CHAPTER IV

Non-Darcy convective heat and mass transfer through a porous medium in a vertical channel with heat generating sources
The analysis of natural convection has been of considerable interest to engineers and scientists. Most studies of natural convection are mainly concerned with heat convection solely. However, Gebhart and Pera [2] indicated that buoyancy effects from concentration gradients can be as important as those from temperature gradients. There are applications of interest in which combined heat and mass transfer by natural convection such as design of chemical processing equipment design of heat exchangers. Formation and dispersion of fog distributions of temperature and moisture over agricultural fields pollution of the environments and thermal protection systems.

Convective flows in porous media are of interest in many varied situations for example in geothermal energy resource and oil reservoir modeling in the analysis of insulating systems and in flows through tobacco rods. There is a plethora of literature covering these situations; most of which concentrates on the
classical Darcy flow case. It is known, however, that at higher flow rates or in highly porous media there is a departure from the linear law and inertial effects become important. In terms of the Reynolds number based on a typical particle diameter (say). It has been found that the flow becomes Non-Darcian when the Reynolds number exceeds unity. Physically, this departure is believed to be due to flow separation within the medium, whilst mathematically, it manifests itself as a nonlinear term in the velocity—pressure gradient relationship.

Non-Darcy effects on natural convection in porous media have received a great deal of attention in recent years because of the experiments conducted with several combinations of solids and fluids covering wide ranges of governing parameters which indicate that the experimental data for systems other than glass water at low Rayleigh numbers do not agree with theoretical predictions based on the Darcy flow model. This divergence in the heat transfer results has been reviewed in detail in Chang [3] and Prasad et al [7]. Among other extensive efforts are thus being made to include the inertia and viscous diffusion term in the flow equations and to examine their effects. In order to develop a reasonable accurate mathematical model for convective transport in porous media. The work of Vafai and Tien [14] was one of the early attempts to account for the boundary and inertia effects in the momentum equation for a porous medium. They found that the momentum boundary layer thickness is of order $K \sqrt{\frac{\varepsilon}{\Sigma}}$. Vafai and Thyagaraja [15] presented analytical solutions for the velocity and temperature fields for the interface region using Brinkmann Forchheimer-extended Darcy equation. Detailed accounts of the recent efforts on Non-Darcy convection have been recently reported in Tien and Hong [12] Chang [4] Prasad et al [8] and Kalidas and Prasad.
Poulikako's and Bejan investigated in inertia effects through the inclusion of Fochhimer's velocity squared term and presented the boundary layer analysis for tall cavities. They also obtained numerical results for a few cases in order to verify accuracy of their boundary layer solutions. Later Prasad and Tuntomo reported an extensive numerical work for a wide range of parameters and demonstrated that effects of Prandtl number remain almost unaltered while the dependence on the modified Grashof number changes significantly with an increase in the Forchheimer number. They also reported a criterion for the Darcy flow limit.

The Brinkmann - Extended Darcy model was considered in Tong and Subramanian and Laurait and Prasad to examine the boundary effects on free convection in a vertical cavity. While Tong and Subramanian performed a Weber type boundary layer analysis, Laurait and Prasad solved the problem numerically for \( A = 1 \) and 5. It was shown that for a fixed Rayleigh number \( Ra \), the Nusselt number decreases with an increase in the Darcy number, the reduction being larger at higher values of \( Ra \).

A scale analysis as well as the computational data also showed that the transport term \( (v \nabla)v \) is of low order of magnitude compared to the diffusion plus buoyancy terms. A numerical study based on the Forchheimer - Brinkmann extended Darcy equation of motion has also been reported recently by Beckermann et al. They demonstrated that the inclusion of both the inertia and boundary effects is important for convection in a rectangular packed sphere cavity. Recently Reddy has discussed non-Darcy convective flow in a circular duct under different conditions.
We consider the flow of a viscous, incompressible, electrically conducting fluid through a porous medium confined in a vertical channel in the presence of heat generating sources. A Cartesian coordinate system \( O(x,y,z) \) is used so that the boundaries are taken at \( y = \pm L \), where \( 2L \) is the distance between the walls. A uniform magnetic field of strength is applied normal to the walls. Boussinesq approximation is used so that the density variation is taken only in the buoyancy force term. The plates are maintained at constant temperature \( T_1, T_2 \) and concentrations \( C_1, C_2 \). We also take dissipation into account in the energy equation. The equations governing the flow, heat and mass transfer in the absence of applied electric field are

Momentum equation:

\[
\frac{\partial P}{\partial x} + \frac{\mu}{\delta} (\frac{\partial^2 u}{\partial y^2}) - \frac{\mu}{k} u - \frac{\rho \delta F}{\sqrt{k}} u^2 - \rho g - (\sigma \mu_l^2 H^2) \delta u = 0
\]  

Energy equation:

\[
\rho_0 C_p \frac{\partial T}{\partial y} = \lambda \frac{\partial^2 T}{\partial y^2} + Q(T_o - T) + \mu (\frac{\partial u}{\partial y})^2
\]

Equation of diffusion:

\[
u \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}
\]

Equation of State:

\[
\rho - \rho_o = -\beta \rho_o (T - T_o) - \beta' \rho_o (C - C_o)
\]

Boundary conditions:

\[ u = 0 \quad \text{on} \quad y = \pm L \]
\[ T = T_1, C = C_1 \quad \text{on} \quad y = -L \]
\[ T = T_2, C = C_2 \quad \text{on} \quad y = +L \quad (2.5) \]

Where \( T \) is the temperature of the fluid, \( \rho \) is the density of the fluid, \( C \) is the specific heat at constant pressure, \( k \) is the permeability of the porous medium, \( \mu \) is the coefficient of viscosity of the fluid, \( \delta \) is the porosity of the medium, \( \sigma \) is the electrical conductivity, \( \beta \) is the coefficient of Thermal expansion, \( \lambda \) is the coefficient of Thermal Conductivity and \( F \) is the function that depends on the Reynolds number and microstructure of porous medium.

We introduce the following non-dimensional variables as

\[ (x', y') = (x, y)/L, u' = u/(v/L), p' = p\delta/(\rho v^2 / L^2) \quad (2.6) \]

\[ \theta = \frac{T - T_2}{T_1 - T_2}, \quad C' = \frac{C - C_1}{C_1 - C_2} \]

Substituting (2.6) the equations in the non-dimensional form are

\[ \frac{dp}{dx} = \frac{d^2u}{dy^2} - \delta(D^{-1} + M^2)u - \delta G(\theta + NC) \quad (2.7) \]

\[ PNu = \frac{d^2\theta}{dy^2} - \alpha\theta + PECu^2 \quad (2.8) \]

\[ ScNg = \frac{d^2C}{dy^2} \quad (2.9) \]

\[ u(\pm 1) = 0, \]
\[ \theta(-1) = 1, C(-1) = 1 \]
\[ \theta(+1) = 0, C(+1) = 0 \quad (2.10) \]
SOLUTION OF THE PROBLEM

Assuming the parameter \( \delta \) to be small we assume the solutions as:

\[
\begin{align*}
    u(y) &= u_0(y) + \delta u_1(y) + \delta^2 u_2(y) + \ldots,
    \\
    \theta(y) &= \theta_0(y) + \delta \theta_1(y) + \delta^2 \theta_2(y) + \ldots.
\end{align*}
\]

\[ C(y) = C_0(y) + \delta C_1(y) + \delta^2 C_2(y) + \ldots \]  \quad (3.1)

Substituting (3.1) in equations (2.7)-(2.10) and equating the like powers of \( \delta \) the equations to the zeroth order are

\[
\frac{d^2 u_0}{dy^2} = \pi \quad (3.2)
\]

\[
\frac{d^2 \theta_0}{dy^2} - \alpha \theta_0 = (P_1N_r)u_0 \quad (3.3)
\]

\[
\frac{d^2 C_0}{dy^2} = +(N_c S_r)u_0 \quad (3.4)
\]

to the first order are

\[
\frac{d^2 u_1}{dy^2} - D^{-1}u_1 = -G(\theta_0 + NC_0) \quad (3.5)
\]

\[
\frac{d^2 \theta_1}{dy^2} - \alpha \theta_1 = (P_1N_r)u_1 \quad (3.6)
\]

\[
\frac{d^2 u_2}{dy^2} - D^{-1}u_2 = \Delta(u_0)^2 - G(\theta_1 + NC_1) \quad (3.7)
\]

\[
\frac{d^2 C_1}{dy^2} = (N_c S_r)u_1 \quad (3.8)
\]

and to the second order are

\[
\frac{d^2 \theta_2}{dy^2} - \alpha \theta_2 = (P_1N_r)u_2 \quad (3.9)
\]

\[
\frac{d^2 C_2}{dy^2} = (N_c S_r)u_2 \quad (3.10)
\]
Boundary Conditions

\[ u_0 (+1) = 0 ; \quad u_0 (-1) = 0 ; \]
\[ \theta_0 (+1) = 0 ; \quad \theta_0 (-1) = 1 ; \]
\[ C_0 (+1) = 0 ; \quad C_0 (-1) = (-1) ; \]
\[ u_1 (\pm 1) = 0 ; \quad \theta_1 (\pm 1) = 0 ; \quad C_1 (\pm 1) = 0 ; \]
\[ u_2 (\pm 1) = 0 ; \quad \theta_2 (\pm 1) = 0 ; \quad C_2 (\pm 1) = 0 ; \]

(3.11)

(3.12)

(3.13)

Solving the equations (3.2)-(3.10) subject to the boundary conditions (3.11)-(3.13) we get.

\[ u_0 = \frac{\pi}{2} (y^2 - 1) \]
\[ \theta_0 = a_1 \cosh(\lambda_1 y) + a_2 \sinh(\lambda_1 y) + b_1 y^2 + b_2 \]
\[ C_0 = a_3 \cosh(\lambda_1 y) + a_4 \sinh(\lambda_1 y) + a_5 y^4 + a_6 y + a_7 \]

\[ u_1 = a_7 \cosh(\lambda_2 y) + a_{10} \sinh(\lambda_2 y) + a_{11} \cosh(\lambda_2 y) + a_{12} \sinh(\lambda_2 y) + a_{13} y^4 + a_{14} y^2 + a_{15} y + a_{16} \]
\[ \theta_1 = (a_{17} + a_{21}) \cosh(\lambda_1 y) + (a_{18} + a_{22}) \sinh(\lambda_1 y) + a_{23} \cosh(\lambda_2 y) + a_{24} \sinh(\lambda_2 y) + a_{25} y^2 \cosh(\lambda_1 y) + a_{26} y^2 \sinh(\lambda_1 y) + a_{27} y \cosh(\lambda_1 y) + a_{28} y \sinh(\lambda_1 y) + a_{29} y^4 + a_{30} y^2 + a_{31} y + a_{32} \]
\[ C_1 = a_{33} \cosh(\lambda_2 y) + a_{34} \sinh(\lambda_2 y) + a_{35} \cosh(\lambda_2 y) + a_{36} \sinh(\lambda_2 y) + a_{37} \cosh(\lambda_2 y) + a_{38} \sinh(\lambda_2 y) + a_{39} \cosh(\lambda_2 y) + a_{40} \sinh(\lambda_2 y) + a_{41} y^2 \cosh(\lambda_2 y) + a_{42} y^2 \sinh(\lambda_2 y) + a_{43} y \cosh(\lambda_2 y) + a_{44} y \sinh(\lambda_2 y) + a_{45} y^4 + a_{46} y^2 + a_{47} y + a_{48} \]
\[ u_2 = a_{49} \cosh(\lambda_2 y) + a_{50} \sinh(\lambda_2 y) + (a_{51} + a_{55}) \cosh(\lambda_2 y) + (a_{52} + a_{54}) \sinh(\lambda_2 y) + a_{53} \cosh(\lambda_2 y) + a_{54} \sinh(\lambda_2 y) + a_{55} y^2 \cosh(\lambda_2 y) + a_{56} y^2 \sinh(\lambda_2 y) + a_{57} y \cosh(\lambda_2 y) + a_{58} y \sinh(\lambda_2 y) + a_{59} y^4 + a_{60} y^2 + a_{61} y + a_{62} \]
\[ \theta_2 = (a_{65} + a_{77}) \cosh(\lambda_1 y) + (a_{65} + a_{77}) \sinh(\lambda_1 y) + a_{73} \cosh(\lambda_2 y) + a_{74} \sinh(\lambda_2 y) + a_{75} \cosh(\lambda_3 y) + a_{76} \sinh(\lambda_3 y) + \ldots \]

\[ \tau = \mu \left( \frac{du}{dy} \right)_{y=\pm L} \]

which in the non-dimensional form is

\[ \tau = \frac{\tau}{\mu(T_1 - T_2)/L} = \left( \frac{du}{dy} \right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (\tau)_{y=+1} = \pi + \delta b_1 + \delta^2 b_3 \]

\[ (\tau)_{y=-1} = -\pi + \delta b_2 + \delta^2 b_4 \]

The rate heat transfer (Nusselt number) on the walls is given by

\[ C_2 = a_{101} \cosh(\lambda_1 y) + a_{106} \sinh(\lambda_1 y) + a_{107} \cosh(\lambda_2 y) + a_{108} \sinh(\lambda_2 y) + a_{113} \cosh(\lambda_3 y) + a_{114} \sinh(\lambda_3 y) + a_{115} \cosh(\lambda_4 y) + a_{116} \sinh(\lambda_4 y) + \ldots \]
\[ q = -k(dT\bigg|_{y=\pm L}/dy) \]

which in the non-dimensional form is

\[ Nu = \frac{qL}{(T_1 - T_2)k} = \left(\frac{d\theta}{dy}\right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (Nu)_{y=\pm 1} = b_3 + \delta b_3 + \delta^2 b_{15} \]
\[ (Nu)_{y=1} = b_4 + \delta b_6 + \delta^2 b_{16} \]

The rate of mass transfer (Sherwood Number) on the walls is given by

\[ J = -D(dC\bigg|_{y=\pm L}/dy) \]

which in the non-dimensional form is

\[ Sh = \frac{JL}{(C_1 - C_2)D} = \left(\frac{dC}{dy}\right)_{y=\pm 1} \]

and the corresponding expressions are

\[ (Sh)_{y=\pm 1} = b_7 + \delta b_{11} + \delta^2 b_{17} \]
\[ (Sh)_{y=1} = b_{10} + \delta b_{12} + \delta^2 b_{18} \]
5. DISCUSSION OF THE NUMERICAL RESULTS

In this analysis we investigate the combined effect of radiation and viscous dissipation on the mixed convective heat and mass transfer flow of viscous fluid confined in a vertical channel bounded by flat wall in the presence of temperature dependent heat sources. We take the Prandtl Number $P = 0.71$. The actual axial velocity is in the vertically downward direction and hence $u > 0$ represents reversal flow. The velocity, temperature and concentration are analysed for different values of $G, D^1, \alpha, S_c, N, N_1$ and $E_c$ and are represented in figures. It is found from fig (1) that the profiles of the velocity rises from prescribed value zero on $y = -1$, reaches maximum at $y = 0$ and then falls to its prescribed value zero $y = 1$. It is found that the reversal flow does not occur anywhere in the region for any value of the governing parameters. The magnitude of $u$ decreases with increase in $|G|$, with maximum at $y = 0$. The variation of $u$ with $D^1$ shows that lesser the permeability of the porous medium larger the magnitude of $u$, everywhere in the fluid region. The behaviour of $u$ with heat source parameter $\alpha$ indicates that $|u|$ enhances with increase in the heat source parameter $\alpha$. The radiation of $u$ with Schmidt Number $S_c$ show that lesser the molecular diffusivity smaller $|u|$ in the flow region. When the molecular buoyancy force dominates over the thermal buoyancy force the magnitude of $u$ depreciates when the buoyancy force act in the same direction while for the forces act in opposite direction $|u|$ enhances in the fluid region. The presence of radiation to heat transfer leads to an increase in the tendency to $|u|$ in entire fluid region. Inclusion of assistance effects enhances $|u|$ in the flow region. (Eq. 6)
Fig. 1 Profiles of axial velocity ($u$) with $G$

$D = 10^3, Sc = 1.3, N = 1, N_1 = 0.5, Ec = 2$

I    II    III    IV    V    VI
$G$  $10^3, 3 \times 10^3, 5 \times 10^3, -10^3, -3 \times 10^3, -5 \times 10^3$

Fig. 2 Variation of $u$ with $D^{-1}$

$G = 10^3, N = 1, Sc = 1.3, N_1 = 0.5$

I    II    III
$D^{-1}$  $10^3, 2 \times 10^3, 3 \times 10^3$
Fig. 3 Variation of $u$ with $\alpha$
$G=10^3, N=1, Sc=1.3, N_1=0.5$
I  II  III
$\alpha$  2  4  6

Fig. 4 Variation of $u$ with $Sc$
$G=10^3, N=1, N_1=0.5$
I  II  III  IV
$Sc$  0.24  0.6  1.3  2.01
Fig. 5 Variation of $u$ with $N$
$G=10^3$, $Sc=1.3$, $N1=0.5$
I II III IV
N 1 2 -0.5 -0.8

Fig. 6 Variation of $u$ with $N1$
$G=10^3$, $N=1$, $Sc=1.3$, $Ec=2$
I II III IV V
N1 0.5 1.5 4 10 100
Fig. 7 Variation of $u$ with $Ec$
$G=10^3, N=1, N_1=0.5, Sc=1.3$

Fig. 8 Profiles of temperature ($\theta$) with $G$
$D'=10^3, Sc=1.3, N=1, N_1=0.5, Ec=2$

$G = 10^3, 3 \times 10^3, 5 \times 10^3, -10^3, -3 \times 10^3, -5 \times 10^3$
Fig. 9 Variation of $\theta$ with $D^{-1}$
$G=10^3, N=1, Sc=1.3, N_1=0.5$

I  II  III
$D^{-1}$ $10^3$ $2 \times 10^3$ $3 \times 10^3$

Fig. 10 Variation of $\theta$ with $\alpha$
$G=10^3, N=1, Sc=1.3, N_1=0.5$

I  II  III
$\alpha$ 2  4  6
Fig. 11 Variation of $\theta$ with Sc
$G=10^3, N=1, N_1=0.5$

I  II  III  IV
Sc  0.24  0.6  1.3  2.01

Fig. 12 Variation of $\theta$ with N
$G=10^3, Sc=1.3, N_1=0.5$

I  II  III  IV
N  1  2  -0.5  -0.8
Fig. 15 Profiles of Concentration (C) with G
$D' = 10^3, Sc = 1.3, N' = 1, N_1 = 0.5, Ec = 2$
I II III IV V VI
G $10^3 \ 3 \times 10^3 \ 5 \times 10^3 \ -10^3 \ -3 \times 10^3 \ -5 \times 10^3$

Fig. 16 Variation of C with $D^{-1}$
$G = 10^3, Sc = 1.3, N = 1, N_1 = 0.5$
I II III
$D^{-1} \ 10^3 \ 2 \times 10^3 \ 3 \times 10^3$
Fig. 17 Variation of $C$ with $\alpha$
$G=10^3, N=1, Sc=1.3, N_1=0.5$

I   II   III
$\alpha$  2   4   6

Fig. 18 Variation of $C$ with $Sc$
$G=10^3, N=1, N_1=0.5$

$Sc$ 0.24 0.6 1.3 2.01
Fig. 21 Variation of C with $E_c$ 
$G=10^3, N=1, N_1=0.5, Sc=1.3$
The non-dimensional temperature (0) is exhibited in the figures ( - ) for different values of governing parameters. We follow the convention that that the non-dimensional temperature '0' is positive or negative according as the actual temperature is greater or lesser than the equilibrium temperature. The variation of \( \theta \) with the buoyancy parameter \( N \) shows that \( \theta \) is positive in the entire flow region except in a narrow region adjacent to right boundary \( y = 1 \). the region of transition from positive to negative enhances with increase in \( G \leq 3 \times 10^3 \) and again reduces for further increase in \( G \), while for an increase in \( |G| \) enhances the transition region towards the left boundary for \( G \leq 3 \times 10^3 \) and for further increase in \( |G| \geq 5 \times 10^3 \), we find that the temperature negative in the entire region. Also it is found that the actual temperature decreases with increase in \( |G| \leq 3 \times 10^3 \) and enhances for further increase in \( G \geq 5 \times 10^3 \), while an increase in \( |G| \) the actual temperature reduces in the entire flow region. The radiation of \( \theta \) with Darcy parameter \( D' \) shows that \( \theta \) is positive in the entire flow region except in a narrow region adjacent to \( y = -1 \). Lesser the permeability of the porous medium larger the actual temperature and for further lowering the permeability smaller the actual temperature in the flow region. The behaviour of \( \theta \) with heat source parameter \( \alpha \) implies that the actual temperature depreciates in the flow region with increase in \( \alpha \). The variation of \( \theta \) with Schmidt number \( Sc \) shows that lesser the molecular diffusivity larger the actual temperature in the flow region and further increase in the molecular diffusivity smaller the actual temperature. When the molecular buoyancy force dominates over the thermal buoyancy force the actual temperature depreciates in the flow region irrespective of the direction of the buoyancy forces (fig. 1) reports the variation of \( \theta \) with radiation parameter \( N_i \). An increase in \( N_i \)
Table 1
Shear Stress(τ) at y = -1
P=0.71, N=1, Sc=1.3, N1=0.5, Ec=0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
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Table 2
Shear Stress(τ) at y = 1
P=0.71, N=1, Sc=1.3, N1=0.5, Ec=0.5

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<th>G/τ</th>
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Table 3

Shear Stress(τ) at y = -1
P = 0.71, D'1 = 5x10^2, N1 = 0.5, Ec = 0.5

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Table 4

Shear Stress(τ) at y = 1
P = 0.71, D'1 = 5x10^2, N1 = 0.5, Ec = 0.5

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<td>3.13882</td>
<td>3.14148</td>
<td>3.14259</td>
<td>3.14696</td>
</tr>
<tr>
<td>3x10^3</td>
<td>3.13273</td>
<td>3.12379</td>
<td>3.14622</td>
<td>3.14893</td>
<td>4.14129</td>
<td>3.13838</td>
<td>3.12701</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1</td>
<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sc</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>0.24</td>
<td>0.6</td>
<td>2.01</td>
</tr>
</tbody>
</table>
### Table 5
Shear Stress ($\tau$) at $y = -1$
$P = 0.71, D'' = 5 \times 10^2, N = 1.0, Sc = 1.3$

<table>
<thead>
<tr>
<th>$G/\tau$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3 \times 10^3$</td>
<td>3.14708</td>
<td>3.14753</td>
<td>3.14762</td>
<td>3.14774</td>
<td>3.14788</td>
<td>3.14385</td>
<td>3.14062</td>
</tr>
<tr>
<td>$3 \times 10^3$</td>
<td>3.13273</td>
<td>3.13556</td>
<td>3.13731</td>
<td>3.13832</td>
<td>3.13909</td>
<td>3.13254</td>
<td>3.13231</td>
</tr>
</tbody>
</table>

### Table 6
Shear Stress ($\tau$) at $y = 1$
$P = 0.71, D'' = 5 \times 10^2, N = 1.0, Sc = 1.3$

<table>
<thead>
<tr>
<th>$G/\tau$</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3 \times 10^3$</td>
<td>3.13273</td>
<td>3.13556</td>
<td>3.13731</td>
<td>3.13831</td>
<td>3.13909</td>
<td>3.13254</td>
<td>3.13231</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_1$</td>
<td>0.5</td>
<td>1.5</td>
<td>4.0</td>
<td>10</td>
<td>100</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$E_c$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.07</td>
</tr>
</tbody>
</table>
reduces the actual temperature entire flow region. From fig ( ) we notice that the actual temperature depreciates with increase in Eckert number \( E_c \). Thus the inclusion of the viscous dissipation enhances the temperature in the flow region.

The non-dimensional concentration \( C \) is shown to figures ( ) for different values of \( G,D^{-1},S_c,N,N_1,\alpha \).

The variation of \( C \) with the buoyancy parameter \( G \) shows that the actual concentrated is greater than the equilibrium concentration except in narrow region adjacent to left boundary. It is found that the actual concentration depreciates with heating case and enhances in the cooling case. Variation of \( C \) with \( D^{-1} \) shows that the lesser permeability of the porous medium higher the actual concentration in the flow region. An increase in \( \alpha > 0 \) enhances the actual concentration. The variation of \( C \) with the buoyancy ratio \( N \) shows that when the molecular buoyancy force dominates over the thermal buoyancy force the actual concentration enhances when the buoyancy forces act in the same direction while for the forces acting in the opposite direction larger the actual concentration in the entire flow region, except in a narrow region adjacent to the right boundary \( y = 1 \).

From figure we notice that the actual concentration experiences an enhancement with increase in \( N_1 \) (Radiation). The influence of the dissipation is to depreciate the actual concentration every where in the fluid region.

The shear stress at the bounding walls \( y = \pm 1 \) is shown in tables ( ) for differ of \( G,D^{-1},N,S_c,\alpha,N_1 \) and \( E_c \). It is found that the shear stress at \( y = \pm 1 \) enhances with increase in \( |G| \) and \( D^{-1} \). Thus the lesser the permeability of the porous media smaller \( |\tau| \) at both the walls. The variation of \( |\tau| \) with buoyancy ratio \( N \) shows that when the molecular buoyancy force dominates over the thermal buoyancy force \( |\tau| \) enhances at \( y = 1 \) and depreciates at \( y = -1 \). when
the buoyancy force act in the same direction and a reversal effect is observed for the forces acting in the opposite directions. The variation of $t$ with $\alpha$ shows that at $y = -1$, $|t|$ enhances with $\alpha \leq 4$ and depreciates with higher $\alpha > 6$ in the cooling case while in the heating case $|t|$ enhances with $\alpha$. At $y = 1$, $|t|$ enhances with $\alpha$ in both heating and cooling cases. The behaviour of $t$ with $S_c$ reveals that lesser the molecular diffusivity larger the magnitude of $t$. The behaviour of $t$ with radiation between $N_1$ shows that an increase in $N_1$ enhances $|t|$ at $y = -1$ for all $G$, except for at $G = -10^3$, where it reduces with $N_1$, while at $y = 1$, $|t|$ experiences an enhances with $N_1$. The effect of dissipation on the stress is to reduces $|t|$ at both the walls except for $G = 10^3$, $|t|$ at $y = -1$ enhances with $E_c$. In general we notice that the stress at the right boundary is less that at left boundary. The Nusselt number (Nu) which measures the rate of heat transfer at boundary $y = \pm 1$ is evaluated for different values of $G, D^{-1}, S_c$ it is found that the rate of heat transfer at $y = \pm 1$ enhances with increase in $|G|$. The evaluation of Nu with $D^{-1}$ shows that the in the cooling case the rate of heat transfer at $y = \pm 1$ depreciates with increase in $D^{-1}$, while in the heating case $|Nu|$ at $y = -1$ depreciates with $D^{-1} \leq 3 \times 10^3$ and enhances for further increase in $D^{-1}$ and $|Nu|$ at $y = -1$ depreciates with $D^{-1}$. 

The variation of Nu with buoyancy shows that the rate of heat transfer increases when the buoyancy forces act in opposite directions, $|Nu|$ depreciates for the forces acting in the same direction. With reference to variation of $\alpha$ we found that the rate of heat transfer at $y = -1$ decrease with $\alpha \leq 4$ and enhances for $\alpha = 6$ while at $|G| = 5 \times 10^3$, $|Nu|$ enhances with $\alpha$. At $y = 1$, the rate of heat transfer depreciates with increase in $\alpha$ for all $G$. Also lesser the
Table 7
Nusselt Number (Nu) at y = -1
P = 0.71, N = 1, S = 1.3, N = 1, E = 0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 \times 10^3</td>
<td>3.40374</td>
<td>1.96626</td>
<td>1.93065</td>
<td>2.14598</td>
<td>2.26905</td>
</tr>
<tr>
<td>-10^3</td>
<td>2.06503</td>
<td>1.92508</td>
<td>1.91271</td>
<td>2.10535</td>
<td>2.25722</td>
</tr>
<tr>
<td>10^3</td>
<td>1.88691</td>
<td>1.86541</td>
<td>1.87687</td>
<td>2.06224</td>
<td>2.23727</td>
</tr>
<tr>
<td>3 \times 10^3</td>
<td>3.04751</td>
<td>1.84691</td>
<td>1.85895</td>
<td>2.05975</td>
<td>2.22914</td>
</tr>
</tbody>
</table>

Table 8
Nusselt Number (Nu) at y = 1
P = 0.71, N = 1, S = 1.3, N = 1, E = 0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 \times 10^3</td>
<td>2.2287</td>
<td>0.79077</td>
<td>0.75515</td>
<td>0.80654</td>
<td>0.77537</td>
</tr>
<tr>
<td>-10^3</td>
<td>0.8895</td>
<td>0.74957</td>
<td>0.73719</td>
<td>0.76576</td>
<td>0.76341</td>
</tr>
<tr>
<td>10^3</td>
<td>0.71137</td>
<td>0.68985</td>
<td>0.70132</td>
<td>0.72251</td>
<td>0.74325</td>
</tr>
<tr>
<td>3 \times 10^3</td>
<td>1.87233</td>
<td>0.67134</td>
<td>0.68340</td>
<td>0.72002</td>
<td>0.73506</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D'</th>
<th>I \times 10^2</th>
<th>II \times 10^3</th>
<th>III \times 10^3</th>
<th>IV \times 10^2</th>
<th>V \times 10^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>
Table 9
Nusselt Number (Nu) at y = -1
P=0.71, D' = 5x10², N1=0.5, Ec=0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3x10³</td>
<td>3.40374</td>
<td>5.35821</td>
<td>1.49189</td>
<td>1.25633</td>
<td>2.21026</td>
<td>2.57505</td>
<td>4.40531</td>
</tr>
<tr>
<td>-10³</td>
<td>2.06506</td>
<td>2.26406</td>
<td>1.83006</td>
<td>1.79221</td>
<td>1.92695</td>
<td>1.97100</td>
<td>2.17035</td>
</tr>
<tr>
<td>10³</td>
<td>1.88386</td>
<td>1.90953</td>
<td>1.91648</td>
<td>1.93148</td>
<td>1.89475</td>
<td>1.90956</td>
<td>1.99425</td>
</tr>
<tr>
<td>3x10⁵</td>
<td>3.04751</td>
<td>4.64906</td>
<td>1.66469</td>
<td>1.53494</td>
<td>2.14658</td>
<td>2.41201</td>
<td>3.85311</td>
</tr>
</tbody>
</table>

Table 10
Nusselt Number (Nu) at y = 1
P=0.71, D' = 5x10², N1=0.5, Ec=0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3x10³</td>
<td>2.2287</td>
<td>4.18505</td>
<td>0.31396</td>
<td>0.07787</td>
<td>1.03491</td>
<td>1.39982</td>
<td>3.23053</td>
</tr>
<tr>
<td>-10³</td>
<td>0.8895</td>
<td>1.08942</td>
<td>0.65331</td>
<td>0.61521</td>
<td>0.75111</td>
<td>0.79561</td>
<td>0.99491</td>
</tr>
<tr>
<td>10³</td>
<td>0.71137</td>
<td>0.73323</td>
<td>0.74211</td>
<td>0.75741</td>
<td>0.71919</td>
<td>0.71401</td>
<td>0.71872</td>
</tr>
<tr>
<td>3x10⁵</td>
<td>1.87233</td>
<td>3.47266</td>
<td>0.49154</td>
<td>0.36224</td>
<td>0.97108</td>
<td>1.23651</td>
<td>2.67817</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Sc</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>0.24</td>
<td>0.6</td>
<td>2.01</td>
</tr>
</tbody>
</table>
Table 11
Nusselt Number (Nu) at \( y = -1 \)
\( P = 0.71, D' = 5 \times 10^2, N = 1.0, Sc = 1.3 \)

<table>
<thead>
<tr>
<th>( \frac{G'}{\tau} )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3 \times 10^3)</td>
<td>2.2287</td>
<td>2.39535</td>
<td>2.96093</td>
<td>3.33025</td>
<td>3.62518</td>
<td>1.44437</td>
<td>7.70407</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>0.8895</td>
<td>1.77316</td>
<td>2.49852</td>
<td>2.91149</td>
<td>3.22679</td>
<td>1.41535</td>
<td>5.15435</td>
</tr>
<tr>
<td>(10^3)</td>
<td>0.7113</td>
<td>1.68537</td>
<td>2.43664</td>
<td>2.85837</td>
<td>3.17874</td>
<td>1.38935</td>
<td>5.08846</td>
</tr>
<tr>
<td>(3 \times 10^3)</td>
<td>1.8723</td>
<td>2.21977</td>
<td>2.83716</td>
<td>3.22401</td>
<td>3.52906</td>
<td>1.38237</td>
<td>7.57227</td>
</tr>
</tbody>
</table>

Table 12
Nusselt Number (Nu) at \( y = 1 \)
\( P = 0.71, D' = 5 \times 10^2, N = 1.0, Sc = 1.3 \)

<table>
<thead>
<tr>
<th>( \frac{G'}{\tau} )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3 \times 10^3)</td>
<td>3.40374</td>
<td>3.72486</td>
<td>4.41598</td>
<td>4.85773</td>
<td>5.20841</td>
<td>3.40373</td>
<td>11.7412</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>2.06503</td>
<td>3.10328</td>
<td>3.95441</td>
<td>4.43998</td>
<td>4.81121</td>
<td>2.06503</td>
<td>3.16229</td>
</tr>
<tr>
<td>(10^3)</td>
<td>1.88691</td>
<td>3.01576</td>
<td>3.89305</td>
<td>4.38756</td>
<td>4.76421</td>
<td>1.88691</td>
<td>2.89664</td>
</tr>
<tr>
<td>(3 \times 10^3)</td>
<td>3.04752</td>
<td>3.64982</td>
<td>4.29325</td>
<td>4.75289</td>
<td>5.11402</td>
<td>3.04751</td>
<td>11.2099</td>
</tr>
</tbody>
</table>

| N1 | 0.5 | 1.5 | 4.0 | 10 | 100 | 0.5 | 0.5 |
| Ec | 0.05 | 0.05 | 0.05 | 0.05 | 0.03 | 0.07 |
Table 13
Sherwood Number (Sh) at y = -1
P=0.71, N=1, Sc=1.3, N1=0.5, Ec=0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3x10³</td>
<td>-4.9827</td>
<td>-4.9432</td>
<td>-4.9366</td>
<td>-4.9391</td>
<td>-4.9309</td>
</tr>
<tr>
<td>-10³</td>
<td>-4.9525</td>
<td>-4.9334</td>
<td>-4.9299</td>
<td>-4.9316</td>
<td>-4.9276</td>
</tr>
<tr>
<td>10³</td>
<td>-4.8982</td>
<td>-4.9161</td>
<td>-4.9196</td>
<td>-4.9176</td>
<td>-4.9213</td>
</tr>
<tr>
<td>3x10³</td>
<td>-4.8742</td>
<td>-4.9085</td>
<td>-4.9157</td>
<td>-4.9111</td>
<td>-4.9183</td>
</tr>
</tbody>
</table>

Table 14
Sherwood Number (Sh) at y = 1
P=0.71, N=1, Sc=1.3, N1=0.5, Ec=0.5

<table>
<thead>
<tr>
<th>G/τ</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3x10³</td>
<td>-3.98201</td>
<td>-3.94292</td>
<td>-3.93622</td>
<td>-3.93837</td>
<td>-3.93028</td>
</tr>
<tr>
<td>3x10³</td>
<td>-3.87485</td>
<td>-3.90865</td>
<td>-3.91583</td>
<td>-3.91175</td>
<td>-3.91894</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>D¹</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>5x10²</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>10³</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3x10³</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5x10²</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5x10²</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Table 15
Sherwood Number (Sh) at \( y = -1 \)
\( P=0.71, D''=5 \times 10^2, \text{N}1=0.5, \text{Ec}=0.5 \)

<table>
<thead>
<tr>
<th>( \frac{G}{\tau} )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 \times 10^3</td>
<td>-4.9827</td>
<td>-4.9838</td>
<td>-4.9809</td>
<td>-4.9805</td>
<td>-1.3275</td>
<td>-2.5687</td>
<td>-7.4314</td>
</tr>
<tr>
<td>-10^3</td>
<td>-4.9525</td>
<td>-4.9526</td>
<td>-4.9523</td>
<td>-4.9522</td>
<td>-1.3221</td>
<td>-2.5551</td>
<td>-7.3841</td>
</tr>
<tr>
<td>10^3</td>
<td>-4.8983</td>
<td>-4.8992</td>
<td>-4.8972</td>
<td>-4.8971</td>
<td>-1.3118</td>
<td>-2.5297</td>
<td>-7.3013</td>
</tr>
<tr>
<td>3 \times 10^3</td>
<td>-4.8742</td>
<td>-4.8765</td>
<td>-4.8707</td>
<td>-4.8702</td>
<td>-1.3071</td>
<td>-2.5182</td>
<td>-7.2656</td>
</tr>
</tbody>
</table>

### Table 16
Sherwood Number (Sh) at \( y = 1 \)
\( P=0.71, D''=5 \times 10^2, \text{N}1=0.5, \text{Ec}=0.5 \)

<table>
<thead>
<tr>
<th>( \frac{G}{\tau} )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3 \times 10^3</td>
<td>-3.98201</td>
<td>-3.98263</td>
<td>-3.98097</td>
<td>-3.98075</td>
<td>-0.32735</td>
<td>-1.56847</td>
<td>-6.43036</td>
</tr>
<tr>
<td>-10^3</td>
<td>-3.95221</td>
<td>-3.95211</td>
<td>-3.95232</td>
<td>-3.05234</td>
<td>-0.32196</td>
<td>-1.55489</td>
<td>-6.38682</td>
</tr>
<tr>
<td>10^3</td>
<td>-3.89863</td>
<td>-3.89954</td>
<td>-3.89724</td>
<td>-3.89696</td>
<td>-0.31192</td>
<td>-1.52989</td>
<td>-6.30178</td>
</tr>
<tr>
<td>3 \times 10^3</td>
<td>-3.87485</td>
<td>-3.87752</td>
<td>-3.87081</td>
<td>-3.86999</td>
<td>-0.30724</td>
<td>-1.51847</td>
<td>-6.26656</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>1</td>
<td>2</td>
<td>-0.5</td>
<td>-0.8</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Sc</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>1.3</td>
<td>0.24</td>
<td>0.6</td>
<td>2.01</td>
</tr>
</tbody>
</table>
### Table 17
Sherwood Number (Sh) at \( y = -1 \)
\( P=0.71, D'=5 \times 10^2, N=1.0, Sc=1.3 \)

<table>
<thead>
<tr>
<th>( G/\tau )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-3 \times 10^3)</td>
<td>-4.9827</td>
<td>-4.9549</td>
<td>-4.9459</td>
<td>-4.9426</td>
<td>-4.9405</td>
<td>-4.9827</td>
<td>-5.0126</td>
</tr>
<tr>
<td>(-10^3)</td>
<td>-4.9526</td>
<td>-4.9392</td>
<td>-4.9349</td>
<td>-4.9339</td>
<td>-4.9324</td>
<td>-4.9525</td>
<td>-4.9665</td>
</tr>
<tr>
<td>(10^3)</td>
<td>-4.8982</td>
<td>-4.9105</td>
<td>-4.9143</td>
<td>-4.9155</td>
<td>-4.9164</td>
<td>-4.8983</td>
<td>-4.8861</td>
</tr>
<tr>
<td>(3 \times 10^3)</td>
<td>-4.8742</td>
<td>-4.8976</td>
<td>-4.9047</td>
<td>-4.9072</td>
<td>-4.9086</td>
<td>-4.8742</td>
<td>-4.8517</td>
</tr>
</tbody>
</table>

### Table 18
Sherwood Number (Sh) at \( y = 1 \)
\( P=0.71, D'=5 \times 10^2, N=1.0, Sc=1.3 \)

<table>
<thead>
<tr>
<th>( G/\tau )</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>N1</th>
<th>0.5</th>
<th>1.5</th>
<th>4.0</th>
<th>10</th>
<th>100</th>
<th>0.5</th>
<th>0.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ec</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
<td>0.03</td>
<td>0.07</td>
</tr>
</tbody>
</table>
molecular diffusivity larger $|\text{Nu}|$ at $y = -1$ and at $y = 1$, larger $|\text{Nu}|$ except $G = 5 \times 10^3$, where $|\text{Nu}|$ depreciates with $S_c \leq 1.3$, and enhances with $S_c \geq 2.01$. An increase in the to radiation parameter $N_1$ results in an enhancement in the rate of heat transfer at $y = \pm 1$. The influence of the dissipation is to enhance $|\text{Nu}|$ at $y = -1$ and depreciates is at $y = 1$ with $E_c \leq 0.2$ except at $G = -10^3$ where $|\text{Nu}|$ enhances at $E_c \geq 0.3$.

The rate of mass transfer at $y = \pm 1$ is represented in tables ( ) for values of governing parameters. An increase in $G < 0$ enhances $|\text{Sh}|$ while an increase $G > 0$ decreases $|\text{Sh}|$ at $y = \pm 1$. The variation of $\text{Sh}$ with $D^{-1}$ shows that lesser the permeability of the porous medium smaller $|\text{Sh}|$ in the cooling case and larger $|\text{Sh}|$ in the heating case. An increase in the heat source parameter $\alpha$ depreciates $|\text{Sh}|$ in the cooling case and enhances in the heating case at $y = \pm 1$. An increase in heat source parameter $\alpha > 0$ depreciates the rate of mass transfer at both walls. An increase in $S_c$ enhances the rate of mass transfer.

The variation of $|\text{Sh}|$ with $N$ shows that the molecular buoyancy force dominates over the thermal buoyancy force the rate of mass transfer enhances at $y = \pm 1$, when buoyancy forces act in the same direction while for the forces acting in opposite direction $|\text{Sh}|$ depreciates at both the walls. The variation $\text{Sh}$ with radiation parameter $N_1$ reveals that the rate of mass transfer depreciates with $N_1$ in the cooling case and enhances in the heating case. The variation of $\text{Sh}$ with Eckert Number $E_c$ indicates that at $y = -1$, $\text{Sh}$ enhances with increases to $E_c$ in the cooling case and depreciates in the heating case at both the boundaries. $(\frac{E_c}{E_c + 1})$

In general we found that the rate of mass transfer at the left boundary is greater than that at the right boundary $y = 1$. 92
6. REFERENCE


\[a_1 := \left( \frac{1 - 2b_1 - 2b_2}{2 \cosh[\sqrt{\nu}]} \right)\]

\[a_2 := \frac{-1}{2 \sinh[\sqrt{\nu}]}\]

\[a_3 := \frac{1}{2} - a_6 \sinh[\sqrt{\nu}]\]

\[b_1 := -\left( \frac{\pi P_1 K}{2 (\sqrt{\nu})^2} + \frac{P_1 \pi^2 E}{(\sqrt{\nu})^2} \right)\]

\[b_2 := \frac{(P_1 K \pi)}{2 (\sqrt{\nu})^2} - \frac{2}{(\sqrt{\nu})^4} \left( \frac{(P_1 K \pi)}{2} + P_1 \pi^2 E \right)\]

\[a_4 := -\frac{1}{2} - a_5 \cosh[\sqrt{\nu}] - a_7 - a_8\]

\[a_5 = a_6 = a_7 = a_8 := 0\]

\[a_9 := -\left( (a_{11} \cosh[\sqrt{\nu}] + a_{13} + a_{14} + a_{16}) / \cosh[\sqrt{L}] \right)\]

\[a_{10} := -\left( (a_{12} \sinh[\sqrt{\nu}] + a_{15}) / \sinh[\sqrt{L}] \right)\]

\[a_{11} := \left( (G a_1 + G J a_5) / \left( (\sqrt{\nu})^2 - (\sqrt{L})^2 \right) \right)\]

\[a_{12} := -\left( (G a_2 + G J a_6) / \left( (\sqrt{\nu})^2 - (\sqrt{L})^2 \right) \right)\]

\[a_{13} := \left( G J a_7 \right) / \left( (\sqrt{L})^2 \right), \quad a_{15} := \left( G J a_3 \right) / \left( (\sqrt{L})^2 \right)\]

\[a_{14} := \left( 12 G J a_7 + G b_2 + G J a_9 \right) / \left( (\sqrt{L})^4 + (\sqrt{L})^2 \right)\]
\[ a_{17} := -\frac{1}{\cosh(\sqrt{v})} \left[ \frac{P_1 K (\sqrt{L})^2 - (\sqrt{v})^2}{4 (\sqrt{v})^2} \right] \left\{ 4 \pi P_1 E_c (\sqrt{L})^2 / \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right)^2 a_9 \cosh(\sqrt{L}) + \left( \frac{\pi P_1 E_c a_{11}}{4 (\sqrt{v})^2} \right) \cosh(\sqrt{v}) + \left( \frac{\pi P_1 E_c a_{11}}{2} \right) \cosh(\sqrt{v}) \right\} \]

\[ + \left( \frac{(P_1 K - \pi P_1 E_c) a_{11}}{2 \sqrt{v}} \right) \sinh(\sqrt{L}) + \left( \frac{(P_1 K + 8 \pi P_1 E_c) a_{13}}{2} \right) \frac{1}{(\sqrt{v})^2} + \left( \frac{(P_1 K + 4 \pi P_1 E_c) a_{14}}{4 (\sqrt{v})^2} \right) \frac{1}{(\sqrt{v})^6} \]

\[ + \left( \frac{(P_1 K + 8 \pi P_1 E_c) a_{13}}{2} \right) \frac{1}{(\sqrt{v})^4} + \frac{P_1 K a_{16}}{(\sqrt{v})^2} \right\} \right) \]

\[ a_{15} := \left( \frac{GJ a_3}{(\sqrt{L})^2} \right) \]

\[ a_{16} := \left( \frac{24 GJ a_7 + 2 (G b_1 + G J a_9) + G b_2 + G J a_4}{(\sqrt{L})^6 + (\sqrt{L})^4 + (\sqrt{L})^2} \right) \]
\[ a_{18} := \frac{-1}{\text{Sinh}[\sqrt{v}]} \left\{ -\left( P_1 K \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right) - 4 \pi P_1 E_c \left( (\sqrt{v})^2 \right) / \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right)^2 a_{10} \text{Sinh}[\sqrt{v}] - \left( \frac{\pi P_1 E_c a_{12}}{4 \left( \sqrt{v} \right)^2} \right) \text{Sinh}[\sqrt{v}] - \left( \frac{2 \pi P_1 E_c \sqrt{L} a_{16}}{\left( (\sqrt{L})^2 - (\sqrt{v})^2 \right)} \right) \text{Cosh}[\sqrt{L}] - \left( \frac{\pi P_1 E_c a_{12}}{2} \right) \text{Sinh}[\sqrt{v}] - \left( (P_1 K - \pi P_1 E_c) / (2 \sqrt{L}) \right) a_{12} \text{Cosh}[\sqrt{v}] \right\} \] 

\[ a_{19} := \left\{ P_1 K \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right) - 4 \pi P_1 E_c \left( (\sqrt{L})^2 \right) / \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right)^2 a_9 \right\} \] 

\[ a_{20} := \left\{ P_1 K \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right) - 4 \pi P_1 E_c \left( (\sqrt{L})^2 \right) / \left( (\sqrt{L})^2 - (\sqrt{v})^2 \right)^2 a_{10} \right\} \] 

\[ a_{21} := \frac{\pi P_1 E_c a_{11}}{4 \left( \sqrt{v} \right)^2} \] 

\[ a_{22} := \frac{\pi P_1 E_c a_{12}}{4 \left( \sqrt{v} \right)^2} \]
\[
\begin{align*}
\alpha_{23} & := \frac{(2 \pi P_1 E_c \sqrt{L} a_{10})}{(\sqrt{L})^2 - (\sqrt{V})^2} \\
\alpha_{24} & := \frac{(2 \pi P_1 E_c \sqrt{L} a_9)}{(\sqrt{L})^2 - (\sqrt{V})^2} \\
\alpha_{25} & := \frac{\pi P_1 E_c a_{11}}{2} \\
\alpha_{26} & := \frac{\pi P_1 E_c a_{12}}{2} \\
\alpha_{27} & := \frac{(P_1 K - \pi P_1 E_c) a_{12}}{2 \sqrt{V}} \quad \alpha_{28} := \frac{(P_1 K - \pi P_1 E_c) a_{11}}{2 \sqrt{V}} \\
\alpha_{29} & := -\left\{\frac{(P_1 K + 8 \pi P_1 E_c) a_{13}}{(\sqrt{V})^2} \right\} \\
\alpha_{30} & := -\left\{\frac{(12 (P_1 K + 8 \pi P_1 E_c) a_{13})}{(\sqrt{V})^4} + \frac{(P_1 K + 4 \pi P_1 E_c) a_{14}}{(\sqrt{V})^2}\right\} \\
\alpha_{31} & := -\left\{\frac{(P_1 K + 2 \pi P_1 E_c) a_{15}}{(\sqrt{V})^2}\right\} \\
\alpha_{32} & := -\left\{\frac{(48 (P_1 K + 8 \pi P_1 E_c) a_{13})}{(\sqrt{V})^6} + \frac{(P_1 K + 4 \pi P_1 E_c) 2 a_{14}}{(\sqrt{V})^4} + \frac{P_1 K a_{16}}{(\sqrt{V})^2}\right\} \\
\alpha_{33} & = a_{34} = a_{35} = a_{36} = a_{37} = a_{38} = 0 \\
\alpha_{39} & = a_{40} = a_{41} = a_{42} = a_{43} = a_{44} = 0 \\
\alpha_{45} & := -\left\{(a_{49} + a_{34}) \sinh[\sqrt{V}] + a_{36} \sinh[\sqrt{L}] + a_{37} \cosh[\sqrt{L}] + a_{39} \cosh[\sqrt{V}] + a_{43}\right\}
\end{align*}
\]
\begin{align*}
  a_{46} & := - \left( (a_{33} + a_{47}) \cosh[\sqrt{v}] + a_{35} \cosh[\sqrt{L}] + a_{38} \sinh[\sqrt{L}] + a_{40} \sinh[\sqrt{v}] + a_{41} + a_{42} + a_{44} \right) \\
  a_{47} & = a_{48} \\
  a_{49} & := - \frac{1}{\cosh[\sqrt{v}]} \left( (a_{61} + a_{51}) \cosh[\sqrt{v}] + a_{60} \sinh[\sqrt{v}] + (a_{53} + a_{57}) \cosh[\sqrt{L}] + a_{56} \sinh[\sqrt{L}] + a_{53} + a_{64} + a_{66} + a_{68} \right) \\
  a_{50} & := \frac{1}{\sinh[\sqrt{v}]} \left( -(a_{62} + a_{52}) \sinh[\sqrt{v}] - a_{54} \cosh[\sqrt{v}] - (a_{54} + a_{58}) \sinh[\sqrt{L}] - a_{55} \cosh[\sqrt{L}] - a_{65} - a_{67} \right) \\
  a_{51} & := \left( - (G (a_{17} + a_{21} + Ja_{33})) / \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 + \right) \\
 & \quad \left( G (a_{28} + Ja_{40}) 2 \sqrt{v} / \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 \right) - \left( G (a_{25} + Ja_{47}) \left( 6 (\sqrt{v})^2 + 2 (\sqrt{L})^2 \right) / \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 \right) \\
\end{align*}
\[ a_{52} := \left( - (G (a_{18} + a_{22} + J a_{34})) \right) \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 + \]
\[ \frac{G (a_{27} + J a_{39})}{2 \sqrt{v}} \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 - \]
\[ \frac{G (a_{26} + J a_{48})}{6 (\sqrt{v})^2 + 2 (\sqrt{L})^2} \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 \]
\[ a_{53} := - \frac{G (a_{24} + J a_{38})}{8 (\sqrt{L})^3} \]
\[ a_{54} := - \frac{G (a_{23} + J a_{37})}{8 (\sqrt{L})^3} \]
\[ a_{55} := - \frac{G (a_{20} + J a_{36})}{2 \sqrt{L}} + \frac{G (a_{23} + J a_{37})}{4 (\sqrt{L})^2} \]
\[ a_{56} := - \frac{G (a_{19} + J a_{35})}{2 \sqrt{L}} + \frac{G (a_{24} + J a_{38})}{4 (\sqrt{L})^2} \]
\[ a_{57} := - \frac{G (a_{24} + J a_{38})}{4 \sqrt{L}} \]
\[ a_{58} := - \frac{G (a_{23} + J a_{37})}{4 \sqrt{L}} \]
\[ a_{59} := - (G (a_{27} + J a_{39})) \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 + \]
\[ 4 G (a_{26} + J a_{48}) \sqrt{v} \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 \]
\[ a_{60} := - (G (a_{28} + J a_{40})) \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 + \]
\[ 4 G (a_{25} + J a_{47}) \sqrt{v} \bigg/ \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)^2 \]
a_{61} := -\left( G (a_{25} + J a_{47}) \right) / \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)

a_{62} := -\left( G (a_{26} + J a_{48}) \right) / \left( (\sqrt{v})^2 - (\sqrt{L})^2 \right)

a_{63} := -\frac{G J a_{41}}{(\sqrt{L})^2}

a_{64} := -\frac{\pi^2 \Delta}{4 (\sqrt{L})^2} + \frac{G (a_{29} + J a_{42})}{(\sqrt{L})^2} + \frac{30 G J a_{41}}{(\sqrt{L})^4}

a_{65} := \frac{G J a_{43}}{(\sqrt{L})^2}

a_{66} := -\frac{3 \pi^2 \Delta}{(\sqrt{L})^2} + \frac{12 G (a_{29} + J a_{42})}{(\sqrt{L})^2} + \frac{360 G J a_{41}}{(\sqrt{L})^6} + \frac{\pi^2 \Delta}{2 (\sqrt{L})^2} + \frac{G (a_{30} + J a_{44})}{(\sqrt{L})^2}

a_{67} := \frac{G (a_{31} + J a_{45})}{(\sqrt{L})^2} + \frac{6 G J a_{43}}{(\sqrt{L})^4}

a_{68} := -\frac{6 \pi^2 \Delta}{(\sqrt{L})^4} + \frac{24 G (a_{29} + J a_{42})}{(\sqrt{L})^4} + \frac{720 G J a_{41}}{(\sqrt{L})^8} + \frac{\pi^2 \Delta}{(\sqrt{L})^2} + \frac{2 G (a_{30} + J a_{44})}{(\sqrt{L})^2} - \frac{\pi^2 \Delta}{4 (\sqrt{L})^2} + \frac{G (a_{32} + J a_{46})}{(\sqrt{L})^2}

a_{69} := \frac{-1}{\text{Cosh}[\sqrt{v}]} \left\{ (a_{71} + a_{93} + a_{87}) \text{Cosh}[\sqrt{v}] + (a_{82} + a_{96}) \text{Sinh}[\sqrt{v}] + (a_{73} + a_{77}) \text{Cosh}[\sqrt{L}] + (a_{76} + a_{80}) \text{Sinh}[\sqrt{L}] + a_{99} \text{Cosh}[2 \sqrt{L}] + a_{91} \text{Cosh}[2 \sqrt{v}] + a_{93} \text{Cosh}[\sqrt{v} + \sqrt{L}] + a_{94} \text{Cosh}[\sqrt{v} - \sqrt{L}] + a_{97} + a_{98} + a_{100} + a_{102} \right\}
\[a_{69} := \frac{-1}{\cosh[\sqrt{v}]} \left\{ (a_{71} + a_{83} + a_{87}) \cosh[\sqrt{v}] + 
(a_{72} + a_{85}) \sinh[\sqrt{v}] + (a_{73} + a_{77}) \cosh[\sqrt{L}] + 
(a_{76} + a_{89}) \sinh[\sqrt{L}] + a_{73} \cosh[2 \sqrt{L}] + 
(a_{91} \cosh[2 \sqrt{v}] + a_{93} \cosh[\sqrt{v} + \sqrt{L}] + 
(a_{94} \cosh[\sqrt{v} - \sqrt{L}] + a_{97} + a_{90} + a_{100} + a_{102}) \right\} \]

\[a_{70} := \frac{-1}{\sinh[\sqrt{v}]} \left\{ (a_{72} + a_{84} + a_{88}) \sinh[\sqrt{v}] + 
(a_{81} + a_{85}) \cosh[\sqrt{v}] + 
(a_{75} + a_{79}) \sinh[\sqrt{L}] + 
(a_{75} + a_{79}) \cosh[\sqrt{L}] + a_{90} \sinh[2 \sqrt{L}] + 
(a_{92} \sinh[2 \sqrt{v}] + a_{96} \sinh[\sqrt{v} + \sqrt{L}] + 
(a_{96} \sinh[\sqrt{v} - \sqrt{L}] + a_{99} + a_{101}) \right\} \]

\[a_{71} := \left( \frac{P_1 K a_{60} + 4 \sqrt{v} P_1 E_c a_{11} a_{14}}{8 (\sqrt{v})^3} \right) + \frac{3 P_1 E_c a_{11} a_{13} - P_1 K a_{61}}{8 (\sqrt{v})^3} \]

\[a_{72} := \left( \frac{P_1 K a_{59} + 4 \sqrt{v} P_1 E_c a_{12} a_{14}}{8 (\sqrt{v})^3} \right) + \frac{3 P_1 E_c a_{12} a_{13} - P_1 K a_{62}}{8 (\sqrt{v})^3} \]
\[ a_{73} := \]
\[ \left( \frac{\sqrt{v}}{\frac{\sqrt{v}}{2}} \left( a_{49} + a_{53} \right) + 2 \sqrt{v} \frac{E_{c} a_{10} a_{15}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} - 2 \sqrt{L} \left( \frac{\sqrt{v}}{2} \left( a_{56} + 4 \sqrt{v} \frac{E_{c} a_{9} a_{14}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right) \right) 
\]
\[ + \frac{P_{1}K a_{57}}{2} \left( \frac{8 (\sqrt{v})^2}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right)^3 - \frac{2}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} + 8 \frac{P_{1}K a_{58}}{2} \left( \frac{24 \sqrt{L}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right)^3 - \frac{48 \sqrt{L}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right) \]

\[ a_{74} := \]
\[ \left( \frac{\sqrt{v}}{\frac{\sqrt{v}}{2}} \left( a_{50} + a_{54} \right) + 2 \sqrt{v} \frac{E_{c} a_{9} a_{15}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} - 2 \sqrt{L} \left( \frac{\sqrt{v}}{2} \left( a_{56} + 4 \sqrt{v} \frac{E_{c} a_{10} a_{14}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right) \right) 
\]
\[ + \frac{P_{1}K a_{57}}{2} \left( \frac{8 (\sqrt{v})^2}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right)^3 - \frac{2}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} + 8 \frac{P_{1}K a_{58}}{2} \left( \frac{24 \sqrt{L}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right)^3 - \frac{48 \sqrt{L}}{\left( \sqrt{L} \right)^2 - (\sqrt{v})^2} \right) \]
\[
a_{75} := \left( (P_1 K a_{55} + 4 \sqrt{v} P_1 E_c a_{10} a_{14}) / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \\
+ 8 \sqrt{L} P_1 E_c a_{10} a_{13} \\
\left[ \left( \frac{24 (\sqrt{L})^2}{((\sqrt{L})^2 - (\sqrt{v})^2)^3} - 6 / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \right] \\
- \left( 4 \sqrt{L} P_1 K a_{57} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{76} := \left( (P_1 K a_{56} + 4 \sqrt{v} P_1 E_c a_{9} a_{14}) / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \\
+ \left( 8 \sqrt{L} P_1 E_c a_{9} a_{13} \right) * \\
\left[ \left( \frac{24 (\sqrt{L})^2}{((\sqrt{L})^2 - (\sqrt{v})^2)^3} - 6 / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \right] \\
- \left( 4 \sqrt{L} P_1 K a_{57} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{77} := \left( (P_1 K a_{57}) / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \\
- \left( 48 (\sqrt{L})^2 P_1 E_c a_{9} a_{13} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{78} := \left( (P_1 K a_{58}) / ((\sqrt{L})^2 - (\sqrt{v})^2) \right) \\
- \left( 48 (\sqrt{L})^2 P_1 E_c a_{10} a_{13} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{79} := \left( 8 \sqrt{L} P_1 E_c a_{10} a_{13} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{80} := \left( 8 \sqrt{L} P_1 E_c a_{9} a_{13} \right) / ((\sqrt{L})^2 - (\sqrt{v})^2)
\]

\[
a_{81} := \left( \frac{1}{2} (P_1 K a_{92} + 2 \sqrt{v} P_1 E_c a_{11} a_{15}) - \\
(P_1 K a_{59} + 4 \sqrt{v} P_1 E_c a_{12} a_{14}) / (4 (\sqrt{v})^2) \right) \\
+ \frac{P_1 K a_{62}}{4 (\sqrt{v})^2} - \frac{3 P_1 E_c a_{12} a_{13}}{(\sqrt{v})^3}
\]
\[ a_{62} := \left( \frac{1}{2} \left( P_1 K a_{51} + 2 \sqrt{v} P_1 E_c a_{12} a_{14} \right) - \right. \]
\[ \left. \left( P_1 K a_{60} + 4 \sqrt{v} P_1 E_c a_{11} a_{14} \right) / \left( 4 (\sqrt{v})^2 \right) \right. \]
\[ + \frac{P_1 K a_{61}}{4 (\sqrt{v})^2} - \frac{3 P_1 E_c a_{11} a_{13}}{(\sqrt{v})^3} \right) \]
\[ a_{63} := \left( P_1 K a_{60} + 4 \sqrt{v} P_1 E_c a_{11} a_{14} \right) / \left( 4 \sqrt{v} \right) - \]
\[ \frac{P_1 K a_{61}}{4 \sqrt{v}} + \frac{3 P_1 E_c a_{11} a_{13}}{(\sqrt{v})^2} \right) \]
\[ a_{64} := \left( P_1 K a_{69} + 4 \sqrt{v} P_1 E_c a_{12} a_{14} \right) / \left( 4 \sqrt{v} \right) - \]
\[ \frac{P_1 K a_{62}}{4 \sqrt{v}} + \frac{3 P_1 E_c a_{12} a_{13}}{(\sqrt{v})^2} \right) \]
\[ a_{65} := \left( \frac{P_1 K a_{62}}{6} - \frac{2 P_1 E_c a_{12} a_{13}}{\sqrt{v}} \right) \]
\[ a_{66} := \left( \frac{P_1 K a_{61}}{6} - \frac{2 P_1 E_c a_{11} a_{13}}{\sqrt{v}} \right) \]
\[ a_{67} := P_1 E_c a_{11} a_{13} \]
\[ a_{68} := P_1 E_c a_{12} a_{13} \]
\[ a_{69} := \left( P_1 E_c (\sqrt{L})^2 \left( a_9^2 + a_{10}^2 \right) \right) \left( 2 \left( 4 (\sqrt{L})^2 - (\sqrt{v})^2 \right) \right) \]
\[ a_{90} := \left( P_1 E_c (\sqrt{L})^2 a_9 a_{10} \right) \left( 4 (\sqrt{L})^2 - (\sqrt{v})^2 \right) \]
\[ \begin{align*}
a_{91} & := \frac{P_1 E_c (a_{11}^2 + a_{12}^2)}{6} \\
a_{92} & := \frac{P_1 E_c a_{11} a_{12}}{3} \\
a_{93} & := \left( P_1 E_c \sqrt{v} \sqrt{L} (a_9 a_{11} + a_{10} a_{12}) \right) / \left( (\sqrt{L})^2 + 2 \sqrt{L} \sqrt{v} \right) \\
a_{94} & := \left( P_1 E_c \sqrt{v} \sqrt{L} (a_{10} a_{12} - a_9 a_{11}) \right) / \left( (\sqrt{L})^2 - 2 \sqrt{L} \sqrt{v} \right) \\
a_{95} & := \left( P_1 E_c \sqrt{v} \sqrt{L} (a_{10} a_{11} + a_9 a_{12}) \right) / \left( (\sqrt{L})^2 + 2 \sqrt{L} \sqrt{v} \right) \\
a_{96} & := \left( P_1 E_c \sqrt{v} \sqrt{L} (a_{10} a_{11} - a_9 a_{12}) \right) / \left( (\sqrt{L})^2 - 2 \sqrt{L} \sqrt{v} \right) \\
a_{97} & := -\frac{16 P_1 E_c a_{13}^2}{(\sqrt{v})^2} \\
a_{98} & := -\frac{480 P_1 E_c a_{13}^2}{(\sqrt{v})^4} - \frac{16 P_1 E_c a_{13} a_{14}}{(\sqrt{v})^2} \\
a_{99} & := -\frac{8 P_1 E_c a_{13} a_{15}}{(\sqrt{v})^2} \\
a_{100} & := \left( \frac{5760 P_1 E_c a_{13}^2}{(\sqrt{v})^6} - \frac{192 P_1 E_c a_{13} a_{14}}{(\sqrt{v})^4} - \frac{4 P_1 E_c a_{14}^2}{(\sqrt{v})^2} \right) \\
a_{101} & := -\frac{48 P_1 E_c a_{13} a_{15}}{(\sqrt{v})^4} - \frac{4 P_1 E_c a_{15} a_{14}}{(\sqrt{v})^2} \\
\end{align*} \]
\[ a_{101} := -\frac{48 P_1 E_c a_{13} a_{15}}{(\sqrt{V})^4} - \frac{4 P_1 E_c a_{15} a_{14}}{(\sqrt{V})^2} \]

\[ a_{102} := \left\{ \frac{11520 P_1 E_c a_{13}^2}{(\sqrt{V})^8} - \frac{384 P_1 E_c a_{13} a_{14}}{(\sqrt{V})^6} - \frac{8 P_1 E_c a_{14}^2}{(\sqrt{V})^4} \right\} \]

\[ -\frac{P_1 E_c}{(\sqrt{V})^2} \left\{ \frac{1}{2} \left( (\sqrt{L})^2 (a_{10}^2 - a_{9}^2) \right) + \frac{1}{2} \left( (\sqrt{L})^2 (a_{12}^2 - a_{11}^2) \right) + a_{15}^2 \right\} \]

\[ a_{103} := -\{ (a_{117} + a_{121}) \cosh[\sqrt{V}] + (a_{106} + a_{116} + a_{120}) \sinh[\sqrt{V}] \}
\]

\[ (a_{109} + a_{113}) \cosh[\sqrt{L}] + (a_{112} + a_{108}) \sinh[\sqrt{L}] \]

\[ + a_{124} \sinh[2 \sqrt{L}] + a_{126} \sinh[2 \sqrt{V}] + a_{129} \sinh[\sqrt{V} + \sqrt{L}] + a_{130} \sinh[\sqrt{V} - \sqrt{L}] \} \]

\[ a_{104} := -\{ (a_{105} + a_{115} + a_{119}) \cosh[\sqrt{V}] + (a_{118} + a_{122}) \sinh[\sqrt{V}] \}
\]

\[ (a_{107} + a_{111}) \cosh[\sqrt{L}] + (a_{110} + a_{114}) \sinh[\sqrt{L}] + a_{123} \cosh[2 \sqrt{L}] + a_{125} \cosh[2 \sqrt{V}] + a_{127} \cosh[\sqrt{V} + \sqrt{L}] + a_{128} \cosh[\sqrt{V} - \sqrt{L}] + a_{131} + a_{132} + a_{134} + a_{135} \}
\]

\[ a_{105} = a_{106} = a_{107} = a_{108} = a_{109} = 0 \]

\[ a_{105} = a_{106} = a_{107} = a_{108} = a_{109} = 0 \]

\[ a_{110} = a_{111} = a_{112} = a_{113} = a_{114} = 0 \]

\[ a_{115} = a_{116} = a_{117} = a_{118} = a_{119} = a_{120} = 0 \]

\[ a_{121} = a_{122} = a_{123} = a_{124} = a_{125} = a_{126} = a_{127} = a_{128} = 0 \]

\[ a_{129} = a_{130} = a_{131} = a_{132} = a_{133} = a_{134} = a_{135} = a_{136} = 0 \]

\[ b_1 = (a_q + a_{11}) L_2 \text{sh}(L_2) + (a_{10} + a_{12}) L_2 \text{ch}(L_2) + 4 a_3 + 2 a_{14} + a_{15} \]

\[ b_2 = -(a_q + a_{11}) L_2 \text{sh}(L_2) + (a_{10} + a_{12}) L_2 \text{ch}(L_2) - 4 a_3 - 2 a_{14} + a_{15} \]

\[ b_3 = a_1 L_1 \text{sh}(L_1) + a_2 L_1 \text{ch}(L_1) \]
\[ b_4 = -a_1 \lambda_1 Sh(\lambda_1) + a_2 \lambda_2 Ch(\lambda_1) \]
\[ b_5 = ((\lambda_1 (a_{17} + a_{21} + a_{22}) + a_{29}) Sh(\lambda_1) + ((\lambda_1 (a_{18} + a_{22} + a_{28}) + a_{29}) Ch(\lambda_1) + \]
\[ + ((\lambda_2 (a_{19} + a_{23}) + a_{29} + 2a_{29}) Sh(\lambda_2) + ((\lambda_2 (a_{20} + a_{24} + a_{26}) + a_{29}) Ch(\lambda_2) + \]
\[ + 4a_{29} + 2a_{30} + a_{31} \]
\[ b_6 = -((\lambda_1 (a_{17} + a_{21} - a_{29}) - a_{28}) Sh(\lambda_1) + ((\lambda_1 (a_{18} + a_{22} - a_{29}) + a_{27}) Ch(\lambda_1) + \]
\[ + ((\lambda_2 (a_{20} - a_{24} + a_{26}) + a_{23} - 2a_{29}) Ch(\lambda_2) + (\lambda_2 (a_{20} + a_{23} - a_{25}) + a_{23}) + a_{24} + 2a_{29}) Ch(\lambda_2) + \]
\[ - 4a_{29} - 2a_{30} + a_{31} \]
\[ b_7 = \lambda_1 (a_5 Sh(\lambda_1) + a_6 Ch(\lambda_1)) + 4a_7 + 2a_8 + a_9 \]
\[ b_8 = \lambda_1 (a_6 Ch(\lambda_1) - a_5 Sh(\lambda_1)) - 4a_7 - 2a_8 + a_9 \]
\[ b_9 = a_2 \lambda_1 Sh(\lambda_1) + a_6 \lambda_2 Ch(\lambda_1) + 4a_7 + 2a_8 + a_3 \]
\[ b_{10} = -a_2 \lambda_1 Sh(\lambda_1) + a_6 \lambda_2 Ch(\lambda_1) - 4a_7 - 2a_8 + a_3 \]
\[ b_{11} = (\lambda_1 (a_{21} + a_{39}) + a_{40} + 2a_{44}) Sh(\lambda_1) + (\lambda_1 (a_{23} + a_{40} + a_{44}) + a_{39} + 2a_{47}) Ch(\lambda_1) + \]
\[ + (\lambda_1 (a_{35} + a_{38} + a_{17})) Ch(\lambda_2) + (\lambda_2 (a_{36} + a_{37}) + a_{38}) Sh(\lambda_2) + 6a_{41} + 4a_{42} + 3a_{43} \]
\[ + 2a_{44} + a_{45} \]
\[ b_{12} = (\lambda_1 (-a_{33} + a_{39} - a_{47}) - a_{40} + 2a_{48}) Sh(\lambda_1) + (\lambda_1 (a_{34} - a_{40} + a_{48}) + a_{39} - 2a_{47}) Ch(\lambda_1) + \]
\[ + (\lambda_2 (-a_{35} + a_{37}) - a_{38}) Sh(\lambda_2) + (\lambda_2 (-a_{38} + a_{35}) + a_{37}) Ch(\lambda_2) - 6a_{41} - 4a_{42} + 3a_{43} \]
\[ - 2a_{44} + a_{45} \]
\[b_{13} = (\lambda_1 (a_{21} + a_{42})) Ch(\lambda_1) + (\lambda_1 (a_{33} + a_{43})) Sh(\lambda_1) + (\lambda_2 (a_{50} + a_{54} + a_{56} + a_{58}) + a_{51})
+ 2a_{61}) Ch(\lambda_2) + (\lambda_2 (a_{55} + a_{35}) + a_{61}) + a_{56} + 2a_{62}) Sh(\lambda_2) + 6a_{63} + 4a_{64} + 2a_{65} + a_{67}
\]

\[b_{14} = (\lambda_1 (a_{21} + a_{42})) Ch(\lambda_1) + (-\lambda_1 (a_{33} + a_{49})) Sh(\lambda_1) + (\lambda_2 (a_{50} + a_{54} - a_{56} + a_{38}) +
+ a_{55} - 2a_{61}) Ch(\lambda_2) + (\lambda_2 (a_{55} - a_{61}) - a_{56} + 2a_{62}) Sh(\lambda_2) - a_{63} - 4a_{64} - 2a_{65} + a_{67}
\]

\[b_{15} = (\lambda_1 (a_{69} + a_{71} + a_{81} + a_{83} + a_{85} + a_{87}) + a_{92} + 2a_{44} + 3a_{86} + 4a_{88}) Sh(\lambda_1) +
+ (\lambda_1 (a_{69} + a_{71} + a_{84} + a_{86} + a_{88}) + a_{81} + 2a_{43} + 3a_{85} + 4a_{87}) Ch(\lambda_1)
+ (\lambda_2 (a_{73} + a_{75} + a_{77} + a_{79}) + a_{76} + 2a_{78} + 3a_{80})) Sh(\lambda_2) +
+ (\lambda_1 (a_{74} + a_{76} + a_{87} + a_{90}) + a_{75} + 2a_{77} + 3a_{79}) Ch(\lambda_2) + (2\lambda_2 a_{89}) Sh(2\lambda_2) + (2\lambda_2 a_{90}) Ch(2\lambda_2) +
+ (2\lambda_1 a_{91}) Sh(2\lambda_1) + (2\lambda_1 a_{92}) Ch(2\lambda_1) + ((\lambda_1 + \lambda_2) a_{93}) Sh(\lambda_1 + \lambda_2) + ((\lambda_1 - \lambda_2) a_{94}) Sh(\lambda_1 - \lambda_2)
+ ((\lambda_1 + \lambda_2) a_{95}) Ch(\lambda_1 + \lambda_2) + ((\lambda_1 - \lambda_2) a_{96}) Ch(\lambda_1 - \lambda_2) + 6a_{97} + 4a_{98} + 3a_{99} + 2a_{100} + a_{101}
\]

\[b_{16} = (\lambda_1 (-a_{69} - a_{71} + a_{81} - a_{83}) + a_{92} + 2a_{44} - 3a_{86} + 4a_{88}) Sh(\lambda_1) +
+ (\lambda_1 (a_{69} + a_{71} - a_{82} + a_{84} - a_{86} + a_{88}) + a_{81} - 2a_{83} + 3a_{85} - 4a_{87}) Ch(\lambda_1)
+ (\lambda_2 (-a_{73} + a_{75} - a_{77} + a_{79}) - a_{76} + 2a_{78} - 3a_{80}) Sh(\lambda_2) +
+ (\lambda_2 (a_{74} + a_{76} + a_{87} + a_{90}) + a_{75} - 2a_{77} + 3a_{79}) Ch(\lambda_2) + (2\lambda_2 a_{89}) Sh(2\lambda_2) + (2\lambda_2 a_{90}) Ch(2\lambda_2) +
+ (2\lambda_1 a_{91}) Sh(2\lambda_1) + (2\lambda_1 a_{92}) Ch(2\lambda_1) - ((\lambda_1 + \lambda_2) a_{93}) Sh(\lambda_1 + \lambda_2) - ((\lambda_1 - \lambda_2) a_{94}) Sh(\lambda_1 - \lambda_2)
+ ((\lambda_1 + \lambda_2) a_{95}) Ch(\lambda_1 + \lambda_2) + ((\lambda_1 - \lambda_2) a_{96}) Ch(\lambda_1 - \lambda_2) + 6a_{97} + 4a_{98} + 3a_{99} + 2a_{100} + a_{101}
\]
\[ b_{17} = (\lambda_1 (a_{105} + a_{106} + a_{122} + a_{84} + a_{118} + a_{116} + a_{120}) + a_{121} + 2a_{83} + 3a_{117} + 4a_{115} + 3a_{119})Ch(\lambda_1) + \]
\[ + (\lambda_1 (a_{121} + a_{83} + a_{117} + a_{115} + a_{119}) + a_{122} + 2a_{84} + 3a_{118} + 4a_{116} + 2a_{120})Sh(\lambda_1) + \]
\[ + (\lambda_2 (a_{108} + a_{114} + a_{112}) + a_{113} + 2a_{111} + 3a_{109} + 3a_{110})Ch(\lambda_2) + \]
\[ + (\lambda_1 (a_{114} + a_{107} + 2a_{112}) + a_{114} + 2a_{112})Sh(\lambda_2) + (2\lambda_2 a_{112})Sh(2\lambda_2) + (2\lambda_2 a_{124})Ch(2\lambda_2) + \]
\[ + (2\lambda_2 a_{125})Sh(2\lambda_1) + (2\lambda_2 a_{126})Ch(2\lambda_1) + ((\lambda_1 + \lambda_2) a_{127})Sh(\lambda_1 + \lambda_2) + ((\lambda_1 - \lambda_2) a_{128})Sh(\lambda_1 - \lambda_2) \]
\[ + ((\lambda_1 + \lambda_2) a_{129})Ch(\lambda_1 + \lambda_2) + ((\lambda_1 - \lambda_2) a_{130})Ch(\lambda_1 - \lambda_2) + 8a_{131} + 6a_{132} + 5a_{133} + 4a_{134} + 3a_{135} + : \]

\[ b_{18} = (\lambda_1 (-a_{105} + a_{122} - a_{83} + a_{117} - a_{115} + a_{119}) - a_{122} + 2a_{84} - 3a_{118} + 4a_{116} + 2a_{120})Sh(\lambda_1) + \]
\[ + (\lambda_1 (a_{107} - a_{122} + a_{84} - a_{114} + a_{116} + a_{120}) + a_{121} - 2a_{83} + 3a_{117} - 4a_{115} + 3a_{119} + 3a_{111})Ch(\lambda_1) \]
\[ + (\lambda_2 (-a_{108} - a_{111} + a_{109} + a_{110}) - a_{114} + 2a_{112} + 3a_{109})Sh(\lambda_2) + \]
\[ + (\lambda_2 (a_{106} - a_{114} + a_{112}) + a_{115} - 2a_{111} + 3a_{109})Ch(\lambda_2) - (2\lambda_2 a_{112})Sh(2\lambda_2) + (2\lambda_2 a_{124})Ch(2\lambda_2) - \]
\[ - (2\lambda_2 a_{125})Sh(2\lambda_1) + (2\lambda_2 a_{126})Ch(2\lambda_1) - ((\lambda_1 + \lambda_2) a_{127})Sh(\lambda_1 + \lambda_2) - ((\lambda_1 - \lambda_2) a_{128})Sh(\lambda_1 - \lambda_2) \]
\[ + ((\lambda_1 + \lambda_2) a_{129})Ch(\lambda_1 + \lambda_2) - ((\lambda_1 - \lambda_2) a_{130})Ch(\lambda_1 - \lambda_2) - 8a_{131} - 6a_{132} + 5a_{133} - 4a_{134} + 3a_{135} - 2a_{136} \]