BASIC CONCEPTS
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Flows of fluids bounded by one or two planes have been extensively studied in literature, in view of the fact that they are ideal models of practical importance and interest in exploring the general features of fluids forces, like viscous force, electromagnetic force, gravitational force and coriolis force. The flow bounded by one or two vertical planes is particularly important in the area of heat transfer because such temperature stratified flows also occur in nature. The following are some of the important areas in fluid dynamics in which flows through such geometries are explored by several authors.

1.1 Newtonian and Non-Newtonian fluid dynamics
1.2 Heat Transfer
1.3 Magneto hydrodynamics
1.4 Flows through porous media
1.5 Mass Transfer
1.6 Rotating Fluid

We make a brief survey of the fundamental aspects concerning these areas of study which in turn will provide a foundation for formulating the problem of this dissertation.

1.1 Newtonian and non-Newtonian fluids

The classical Navier-Stokes equation of motion is derived by assuming a linear relationship between stress tensor and the strain rate tensor in the fluid. Fluids which obey this relationship are known as Newtonian fluids. They possess a single rheological property called viscosity. Water, air, mercury, engine oil are some of the examples of Newtonian fluids.

Many important industrial fluids are non-Newtonian in their flow characteristics. These include paints, various suspensions, glues, printing inks, food materials, soap and detergent slurries, polymer solutions and many others. Because such fluids have more complicated equations that relate the stress to the
velocity gradient than is the case with Newtonian fluids, new branches in the fields of fluid mechanics and heat transfer are developed.

Another important characteristic of such fluids, because of their large apparent viscosities, is that they have a tendency towards low Reynolds and Grashof numbers and high Prandtl numbers. Thus laminar flow situations are encountered more often in practice than with Newtonian fluids.

The fundamentals of non-Newtonian laminar flow and heat transfer include an examination of the classification system for such fluids, the development of a method to predict the fully developed pressure drop in ducts both circular and non-circular in cross-sectional shape and a consideration of some aspects of the heat transfer processes.

The subject of thermo physical properties and their measurements is an important one when dealing with non-Newtonian fluids and includes the consideration of both classical methods for making such measurements as well as several approaches which are unique to such fluids. It is unfortunate and time consuming that these property measurements must be made continuously when dealing with non-Newtonian fluids because they are not pure substances and vary in their properties because of different preparation methods. The non-Newtonian fluids can in turn be divided into purely viscous and viscoelastic fluids. The purely viscous time-independent fluids are defined as those whose shear stress depends only upon some function of the shear rate, and sometimes an initial yield stress. Viscoelastic fluids are those which possess properties of both viscosity and elasticity. We list below various non-Newtonian fluids and some examples.

<table>
<thead>
<tr>
<th>Type</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newtonian</td>
<td>Water, air, mercury, engine oil</td>
</tr>
<tr>
<td>Pseudo plastic</td>
<td>Paints, glues, blood, suspensions</td>
</tr>
<tr>
<td>Dilatants</td>
<td>Wet sand, sugar and borax solutions</td>
</tr>
<tr>
<td>Bingham Plastic</td>
<td>Certain emulsions land paints</td>
</tr>
<tr>
<td>Thixotropic</td>
<td>Printing inks, food materials, paints</td>
</tr>
</tbody>
</table>
The simplest type of non-Newtonian fluids are the pseudoplastic and dilatant fluids whose relation between shear stress and shear rate can be expressed by an equation of the form $\tau_\gamma = k (\gamma)^n$ where $\gamma$ is strain rate.

Because of this functional relation such fluids are called power law fluids and unlike Newtonian fluids (which have only a single rheological property, i.e. viscosity) two independent properties are required to specify the relation between the shear stress and the shear rate. The term $k$ is called the consistency and $n$ is called flow index. If $n$ is less than one, the fluid is pseudo plastic and if greater than one, it is dilatant. The Couette flow problem helps researchers to investigate the interactions of various forces like viscous force, buoyancy force, electromagnetic force etc., in Newtonian and non-Newtonian fluids.

1.2 Heat Transfer

Heat Transfer is a phenomena associated with both Newtonian and non-Newtonian fluids. The problems concerning heat transfer have many applications in engineering sciences such as the design of cooling systems for motors, generators and transformers. Chemical engineers are concerned with the evaporation, condensation, and heating and cooling of fluids. An understanding of the laws of heat flow is important to the civil engineers in the construction of dams. In almost every branch of engineering, heat transfer problems arise and many of them can be solved lonely with a combination of the thermodynamic laws and the laws of fluid flow Heat transfer is associated with the process of transmission of energy from one region to another as a result of temperature difference between them. It is customary to categorize the various heat transfer processes into three basic types or modes, although, as will become apparent as one studies the subjects, it is certainly a rare instance when one encounters a problem of practical importance which does not involve at least two, and
sometimes all three of these modes occurring simultaneously. The three modes are conduction, convection and radiation.

Heat conduction is the term applied to the mechanism of internal energy exchange from one body to another, or from one part of a body to another body, by the exchange the kinetic energy of the molecules by direct communication or by the drift of free electronics in the case of heat conduction in metals. This flow of energy or heat passes from the higher energy molecules to the lower energy ones (i.e. from a high temperature region to a low temperature region.) The distinguishing feature of conduction is that it takes places within the boundary of a body, or across the boundary of a body into another body placed in contact with the first, without an appreciable displacement of the matter compressing the body.

A metal bar heated on one end will, in time, becomes hot at its other end. This is the simplest illustration of conduction. The laws governing conduction can be expressed in concise mathematical terms, and analysis of the heat flow can be treated analytically in many instances.

Convection is the term applied to the heat transfer mechanism which occurs in a fluid by the mixing of one portion of the fluid with another portion due to gross moments of the mass of fluid. The actual process of energy transfer from one fluid particle or molecule to another is still one of conduction, but the energy may be transported from one point in space to another by the displacement of the fluid itself.

The fluid motion may be caused by external mechanical means in which case the process is called forced convection. If the fluid motion is caused by density differences which are created by the temperature differences existing in the fluid mass, the process is termed free convection or natural convection. The circulation of water in a pan heated on a stove is an example of free convection. The important heat transfer problems of condensing and boiling are also examples of convection – involving the additional complication of a latent heat exchange.
It is virtually impossible to observe pure heat conduction in a fluid because as soon as temperature difference is imposed on a fluid, natural convection currents will occur as a result of density differences.

The basic laws of heat conduction must be coupled with those of fluid motion in order to describe, mathematically, the process of heat convection. The mathematical analysis of the resulting system of differential equations is perhaps one of the most complex fields of applied mathematics. Thus, for engineering applications, convection analysis will be seen to be a suitable combination of powerful mathematical techniques and the intelligent use of empiricism and experience.

We have described convection as the term applied to the heat transfer mechanism which takes places in a fluid because of a combination of conduction within the fluid and the energy transport which is due to the fluid motion itself—the fluid motion being produced either by artificial means or by density currents.

Since fluid motion is the distinguishing feature of heat convection, it is necessary to understand some of the principles of fluid dynamics in order to describe adequately the processes of convection. When any real fluid moves past a solid surface it is observed that the fluid velocity varies from a zero value immediately adjacent to the wall to a finite value at a point some distance away. Consider the case of a flow past a plain surface where the fluid velocity varies from a uniform value at points away from the wall to zero at the wall. For fluids of low viscosity, such as air or water, the region near the surface, in which most of the velocity variation occurs may be quite thin—depending on the free stream velocity of the fluid. In many applications—such as low speed aerodynamics, hydraulics, etc. It is possible to obtain satisfactory results by assuming that the fluid is in viscid. Hence, the flow may be treated as through it slips past the surface with no viscous radiation. However, since the process of convection of heat away from the wall (if the wall is at a temperature different from the free
stream of the fluid) is intimately concerned with thermal conduction and energy transport due to motion in the fluid layers in the vicinity of the wall, the simplification of assuming the fluid to be in viscid may not be made when analyses of heat convection are undertaken.

Since the region in which the retarding effect of the fluid viscosity plays a dominant role will often be a very thin layer near the wall, it is possible to simplify the description of the convection process by introducing the concept of the velocity boundary layer. The velocity boundary layer is defined as the thin layer near the wall in which one assumes that viscous effects are important within this region the effect of the wall on the motion of the fluid is significant. Outside the boundary layer it is assumed that the effect of the wall may be neglected. The exact limit of the boundary layer cannot be precisely defined because of the asymptotic nature of the velocity variation. The limit of the boundary layer is usually taken to be at the distance from the wall at which the fluid velocity is equal to a predetermined percentage of the free steam value. This percentage depends on the accuracy desired, 99 or 95 per cent being customary. Outside the boundary layer region the flow is assumed to be in viscid. Inside the boundary layer the viscous flow may be either laminar or turbulent. In the case of laminar boundary layer flow, adjacent fluid layers slide relative to one another but do not mix in the direction normal to the fluid streamlines. Thus, any heat that flows from the surface away from it does so mostly by conduction—although a transport of energy is also accomplished by virtue of the fact that the fluid has a velocity component normal to the surface.

In the presence of a temperature distribution in the fluid, the velocity field and the temperature field mutually interact, which means that the temperature and the velocity depend on one another. In the special case when buoyancy force $\rho \vec{G}$ may be disregarded, and when the properties of the fluid may be assumed to be independent of temperature, mutual interaction ceases, and the velocity field no longer depends on the temperature field, although the converse dependence of the temperature field on the velocity field still persists. This
happens at large velocities (large Reynolds numbers) and small temperature differences, such flows being termed forced. The process of heat transfer in such flows is described as forced convection. Flows in which buoyancy forces are dominant are called free, the respective heat transfer being known as free convection. The density difference will produce a positive or negative buoyant force (depending on whether the surface is hotter or colder than the fluid) in the fluid near the surface. The buoyant force results in a fluid motion, substantially in the vertical direction, past the surface with consequent convective heat transfer taking place. The force of gravity is, then, the driving force which produces the fluid motion and maintains the convective process.

The heating of rooms and buildings by the use of ‘radiators’ is a familiar example of heat transfer by free convection. Heat losses from hot pipes, ovens, etc., surrounded by cooler air are due to free convection, at least in part. The state of motion which accompanies free convection is evoked by buoyancy forces in the gravitational field of earth, the latter being due to density differences and gradients. It has been established that in several practical problems of heat transfer, a linear density temperature relation is used as an equation of state

**Radiation:**

All substances (solids, liquids and gases) at normal and especially at elevated temperatures emit energy in the form of radiation and are also capable of absorbing such energy. This shows that all heat transfer processes are accompanied by a heat exchange by radiation. However, in some cases heat exchange by radiation may be very small fraction of the total quantity of heat exchanged, as such it may be neglected. In case significant amount of heat transfer occurs by radiation, then use may be made of the various laws of radiation.

The relative importance of the various modes of heat transfer differs considerably with the temperature. Heat transfer by conduction and convection depends basically on the temperature difference and is little affected by the
temperature level. For example, other factors remaining constant, heat transfer by 
conduction or convection from a body at 1000°C to a body at 200°C remains the 
same as that from same body at 900°C to a body at 100°C. In case of radiation this, 
however, does not hold good. There may be about 35% more heat transfer at 
higher temperature even for the same temperature difference assuming all other 
factors as constant. Another difference between the radiation and the other modes 
of heat transfer lies in the fact that radiation heat transfer does not require any 
intermediate medium where as in case of conduction, and convection, medium for 
heat transfer is essentially required. Moreover, in case of conduction, heat flows 
from a body at high temperature to a body at low temperature if a third body, 
colder than either of the two, is interposed at any point between the two bodies, 
then heat will flow from both bodies to colder body. In case of radiation, 
however, this does not necessarily hold good. Radiative heat transfer may occur 
from a hot body, through a cold non-absorbing medium leaving it unaffected, and 
then reach a 'warmer' body.

Several theories have been proposed to explain the transport of energy by 
radiation. Whichever theory is used, radiant energy is the same type of wave 
motion as radio waves, X-rays and light waves except for the wave length. In 
fact, there is a whole spectrum of electromagnetic radiation in which the various 
arbitrary divisions are referred to by names reflecting the methods of origin or 
some characteristic quality. All forms have the same velocity of propagation but 
different wave lengths and sources of origin. All forms produce heat when 
absorbed. Never-the less, it is only the electromagnetic radiation produced by 
virtue of the temperature of the emitter that we call thermal radiation. Table 11-A 
gives the approximate ranges of wave length of some forms of radiation.

The amount of thermal radiation emitted by a body depends on its 
temperature and surface condition. Radiant energy emitted by a hot body is not 
confined to the visible range of wave length.
Table 1

Characteristic wave lengths of Radiation

<table>
<thead>
<tr>
<th>Name</th>
<th>Wave length Range in Micons *</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cosmic rays</td>
<td>upto ((10^{-6}))</td>
</tr>
<tr>
<td>Gamma rays</td>
<td>((10^{-6})) to ((140\times10^{-6}))</td>
</tr>
<tr>
<td>X-rays</td>
<td>((6\times10^{-6})) to 100,000((10^{-6}))</td>
</tr>
<tr>
<td>Ultraviolet rays</td>
<td>0.014 to 0.4</td>
</tr>
<tr>
<td>Visible or light rays</td>
<td>0.4 to 0.8</td>
</tr>
<tr>
<td>Infrared rays</td>
<td>0.8 to 400</td>
</tr>
<tr>
<td>Radio</td>
<td>((10\times10^{6})) to 30,000((10^{6}))</td>
</tr>
</tbody>
</table>

*1 micron = 10^{-6} metre.

But extends itself on both sides somewhat beyond this region. A thermometer placed in the dark or invisible region beyond the red end of a solar spectrum will detect a temperature rise.

Laws of Thermal-Radiations

(a) Planck's Law:

In 1900, Max Planck developed the quantum theory of electromagnetic waves and with the help of this, he has suggested the following formula for the monochromatic emissive power \( E_{\text{bd}} \) of a black-body which is based on his theoretical analysis and it is given as

\[
E_{\text{bd}} = \frac{c_1\lambda^{-5}}{\left(e^{c_2/\lambda T} - 1\right)}
\]

where the values of \( c_1 \) and \( c_2 \) are given as follows:

\( c_1 = 3.21 \times 10^{-16} \) kcal-m²/hr
As \( 1 \mu = 10^{-4} \text{cm} \)

\[
\because \quad 1 \text{cm} = \frac{\mu}{10^{-4}}
\]

\[
\therefore \quad \text{cm}^4 = \frac{\mu^4}{10^{-16}}
\]

Substituting this in the above equation

\[
c_1 = \frac{3.21 \times 10^{-8}}{10^{-16}} \frac{kcal - \mu^4}{m^2 - hr}
\]

\[
= 3.21 \times 10^8 \frac{kcal - \mu^4}{m^2 - hr}
\]

and

\[c_2 = 1.438 \text{ cm-k} \]

\[= 14380 \mu-k\]

(b) **Rayleigh-Jean’s Law:**

Planck’s law has two limiting cases one of which is that when the product \( \lambda T \) is large compared with the constant \( c_2 \). With this provision we can confine ourselves to only two terms of the exponential function (1) expanded into a series with the \( \frac{c_2}{\lambda T} \) exponent:

\[
e^{\frac{c_2}{\lambda T}} = 1 + \frac{c_2}{\lambda T} + \frac{1}{2!} \left( \frac{c_2}{\lambda T} \right)^2 + - - - - - -
\]

The equation (1) become

\[
E_{\text{BA}} = \frac{2\pi c T}{c_2 \lambda^3}
\]

(2)

This relationship expresses Rayleigh-Jean’s law.
(c) Wien’s Law of Deviation:

The second extreme case corresponds to a small value of the product $\lambda T$ as compared to constant $c_2$. Then, the unity present in the denominator of equation (1) can be neglected, and the relationship becomes Wien’s law (1893).

$$E_{0\lambda} = \frac{2\pi c_2}{\lambda^5} e^{-\frac{c_2}{\lambda T}}$$  \hspace{1cm} (3)

coordinates of the maximum values of the emissive power can be obtained from the extreme value of (1). For this purpose the derivative of the function is found for the wave length. Equating the derivative to zero, we obtain the following transcendental equation:

$$e^{\frac{c_2}{\lambda_{\text{max}} T}} + \frac{c_2}{5\lambda_{\text{max}} T} - 1 = 0$$

whose solution is

$$\frac{c_2}{\lambda_{\text{max}} T} = 4.965$$

from which

$$\lambda_{\text{max}} T = 2.8978 \times 10^{-3}$$  \hspace{1cm} (4)

where $\lambda_{\text{max}}$ is the wave length corresponding to a maximum intensity of radiation; the product $\lambda_{\text{max}} T$ is measured in units of m.k.

(d) Planck’s Law in Dimensionless Form:

Wien’s law of deviation,

$$(E_{0\lambda})_{\text{max}} = c_3 T^3$$  \hspace{1cm} (5)

Permits Planck’s law (1) to be expressed in dimensionless form:
If we substitute the value of $T$ from (4) in (6), then the latter acquires the form of:

$$\frac{E_{\lambda}}{(E_{\lambda})_{\text{max}}} = f\left(\frac{\lambda}{\lambda_{\text{max}}}\right)$$  \hspace{1cm} (7)

Planck's law is represented here graphically not by a family of isotherms, as illustrated. But by a single curve that holds for any wavelength and temperature of the body. The maximum of this relationship corresponds to the values

$$\frac{E_{\lambda}}{(E_{\lambda})_{\text{max}}} = 1 \quad \text{and} \quad \left(\frac{\lambda}{\lambda_{\text{max}}}\right) = 1$$

(e) **Stefan-Boltzmann's Law**

The Stefan-Boltzmann law relates the hemispherical total emissive power and temperature. This relationship was first established by Stefan in 1879, measuring the radiation emitted from a model of a black body, well before the appearance of Planck's quantum theory. Later, in 1884, the relationship was found theoretically (departing from the laws of thermodynamics) by Boltzmann and that is why the law is called Stefan-Boltzmann's law. Stefan-Boltzmann's law can also be obtained from Planck's law. For the emissive power $E_0$, W/m$^2$, Stefan-Boltzmann's law may be presented in the following form:

$$E_0 = \int_0^\infty E_\lambda d\lambda = \sigma_0 T^4$$  \hspace{1cm} (8)

where $\sigma_0$ is Stefan-Boltzmann's constant.

To facilitate practical calculations, equation (8) is usually presented in the following form:
\[ E_0 = c_0 \left( \frac{T}{100} \right)^4 \]  

(9)

Where \( c_0 = 5.6687 = 5.67 \) is the radiation constant of a black body, measured in W/m\(^2\)k\(^4\).

The Stefan-Boltzmann law also applies to grey bodies. It is then assumed that, as with black bodies, the inherent radiation of a grey body is proportional to the fourth power of the absolute temperature, but the emissive power is smaller than that of black bodies at the same temperature, as illustrated. The law thus acquires the following form for grey bodies:

\[ E = \varepsilon E_0 = \varepsilon c_0 \left( \frac{T}{100} \right)^4 = c \left( \frac{T}{100} \right)^4 \]  

(10)

where \( \varepsilon = E / E_0 = c / c_0 \) = integral emissivity of a grey body;

\( c \) = its radiation constant (or factor), W/m\(^2\).K\(^4\).

Thus, by total emissivity is meant the ratio of the total radiation density of a surface to that of a black body at the same temperature.

Stefan-Boltzmann's law is strictly valid for a grey body to the same extent as the assumption that emissivity remains strictly constant, independent of temperature.

(f) **Kirchhoff's Law of Radiation**

Kirchhoff's law (1882) establishes the quantitative relation between the emissivity and absorptivity of grey and black bodies. It can be derived from the heat balance of an emitting system consisting of a relatively large enclosure with heat-insulated walls, with two bodies inside it. Under conditions of
thermodynamics equilibrium the radiant energy of each of the three bodies is equal to the absorbed energy

\[ E_1 = E_{abl} = A_1E_{in} = A_1E_0 \]
\[ E_2 = E_{ab2} = A_2E_{ab2} = A_2E_0 \]

When we get

\[ \frac{E_1}{A_1} = \frac{E_2}{A_2} = \cdots = E_0 = f(T) \]  \hspace{1cm} (11)

Relationship (11) expresses Kirchhoff's law according to which the ratio of radiant energy to absorbed energy is independent of the nature of bodies and is equal to the radiation of a black body at the same temperature.

1.3 Magnetohydrodynamics

Magnetohydrodynamics is another important area of research in fluid dynamics. This branch has several applications in Geophysics and Astrophysics apart from its practical importance in the development of MHD generators, pumps, flow meters and bearings. MHD is the study of the flow of electrically conducting fluids in the presence of a magnetic field under certain assumptions. Most fluid flow which undergoes an interaction with magnetic field is of a natural that allows the so-called MHD approximations. These approximations are additional simplifications that can be made for a flow which is quasi-steady (steady or low frequency oscillatory) and in which that electric field is of the order of magnitude of the induced quantity \( (\mathbf{V} \times \mathbf{B}) \), where \( \mathbf{V} \) is the fluid velocity and \( \mathbf{B} \) is the magnetic flux density. These approximations are

(i) \( \mathbf{V}^2 \ll C^2 \) (C is the speed of the light)
(ii) The electric fields are of order of magnitude of induced effects.
(iii) The phenomena involving high frequency are not considered so that the displacement current is neglected in comparison with the conduction current \( \mathbf{J} \). In Ohm's law the space charge \( \rho_e \) may be neglected in liquid conductors.
(iv) The electric energy is negligible compared to the magnetic energy.
(v) The magnetic force density is represented by
\[ \vec{F} = \rho_e \vec{E} + (\vec{J} \times \vec{B}), \]
where \( \vec{E} \) is the electric field.

The term \( \rho_e \vec{E} \) is usually negligible compared to \( (\vec{J} \times \vec{B}) \) except for some gases.

The equations of the motion under the MHD approximation are the Maxwell's equations and the Navier-Stokes equation with the magnetic body force term \( (\vec{J} \times \vec{B}) \). The term \( (\vec{J} \times \vec{B}) \) must be added to Navier-Stokes equation as an external force term, just as we add \( \rho \vec{g} \) as the buoyancy force term in density varying fluids. The electromagnetic force is usually non-conservative (rotational) and not derivable from a scalar potential function except in some rare cases.

This is evident if we write
\[ \vec{J} \times \vec{B} = \frac{1}{\mu_e} (\vec{B} \cdot \nabla) \vec{B} - \frac{1}{\mu_e} \nabla (\vec{B}^2/2) \]

The irrotational part is derived from a potential \( B^2/2\mu_e \) which may be combined with pressure. In MDH flows the fluid equations and the Maxwell’s equations are coupled and must be solved simultaneously. Boundary conditions at ideal insulators and perfect conductors are derived using the principles of electrodynamics. The Hartmann flow (Poiseuille flow) is the flow model used in MHD generators. The magneto hydrodynamics supports several interesting properties like the propagation of transverse waves (Alfven waves) and the MHD pinch effect, concepts similar to which are not normally seen in viscous fluids.

**Flows through a Porous Media**

Flow through a porous medium is a topic encountered in many branches of engineering and science, e.g., ground water hydrology, reservoir engineering, soil science, soil mechanics and chemical engineering. Flows through porous media are important in petroleum engineering to study the movement of natural gas, oil and water through the oil reservoirs and in chemical engineering for filtration and water purification processes. A porous medium is a continuous solid phase with many void spaces, or pores, in it. Examples are sponges, cloths, wicks, paper, sand and gravel, filters, concrete, bricks, plaster...
wall, many naturally occurring rocks (e.g., sandstones and some limestone), and the packed beds used for distillation, absorption etc. In many such porous solids the void spaces are not connected, so there is no possibility that fluid will flow through them. For example, foamed plastic hot-drink cups, and ice-boxes have many pores, but because of the 'closed cell' structure of the plastic these pores are not interconnected. Thus these porous media form excellent barriers to fluid flow. On the other hand a pile of sand has fewer pores than a foamed polystyrene drinking cup, but its pores are all connected, so that fluids can easily flow through it. The porous media with no interconnected pores are described as impermeable to fluid flow and those with interconnected pores as permeable. The velocity in porous medium is usually not large and the flow passages so narrow that laminar flow may be assumed without hesitation. The permeability of a porous medium is its most useful fluid flow property. The permeability is a measure of the ease with which a fluid flows through the medium. If the permeability is higher, then the flow rate will be higher for a given pressure gradient. The porosity of a porous medium is another important property defined as the void volume, or volume of pore space divided by the total volume of the medium. The porosity of consolidated materials depends mainly on the degree of concentration. The porosity of unconsolidated materials depends on the packing of the grains, their shape, arrangement and size distribution. The specific surface of a porous material is defined as the total interstitial surface area of the pores per unit bulk volume of the porous medium.

The foundation on which laminar flow of fluids through porous media rests is Darcy's law. Darcy (1856). While experimenting with flow of water through sand filters, noted that the flow rate of water was proportional to the difference in head of water across the filter. His basic equation was \( V = -K(h_2-h_1)/L \), where \( V \) is flow rate of water per unit cross-section area, \( k \) is a constant for the system, \( h_2-h_1 \) is the difference in fluid head across length \( L \). Since the work of Darcy, a number of experiments have studied the flow of various fluids through many types of porous solids. The basic relation of Darcy has been extended to cover flow of any fluid, and as with any law, its limitations and range of
applicability have been defined. The use of Darcy's law is restricted to cases in which the flow is luminary or streamlined. The Darcy's law expresses that the seepage velocity is proportional to the pressure gradient and it does not have convective acceleration of the fluid. This law is therefore considered to be valid for low speed flows.

To study the flows through porous media we must introduce a simplified porous medium model that will be amenable to a mathematical treatment and that will incorporate the main features of a porous medium described so far. One of the essential features of a porous medium described so far. One of the essential features of a porous medium, in connection with the flow of a fluid through it, is that it restricts the transport of the fluid to well defined channels. The porous medium is in fact a non-homogeneous medium. For the sake of mathematical analysis it is replaced by a homogeneous fluid which has dynamical properties equal to those of non-homogeneous continuum. Thus one can study the flow of a hypothetical homogeneous fluid under the action of properly averaged external force (additional resistance $-\frac{\mu V}{k}$ due to the medium). The complicated problem of the flow through a porous medium thus reduces to the flow problem of a homogeneous fluid with resistance of the medium taken into account. When the porosity of the medium is very close to unity, the fluid occupies almost all parts of the porous medium and a generalized Darcy's law must be used to represent the flow.

The two simple concepts - Darcy's law and the law of conservation of mass are used to describe the laminar flow of a fluid through porous medium.

The equations governing fluid motion in the presence of a temperature distribution in the fluid are as follows. The general form of the Darcy law governing the motion of an incompressible fluid through a homogeneous isotropic porous medium namely
\[
\frac{\rho}{\varepsilon} \frac{D\vec{v}}{Dt} = -\nabla p - \frac{\mu}{k} \vec{v} + \mu \nabla^2 \vec{v} + \rho \vec{X}
\]

Where \(\rho\), \(\varepsilon\), \(\vec{V}\), \(p\), \(\mu\), \(k\) and \(\vec{X}\) denote fluid density, porosity, of the medium, filtration velocity, fluid pressure, dynamic viscosity coefficient, permeability and body force per unit mass respectively. For very fluffy metal materials of fibrous materials, \(\varepsilon\) is very close to unity and in beds of packed spheres, \(\varepsilon\) is in the range of 0.25 – 0.50. Although the viscous term \(\mu \nabla^2 \vec{v}\) is generally neglected in slow motion through a porous medium, it should be taken into account for the general flow particularly in the case \(\varepsilon \approx 1\), when the fluid occupies all parts of the porous medium. The equation of continuity is \(\nabla \cdot \vec{V} = 0\) while the energy equation is

\[
(\rho c) \frac{\partial T}{\partial t} + (\rho c)_f \vec{v} \cdot \nabla T = \lambda \nabla^2 T
\]

The porous medium formed by the porous matrix and the interstitial fluid (which is the fluid in the pores) is regarded as a fictitious isotropic fluid with heat capacity \((\rho c)\)' = \(\varepsilon (\rho c)_f + (1 - \varepsilon) (\rho c)_s\), denotes the heat capacity of the fluid and the solid respectively. The physical properties of the medium with an effective thermal conductivity \(\lambda\) are assumed constant in particular with respect to temperature dependence. A linear density-temperature relation is assumed as the equation of state. The variations in density with temperature may be neglected except with regard to their influence on the buoyancy force. This is the well known Boussinesq approximation.

Non-Darcy Law

In many practical problems, the flow through porous media is curvilinear and the curvature of the path yields the inertia effect, so that the streamlines become more distorted and the drag increase more rapidly. Lapwood
was the first person who suggested for the inclusion of convective inertial term \((q \cdot \nabla)q\) in the momentum equation. Subsequently many research articles have been appeared on the non-Darcy model \((2,9,13,15,16,17)\). Now (1.8) with the usual inertia term \(\frac{1}{\delta^2} (q \cdot \nabla)q\) can be written as

\[
\frac{1}{\delta^2} (q \cdot \nabla)q = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_s (\nabla^2 q)
\]

(1.9)

However, equation (2.9) does not take care of possible unsteady nature of velocity. The flow pattern in a certain region may be unsteady and one has to consider the local acceleration term \(\frac{1}{\delta^2} \frac{\partial q}{\partial t}\) also. Adding this term equation (1.9) becomes

\[
\rho \left( \frac{1}{\delta^2} \frac{\partial q}{\partial t} + \frac{1}{\delta^2} (q \cdot \nabla)q \right) = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_s (\nabla^2 q)
\]

(1.10)

This equation is known as Darcy-Lapwood-Brinkman equation (Rudraiah (21a)).

For anisotropic porous medium equation (1.10) takes the form

\[
\rho \left( \frac{1}{\delta^2} \frac{\partial q}{\partial t} + \frac{1}{\delta^2} (q \cdot \nabla)q \right) = -\nabla p + \rho g - \frac{\mu_f}{k} q + \mu_s (\nabla^2 q)
\]

(1.11)

**Forchheimer-Brinkman-Extended-Darcy Equation**

Brinkman's modification to the Darcy equation of motion has not been considered to be important by the porous media researchers except in the case of sparsely packed beds, primarily because the drag introduced by the boundary is generally small compared to that due to the ensemble of solid particles. However, since the convective heat transfer is mostly a boundary phenomenon, this modification to include the viscous diffusion effects can be of great importance to the energy transport processes. For an isotropic, homogeneous, fluid-saturated porous medium, the governing equations which include both the Forchheimer and Brinkman modifications in vector form are
\[ \nabla \cdot \mathbf{q} = 0 \]  
(Equation of continuity)  
(1.12)

\[
\frac{\rho}{\delta^2} \frac{\partial q}{\partial t} + \frac{\rho}{\delta^2} (q \cdot \nabla) q = -\nabla p + \rho \beta \left( T - T_0 \right) - \nu \nabla^2 \mathbf{q} + \mu \nabla^2 \mathbf{q}
\]  
(Equation of linear momentum)  
(1.13)

\[
\rho c_p \left[ \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T \right] = \lambda \nabla^2 T
\]  
(Equation of energy)  
(1.14)

\[
\rho - \rho_0 = -\beta g (T - T_0)
\]  
(Equation of state)  
(1.15)

Where \( \mathbf{q} = (0, 0, u) \) is the velocity, \( T \) is the temperature, \( p \) is the pressure, \( \rho \) is the density of the fluid, \( c_p \) is the specific heat at constant pressure, \( k \) is the permeability of the porous medium, \( \mu \) is the coefficient of viscosity of the fluid, \( \delta \) is the porosity of the medium, \( \beta \) is the co-efficient of thermal expansion, \( \lambda \) is co-efficient of thermal conductivity and \( F \) is a function that depends on the Reynolds number and the microstructure of porous medium. Here, the thermophysical properties of the solid and fluid have been assumed to be constant except for the density variation in the body force term (Boussinesq approximation), and the solid particles and fluid are considered to be in local thermal equilibrium.

**Governing equations of the flow**

Based on the experimental research of Darcy inflow through porous medium, Navier-stoke's equations are replaced by linear partial differential equations. Suitable approximations are to be made to get the solution, as the governing Equations of porous media is partial differential equation. In 1856, Henri Darcy formulated the law which governs the flow through a porous medium. Darcy's law is given by

\[
\mathbf{q} = \text{const} \left( -\nabla p + \rho g \right)
\]  
(4)

Where the \( p \) is the pressure.

Equation (4) expresses that Darcy's velocity \( \mathbf{q} \) is proportional to the sum of the pressure gradient and the gravitational force. Moreover, \( \mathbf{q} \) is inversely proportional to viscosity. This Darcy law is macroscopic equation of motion for
Newtonian fluids in porous media at small Reynolds numbers. Many researchers verified this law, experimentally. The constant in equation (4) is replace by the permeability k by Musket(18)

Now equation (4) becomes \( \bar{q} = \frac{-K}{\mu} \nabla p \)  

(5)

This law is valid for the flows through isotropic porous media.

By using Darcy's law various flows through porous media have been investigated by Musket (18), De wiest (11), Bear (1) and many other researchers. The most general form of Darcy's law is given by

\[
\frac{D}{Dt} u_i = -\frac{\partial p}{\partial x_i} + \rho \alpha_i - \frac{\mu}{k} u_i
\]

where

- \( x_i = \) the \( i^{th} \) component of body force per unit mass
- \( u_i = \) The \( i^{th} \) component of velocity.
- \( E = \) Porosity
- \( \frac{D}{Dt} = \) Substantial derivative.

Darcy law is valid when the flow takes place at low speeds. But for high speed flows, Darcy law is not valid. Also Darcy law fails to describe the flows with high speeds or the flow near surfaces which are either permeable or rigid. In such cases, Brinkman (2) equation will be useful. Brinkman obtained the governing equation for the flow through porous media as

\[
-\nabla p - \frac{\mu}{K} \bar{V} + \mu \nabla^2 \bar{V} = 0
\]

(6)

Where \( \bar{V} \) is the velocity vector

Rudraiah, Chennabasappa, Ranganna, Sacheti, Vijayakumar Varma and Syam Babu and many others studied numerous flow models past porous media applying Brinkman model.
Boundary Conditions

i. **No-Slip Condition**

When the flow takes place over a rigid plate, the velocity component vanish at the boundaries. This is called no slip condition.

ii. **Free Surface Boundary Condition**

Vertical component of velocity vanishes at a horizontal free surface. Further if there is no surface tension, the free surface will be free from shear stress.

iii. **Beavers and Joseph slip condition**

When a fluid flows, an impermeable surface, the no-slip condition is valid on the boundary. But when a fluid flows over a permeable surface, it is necessary to specify some condition on the tangential component of the velocity of the free fluid at the permeable interface. In this case, there will be a migration of fluid tangential to the boundary within the permeable surface. In this case, there will be a migration of fluid tangential to the boundary within the permeable surface. That is, there will be a net tangential drag due to transfer of forward momentum across the permeable interface. The velocity inside the permeable bed will be different from the velocity of the fluid past/over the permeable bed. These two velocities are to be matched at the nominal boundary (surface) of the permeable bed. The nominal boundary of a permeable bed is defined as a smooth geometric surface with the assumption that the outermost perimeters of all surface pores of the permeable material are in this surface. Thus if the surface is filled with solid material to the level of their respective perimeters, a smooth rigid boundary of the assumed shape results.

When Newtonian fluid flows past a permeable bed the no slip condition is not valid there. Firstly, Beavers and Joseph proved that there exists a slip on the velocity at the surface of the porous bed. This slip boundary condition is given by

\[
\frac{du}{dy} = \frac{\alpha}{\sqrt{k}} (u_b - Q)aty = B
\]

Where

\[
u = \text{Velocity parallel to the nominal surface}
\]
\[ \alpha = \text{Slip parameter} \]
\[ U_B = \text{Slip Velocity} \]
\[ Q = \text{Darcy Velocity} \]
\[ y = B \text{ is the nominal surface.} \]

1.5 Mass Transfer

Mass transfer is defined as the transfer of matter by virtue of species concentration difference in a system. The difference in concentration provides a driving force for the transfer of mass. Mass transfer always occurs in the direction of redacting concentration gradient.

The phenomena of mass transfer are very common in the theory of stellar structure and observable effects are detectable at least on the solar surface. The effect of mass transfer on the Stokes problems was first studied by Soundalgekar.

The involvement and application of mass transfer process goes to greater lengths in numerous fields of science, engineering and technology. Mass transfer operations quite often occur in the fields of electric engineering, civil engineering, aeronautics, metallurgy, environmental engineering, refrigeration, air conditioning, biological and industrial processes. The study of geophysics, astronomy, meteorology, agricultural oceanography and food processing demands the knowledge of heat and mass transfer. Mass transfer flows are highly significant for their varied practical importance many examples of mass transfer applications can be cited from the environment.

Mass transfer broadly occurs in biological chemical physical and engineering fields. It involves in biological functions or process like respiratory mechanisms, oxygenation (or) purification of blood, kidney functions, osmosis and assimilation of food and drugs. Evaporation of clouds, smoke formation, dispersion of fog, distribution of temperature and moisture over agricultural fields and grooves of fruit trees, damages of crops due to freezing and pollution of the nature. Mass transfer finds its place in ablative coding transpiration and film
cooling of rocket and jet engines. Mass transfer applications are widely found in chemical engineering processes like distillation, absorption of gases, interaction of solids and liquids from their mixtures, crystallization adsorption (solid taking up vapor on its surface) and chromatography processes like air humidification, cooling of water, ion exchange involve mass transfer.

Mass transfer occurs by two mechanisms.
1. Diffusion mass transfer
2. Convective mass transfer.

a) Diffusion Mass Transfer
In diffusion mass transfer, the transfer of matter occurs by the movement of molecules or species or particles of one component to another. Diffusion mass transfer may occur either due to concentration gradient or temperature gradient or pressure gradient (pressure diffusion).

b) Convective Mass Transfer
Convective mass transfer is a mechanism in which mass is transferred between the fluid and the solid surface as a result of movement of matter from the fluid to the solid surface or fluid. Convective mass transfer is again classified into
a) Natural or free convective mass transfer
b) Forced convection mass transfer

In natural convection mass transfer, the transfer of mass occurs by the motion of species due to the density differences resulting from temperature or concentration differences or mixture of varying composition.

Mass Flux:
The amount of mass transfer per unit area of the flow is called mass flux. If \( m \) is the amount of mass flow and \( A \) is the area normal to the direction of mass flow, then the mass flux is \( G = \frac{m}{A} \).
**Fick's Law of Diffusion:***

Fick's law relates the diffusion rate or mass flux of the species to its driving potential or the concentration gradient responsible for the flow. It states that “the mass flux of a component of a system in any direction is proportional to its concentration gradient in that direction”.

\[ G_A = \frac{dC_A}{dx} \]

\[ G_A = -D_{AB} \frac{dC_A}{dx} \]

where \( G_A \) is the mass flux of a component A,

\( C_A \) is the mass concentration of component A

\( \frac{dC_A}{dx} \) is the concentration gradient in the X-direction opposite to the direction of mass flow, \( D_{AB} \) is the coefficient of mass diffusivity for a system of components A and B.

**Control Volume:**

An arbitrary and fixed region in space across the boundaries of which the matter, momentum and energy flow, within which changes of matter, momentum and energy take place and on which the external forces act is termed as control volume.

**Control Surface:**

The closed surface around the control volume is termed as control surface.

**Law of Conservation of Mass**

This principle states that in any control volume, the rate of creation of mass or matter is zero. Mass flowing from control volume - mass flowing into control volume + mass change inside control volume = 0.
Law of Conservation of Momentum

This law states that in any control volume, the rate of change of momentum is proportional to the external forces acting on the control volume. Rate of change of momentum = Fg.

where \( g \) is the acceleration due to gravity and \( F \) is the external force acting on the control volume.

According to this law, rate of momentum flow from control volume - rate of momentum flow into the control volume + rate of change of momentum inside the control volume = acceleration due to gravity \( \times \) external forces.

Law of Conservation of Energy

This law states that energy in a control volume is neither created nor destroyed. Energy flow from control volume - energy flow into control volume + change of energy inside control volume = 0.

Thermo-Diffusion Effect (Soret Effect) (Thermophoresis effect)

When heat and mass transfer occur simultaneously in a moving fluid, the relations between the fluxes and the driving potentials are of more intricate nature. Mass fluxes can be created by temperature gradients and this is the Soret effect or the thermo-diffusion effect. The Soret effect dramatically lowers the thermal convection threshold, since concentration gradients release much more slowly than temperature gradients due to the disparate values of the mass diffusion coefficient and of the thermal diffusivity.

The name ‘Soret effect’ is usually attributed to mass separation induced by temperature gradients. The effect was discovered in 1879 by the swiss scientist Charles Soret who noticed that a salt solution contained in a tube with two ends at different temperatures did not remain uniform in composition. The salt was more concentrated near the cold end than near the hot end of the tube. Charles Soret concluded that a flux of salt was generated by a temperature gradient resulting, in steady state conditions, in a concentration gradient.
Although the German C. Ludwig described the same phenomenon several years before in 1856 in a short communication, the phenomenon bears his name because Soret studied the effect rather in detail and formulated the fundamental equations describing the phenomenon.

The Soret effect plays an important role in the operation of solar ponds, biological systems and the microstructure of the world oceans. In biological systems, mass transport across biological membranes induced by small thermal gradients in living matter is an important factor. One of the challenges in optimizing exploitation of oil reservoirs is a good knowledge of the fluid physics in crude oil reservoirs. Today, the modeling methods are based on pressure-temperature equilibrium diagrams and on gravity segregation of the different components of crude oil. However, improved models which more accurately predict the concentration of the different components are necessary. The concentration distribution of the different components in hydrocarbon mixtures is mainly driven by phase separation and diffusion and the Soret effect plays an important role.

1.6 Rotating Fluids

The study of magnetohydrodynamics of rotating fluids is applicable to several situations of geophysical and astrophysical interest. It is likely to explain the observed phenomenon of secular variations and maintenance of the geometric field. It may also establish the degree of electromagnetic coupling of the electric currents flowing in the convective envelope of some of the stars to their interior. Thus several complicated problems of practical interest can be understood by considering the relatively simple problems concerning the manner in which a rotating fluid bounded by one or two disks or confined in a cylinder, adjusts from one state of rigid rotation to another. In these problems it has been noted that near a rigid boundary of a rotating system, the important forces are the coriolis force and the stress. The structure of the boundary layers in rotating fluids was discussed by several authors like Ekman (1905), Stewartson.

27
(1957) and linear theory of rotating fluids is based heavily on the well understood non-magnetic problem of Greenspan and Howard which deals with the transient establishment of the boundary layer. It is well established that shear layers are located along the bounding surfaces and are the regions where the tangential velocity is adjusted to its proper wall value by viscosity. The structure of the boundary layer depends on the underlying force balance and in general, is different from case to case. Boundary layers of thickness of order $LE^{1/2}$, $LE^{1/3}$, $LE^{1/4}$, (where $E$ is the Ekman number and $L$ is the characteristic length) all occur in rotating fluid problems. A concentration of viscous action into narrow layers means that elsewhere the fluid behaves in an essentially inviscid manner, as if $E=0$.

Fluid motion in a rotating system is often induced by moving sections of the bounding surface at slightly different angular velocities. Although other means are available, the dimensional analysis of the fundamental equations is based on the supposition that density stratification has a major effect only in so far as the buoyancy force is important. The velocity excess over rigid rotation with angular velocity $\Omega$ is characterised by $\varepsilon \Omega L$ to make the variation of the dimensionless velocity, of unit magnitude. Here $\varepsilon$ is a small parameter. The scaling rule is $[L, \Omega^{-1}, \varepsilon \Omega L]$. Three different quantities are needed to describe the density structure.

\[
\begin{align*}
\rho_0 & \quad \text{the average value over the whole field,} \\
\Delta \rho & \quad \text{amount of stratification in the near equilibrium state,} \\
\frac{\varepsilon \Omega^2 L \rho_0}{g} & \quad \text{deviation produced by rotational processes}
\end{align*}
\]

The last scale is arrived at by equating the size of the buoyancy and coriolis forces. Therefore we write,

\[
\frac{\rho}{\rho_0} = 1 + \frac{\Delta \rho}{\rho_0} \int_{\varepsilon}^2(z) + \frac{\varepsilon \Omega^2 L}{g} \rho
\]
where $\rho$ is the dimensionless density perturbation. The first two terms, with the scale factor, constitute $\rho_e$. The temperature distribution is resolved in a similar fashion as

$$T(x, z) + \frac{\alpha L^2}{\rho_e g} T$$

$$T = T_0 + \Delta T$$

Usually $\Delta T$ is given and the value of $\Delta T$ determined from the relationship involving the known expansivity.

$$\alpha = \frac{\Delta \rho}{\Delta T} = \left(\frac{\partial \rho}{\partial T}\right)_P$$

Finally, the pressure is

$$P = \rho_0 e L P_e + \Sigma \rho_0 \Omega^2 L^2 p$$

The following parameters appear (with the usual rotation)

- **Ekman Number**, $E = \frac{\nu}{\Omega L^2}$
- **Rossby Number**, $\epsilon = \frac{\nu}{\Omega L}$
- **Froude number**, $F_R = \frac{\Omega^2 L}{\Delta \rho}$
- **Density ratio**, $H = \frac{\Delta \rho}{\rho_0}$
- **Internal Froude Number**, $\sigma_p = \frac{\mu C_p}{K}$
- **Prandtl number**, $D = \frac{\nu}{C_p \Delta T}$
- **Diffusivity coefficient**, $f_R = \frac{\Omega^2 L}{\rho_e \frac{\Delta \rho}{\rho_0} g} = \frac{F_R}{H}$

of these $\epsilon$, $E$, $F$, $D$, $H$ are assumed to be small compared to unity (though in varying degree), $f_R$ is of order one and $\sigma_p$ can be large. These assumptions restrict
the discussion to a very small range of the parameter space but one that covers a wide variety of interesting and important physical processes.

The Ekman number is a gross measure of how the typical viscous force compares to the coriolis force and it is, in essence, the inverse Reynolds number for the flow. The Ekman number is very small in most cases of interest where primary effects of rotation are displayed. Practical values of $10^{-5}$ are usual and henceforth the assumption $E_k \ll 1$ is made without further statement. Also, we have explicitly assumed that it is permissible to use a linear relationship between $\rho - \rho_e$ and $T - T_e$ as the equation of state.

The scaled variables are substituted into the general equations of mass, momentum and energy to cast the theory into a dimensionless form. If all terms multiplied by small parameters are discarded, the equation reduces to the form $\nabla \cdot \mathbf{q} = 0$

$$f_r \frac{dT}{dt} + \vec{q} \cdot \vec{v} = 0 \left\{ + \frac{E \nu}{\sigma} V^2 T \right\}$$

$$\frac{\partial \vec{q}}{\partial t} + 2 \vec{h} \times \vec{q} = - \nabla p - \rho \frac{\partial \vec{h}}{\partial t} + \{ E \nu^2 \vec{q} \}$$

$$\rho = -T$$

The theory of rotating fluids has tremendous applications in the geophysical and astrophysical sciences. It is well known that when a vast expanse of viscous liquid bounded by an infinite flat plate is rotating about an axis normal to the plate, a layer known as Ekman layer is formed near the plate where the viscous and coriolis forces are of the same order of magnitude. Interest in the study of MHD of rotating fluids has been driven by several important problems such as maintenance and secular variations of the earth’s magnetic field, the internal rotation rate of the sun, structure of rotating magnetic stars, the planetary and solar dynamo and the centrifugal machines etc.