model for vibration of a visco-elastic orthotropic parallelogram plate with linear thickness variation in both directions. Lopatin and Morozov [18] used generalised Galerkin method to evaluate the fundamental frequency of an orthotropic rectangular plate with a centrally located point support and free edges. Leissa [23] used Rayleigh-Ritz technique with a three-term deflection function to obtain fundamental frequencies of a simply-supported elliptical plate. Leissa and Narita [24] worked on natural frequencies of a simply supported circular plate.
Approximate method is used to calculate large amplitude free vibrations of an elliptic plate by Banerjee [25]. Singh and Chakraverty [26] investigated transverse vibration of completely-free elliptic and circular plates of uniform thickness with the help of characteristic orthogonal polynomials. Singh and Saxena [27] studied transverse vibrations of a rectangular plate of variable thickness with different combinations of boundary conditions at the four edges. Warburton [28] investigated vibrational behaviour of rectangular plates.

Filipich et. al. [29] gave analytical and numerical results about vibration analysis of rectangular plates with different boundary conditions. Ma and Huang [30] used finite element method to calculate frequencies of transverse vibration of plates. Chen and Fung [31] used Runge–Kutta method to obtain the non-linear frequencies of initially stressed hybrid composite plates. Hwu et. al. [32] investigated smart composite sandwich beams with surface bonded piezoelectric sensors and actuators. Huang and Aurora [34] presented computational analysis of the non-linear oscillations of elastic orthotropic annular plates of variable thickness. Huang and Ho [35] used the Frobenius method for analytical solution for vibrations of a polarly orthotropic Mindlin sectorial plate with simply supported radial edges.

Alijani and Amabili [45] investigated nonlinear forced vibrations of moderately thick functionally graded (FG) rectangular plates by considering higher-order shear deformation theories. An appreciable work is done by Alijani and Amabili [46] to observe nonlinear vibrations of imperfect rectangular plates with free edges. Ginesu et. al. [48] evaluated natural frequencies of polar orthotropic annular discs with use of finite element approach. Hearmonr [49] discussed the frequency of isotropic rectangular plates. Fancnneau and Marangoni [51] solved the problem of thermal effect on natural frequencies of a rectangular plate. Conway et. al. [52] found resonant axisymmetrical frequencies of clamped tapered bars and circular plates. Authors also calculated transverse vibrational resonant frequencies for truncated-cone cantilever beams for a number of geometries. Conway et. al. used different boundary conditions such as clamped/free, clamped/simply supported, and clamped/clamped. Golden et. al. [53] gave an approach to characterize non linear visco elastic material behaviour using dynamic analysis. Woo et. al. [54] worked on vibration of orthotropic circular plates with a concentric isotropic core. Lurie [55] discussed vibrations of rectangular plates. Wang and Chen [56] observed axisymmetric vibration and damping of rotating annular plates with constrained damping layer treatments.

Alshaikh et. al. [57] observed propagation of two-dimensional transient waves in multilayered viscoelastic media. Elishakoff [58] presented an approximation method for determination of natural frequencies and mode shapes of square orthotropic plate with all sides clamped. Ebirim [59] studied free vibration of simply supported isotropic rectangular plate with one free edge and derived Ibearugbulem’s shape function. Kang and Shim [60] provided exact solutions procedure for the free vibration analysis of rectangular plates having two opposite edges simply supported when these edges are subjected to linearly varying normal stresses causing pure in-plane moments. Fletcherh [61] obtained frequency of vibration of rectangular isotropic plates. Du et. al. [62] investigated vibrations of two plates which are generally supported along the boundary edges and elastically coupled together at an arbitrary angle. The interactions of all four wave groups (bending waves, out-of-plane shearing waves, in-plane
longitudinal waves, and in-plane shearing waves) have been taken into account at the junction via four types of coupling springs of arbitrary stiffnesses. Heo and Chung [63] examined the dynamic characteristics and responses of a flexible rotating disk when the disk has angular misalignment that is defined by the angle between the rotation and symmetry axes. Seok and Tiersten [64], Seok et al. [65] analyzed the free transverse vibrations of a cantilevered plate by means of a variational approximation procedure. Kuttler and Sigillito [66] gave upper and lower bounds are given for the three lowest frequencies of vibration of clamped rectangular orthotropic plates. Hewitt and Mazumdar [67] discussed the vibration of visco-elastic plates by constant deflection lines method.

Raju and Rao [69] used finite element method to evaluate non-linear vibrations of orthotropic plates. Khorshid and Farhadi [70] used Rayleigh–Ritz method to investigate hydrostatic vibration analysis of a laminated composite rectangular plate partially contacting with a bounded fluid. Dai et al. [72] approached a mesh-free method to analyze the static deflection and natural frequencies of thin and thick laminated composite plates using high order shear deformation theory.

Jeong et al. [73] presented a theoretical study to examine coupled natural frequencies of hydroelastic vibration of two identical rectangular plates coupled with a bounded fluid. Rao and rao [74] presented an analytical study for the vibration characteristics of thin circular plates with rotational elastic edge support and resting on an elastic foundation. Dokainish and Kumar [76] solved the problem of vibration of orthotropic parallelogramic plate with variable thickness. Amabili and Garziera [77] discussed transverse vibrations of circular, annular plates with several combinations of boundary conditions. Biancolini et al. [78] calculated fundamental frequency for free vibrations of thin orthotropic rectangular plate. Huang and Sakiyama [80] provided an approximate method for analyzing the free vibration of rectangular plates with a hole of different shapes. The shapes of the holes are circular, semi-circular, elliptic, square, rectangular, triangular, rhombic, etc. Maurizi and Laura [82] determined frequencies of vibration of clamped rectangular plates of uniform thickness by using a simple polynomial approximation. Banerjee
analyzed large amplitude vibrations of non-uniform rectangular plates. Non-linear vibration of circular plate of variable thickness discussed by Banerjee [84]. Malik and Bert [85] investigated three-dimensional elasticity solutions for free vibrations of six types of plates having free lateral surfaces, two opposite sides simply supported, and two other sides having combinations of simply supported, clamped and free boundary conditions. Dhotarad and Ganesan [86] observed dynamic free response of thin rectangular plates subjected to one and two dimensional steady state temperature distributions satisfying Laplace's equation.

Nagaraj and Rao [87] observed vibrations of rectangular plates. Ganesan [88] presented vibration analysis of the family of rectangular plates with two opposite sides simply supported. Ganesan and Jagadeesan [89] used the finite difference method to analyze the vibration of rectangular plate with the opposite edges simply supported. Thomas and Bilbao [90] studied flexural vibrations of plates with different boundary conditions. Laura and Luisoni [91] used polynomial approximations and a variational approach for determination of the fundamental frequency of clamped rectangular plates of variable thickness. Gagnon et al. [92] provided generalized Fourier series solution for the free vibrations of moderately thick rectangular plates with variable thickness and arbitrary boundary conditions.

Jacquot and Lindsay [93] evaluated natural frequencies on circular plates with varying Poisson’s ratio. Gutierrez and Laura [94] observed transverse vibration of a rectangular, anisotropic plate of discontinuously varying thickness. Gutierrez and Laura [95] used differential quadrature method for transverse vibrations of a rectangular plate subjected to a non-uniform stress distribution field. Gutierrez et al. [96] investigated transverse vibration of rectangular orthotropic plate with a free edge hole. Gutierrez et al. [97] observed transverse vibration of circular, annular plates of polar orthotropy with a free edge hole. Free transverse vibrations of non-homogeneous orthotropic rectangular plates of varying thickness with two opposite simply supported edges \( y = 0 \) and \( y = b \) and resting on two-parameter foundation (Pasternak-type) is studied by Lal and Dhanpati [104]. Classsen and Thorne [105] studied first ten natural frequencies of thin isotropic clamped rectangular plate.
Sheikholeslami and Saidi [107] observed free vibration of simply supported functionally graded rectangular plates resting on two-parameter elastic foundation using the higher-order shear and normal deformable plate theory. Algazin [109] derived numerical algorithm without saturation of free vibrations of a free-edge variable-thickness plate of arbitrary shape in plan. Farhadi and Hshemi [112] used a finite element formulation to investigate active vibration suppression of moderately thick rectangular plates by means of piezoelectric actuators. Kukla [114] used quadrature method for free vibration of a system of two rectangular plates connected by a non-homogeneous elastic layer. A sine series solution of free vibration of orthotropic rectangular plates has been studied by Dickinson [115].

Irie et. al. [116] presented natural frequencies of stepped thickness square and rectangular plates together with the mode shapes of vibration. Sakata [117] evaluated fundamental natural frequencies of orthotropic rectangular plates with various boundary conditions and of a clamped orthotropic elliptical plate. Asymmetric vibration of polar orthotropic circular plates of linearly varying thickness subjected to hydrostatic in-plane force is studied by Gupta and Ansari [118]. Gupta and Ansari [119] used Ritz method to present deflection function and the bending moments for forced vibrations of orthotropic linearly tapered circular plates. Li [120] analyzed vibration of visco-elastic plates with general elastic boundary supports.

A novel method is used by Wang and Xu [121] to analyse the behaviour of free vibration of beams, annular plates, and rectangular plates with free boundaries using the discrete singular convolution (DSC). Narita [123, 124] worked on natural frequencies of free vibrations of orthotropic elliptical plates. Narita and Lwato [125] used ritz method to evaluate natural frequencies and mode shapes symmetrically laminated fibre composite circular and elliptical plates resting on elastic point supports. Kim [126] developed a theoretical method to investigate vibration characteristics of initially stressed functionally graded rectangular plates made up of metal and ceramic in thermal environment. A Kirchhoff-type theory by Stavsky and Loewy [127] is established for axisymmetric motions of heterogeneous isotropic composite circular

4. **Aim and Objective of the Thesis**

Literature survey shows that a lot of work has been done in the field of vibration of tapered plates but effect of non-homogeneity on vibration of tapered plates under bi-directional thermal effect is not investigated yet. Hence, it is still necessary to develop some mathematical models to investigate that how two-dimensional temperature variation affects the vibrational properties of non-homogeneous tapered rectangular plates. With the help of these mathematical models; practitioners, scientists or researchers who are working on such kind of plates or structures are advised to analyze the findings and suitability of the model with their requirements to provide much better and authentic structures and machines with more strength, durability and efficiency.

Therefore, main aim of the present work is:

- To develop some mathematical models on vibrational properties of a non-homogeneous tapered rectangular plate under the influence of two dimensional temperature variations.
- To study the effect of two dimensional temperature variation on the vibration of non-homogeneous rectangular plate.
- To analyze that how one dimensional thickness variation affects the vibrational properties of non-homogeneous rectangular plate.
- To investigate the behaviour of vibrational properties of rectangular plate with respect to various plate’s parameters.

In order to achieve the above objectives, it becomes necessary to investigate the following problems or mathematical models:

**P-1) Effect of bi-parabolic temperature variations on the vibration of non-homogeneous rectangular plate with linear thickness variations.**
**P-2)** Effect of exponential temperature variations on the vibration of non-homogeneous rectangular plate with exponential thickness variations.

**P-3)** Effect of bi-linear temperature variations on the vibration of non-homogeneous rectangular plate with linear thickness variations.

**P-4)** Effect of bi-linear temperature variations on the vibration of non-homogeneous rectangular plate with parabolic thickness variations.

**P-5)** Effect of bi-parabolic temperature variations on the vibration of non-homogeneous rectangular plate with parabolic thickness variations.

**P-6)** Effect of bi-directional temperature variations (linear in $x -$ direction and parabolic in $y -$ direction) on the vibration of non-homogeneous rectangular plate with linear thickness variations.

**P-7)** Effect of bi-directional temperature variations (linear in $x -$ direction and parabolic in $y -$ direction) on the vibration of non-homogeneous rectangular plate with parabolic thickness variations.

Therefore, the work presented in this thesis is about the vibrational behaviour of visco-elastic isotropic non-homogeneous rectangular plate whose thickness varies in one direction under the influence of bi-directional temperature variations. Vibrational properties of rectangular plates are investigated for Duralumin, an alloy of Aluminium. Duralumin is a combination of four different materials such as Aluminium (95%), Copper (4%), Magnesium (0.5%) and traces of manganese (0.5%). Due to an alloy of Aluminium and Copper, Duralumin becomes as strong as steel with light weight and cheap in cost.

5. **Thesis Outline**

The whole content of the thesis is divided into 6 chapters along with four sections in last i.e. Comparison, Conclusions, References and Appendix. Chapter 1 to chapter 6 are basically a collection of numerical investigation about the vibrational properties of rectangular plate as mentioned in all above stated problems i.e. P-1 to P-7. A chapter wise summary is given as follows:
Chapter 1: In this chapter, problem-1 (P-1) is discussed carefully. Effect of bi-parabolic temperature variations on vibrations of non-homogeneous visco-elastic rectangular plate with linearly varying thickness is analyzed with great accuracy. Boundary of the plate is considered as clamped and Poisson ratio of the plate’s material is assumed to vary exponential in one direction due to non-homogeneity. Rayleigh-Ritz method is used to solve the frequency equation of motion and to obtain corresponding natural frequencies in the form of explicit formulae. Frequency, deflection and logarithmic decrement for first two modes of vibration are calculated with the help of Mathematica with accuracy and tabulated in tables 1.1 to 1.8 at different values of plate’s parameters.

Chapter 2: Numerical solutions for problem-2 (P-2) are systematically presented in this chapter. Here, effect of exponential thermal gradient on vibration of non-homogeneous visco-elastic rectangular plate having exponential thickness variation is investigated with the help of Rayleigh-Ritz method. Also, it is considered that all sides of the rectangular plate are clamped and effect of non-homogeneity is characterized in Poisson ratio i.e. as exponential variation of Poisson ratio in one direction. Numeric values of frequency, deflection and logarithmic decrement for first two modes of vibration are summarized in tables 2.1 to 2.8 at various values of plate’s parameters.

Chapter 3: Vibrational behaviour of linearly tapered visco-elastic rectangular plate subject to bi-linear thermal condition is analyzed for problem-3 (P-3) in this chapter. Effect of non-homogeneity is considered in Poisson ratio which is assumed to vary exponentially in one direction. Boundary condition is considered four sided clamped. Vibrational properties are investigated for various values of plates parameters and reported in tables 3.1 to 3.8. Frequency for both the modes of vibration is given in tables 3.1 to 3.4. Deflection and logarithmic decrement for first two modes of vibration are tabulated in tables 3.5 to 3.7 and in table 3.8 respectively.

Chapter 4: As mentioned in P-4, this chapter deals with vibrational response of four sided clamped visco-elastic rectangular of plate under bi-linear temperature variations. It is
assumed that thickness of the plate varies parabolically in one direction with exponential Poisson ratio variation due to non-homogeneity present in material of the plate. For various values of plate’s parameters; frequency, deflection and logarithmic decrement for first two modes of vibration are investigated and placed in tables 4.1 to 4.8.

**Chapter 5:** Effect of bi-parabolic temperature variations along with parabolic thickness variation in one direction on vibration of visco-elastic non-homogenous rectangular plate which all sides are assumed clamped is investigated systematically and presented in chapter-5. This chapter represents the solution of problem-5 i.e. P-5. Tabular representation of results for first two modes of frequency, deflection and logarithmic decrement are provided in tables 5.1 to 5.4 (for frequency), tables 5.5 to 5.7 (for deflection) and in table 5.8 (for logarithmic decrement).

**Chapter 6:** Two mathematical models i.e. P-6 and P-7 are investigated and presented in this chapter simultaneously. This chapter shows the combined numerical investigation of P-6 and P-7 about the vibrational response of rectangular visco-elastic plate with clamped boundary. Plate’s thickness is considered non-uniform i.e. linear variation in thickness is assumed for P-6 and parabolic variation in thickness is taken for P-7. Here, two dimensional temperature variations i.e. linear in $x$–direction and parabolic in $y$–direction are considered with exponential variation in Poisson ratio. Results for frequency, deflection and logarithmic decrement for both the modes of vibration are carefully calculated for various values of plate’s parameter and tabulated in tables 6.1 to 6.8 (for P-6) and tables 7.1 to 7.8 (for P-7).

A **comparison** is an important tool in a research project to show that how the various mathematical models or phenomenon used in research are different and alike. The process i.e. comparisons helps readers to clarify concepts and makes the information memorable. To explore the importance of the present research work, a brief comparison between the findings of all the chapters 1-6 i.e. findings of P-1 to P-7 are provided in tables 1 to 8. For better understanding, graphs are also provided corresponding to all tables from 1 to 8.
Conclusions: A synthesis of key points is provided with several important further opportunities in the field of vibration of plates in this section.

References: In this section, a standardized way of acknowledging the sources of information and ideas with the record of full bibliographic details to complete the whole work of this thesis is provided.

Appendix: At last, an appendix is provided with some supplementary material that is not an essential part of the thesis itself but which may be helpful in providing a more comprehensive understanding of the research problems.