INTRODUCTION
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The well known British Physicist P.M.A.Dirac in 1920 introduced the dirac delta function. To give it a solid mathematical foundation Schwartz developed the theory of distributions. With the work of L.Schwartz [24,25] and S.Sobolev[26] as basis, a number of attempts have been made in the literature to generalize the concept of distributions.

Motivated by a desire to define a product of distributions (L.Schwartz in 1954 established the impossibility of defining multiplication within the space of distributions) J.F. Colombeau [2] constructed a new differentiable algebra of generalized functions containing this space of distributions and extended the properties of distributions to this general setup. While the classical theory of integral transforms have been independently studied in the literature for about two centuries, most recently the theory of generalized functions and the theory of integral transforms have been successfully combined to develop the theory of integral transforms of generalized
functions and are used in various problems of mathematical physics and applied mathematics. One of the recent generalization of the theory of generalized functions and their integral transforms is known as the theory of Boehmians and their integral transforms. In recent years this area has become an active and important part in the theory of generalized integral transforms. In the literature several integral transforms for various spaces of Boehmians are defined and their properties investigated in [6, 7, 8, 13, 14, 16, 17, 18, 22].

Further Zemanian in [32] describes in detail how the theory of vector-valued generalized functions are useful in applications. Motivated by his ideas, in this thesis we develop the following integral transforms for vector-valued Boehmian spaces. 1) Fourier Transform. 2) Laplace Transform. We shall extend the theory of Fourier-Plancherel transform to vector-valued functions and Boehmians and also discuss the Laplace transform in the context of vector-valued Boehmians.

The first chapter contains all preliminary concepts and results that are used elsewhere in the thesis. Whenever the
results are taken from the existing literature proper references are also given.

In the second chapter we consider a possible extension of Plancherel theorem in the context of vector-valued Boehmians. Two types of Boehmian spaces are constructed each of which contains vector-valued square integrable functions on \( \mathbb{R} \) and the theory of Fourier transform is extended to this set up. It is proved that the Plancherel transform is a one-to-one continuous linear map from one space of Boehmian onto the other. A comparison with the existing literature on Fourier transform of Boehmians is also under taken.

In the third chapter the theory of Laplace transform on operator-valued distributions is extended to vector-valued Boehmians. An inversion theorem and a characterization for Laplace transformable Boehmians are obtained. A comparison with the existing literature on Laplace transform of Boehmians is also under taken.

In the fourth chapter a characterization for Laplace transformable distribution on \( \mathbb{R}^2 \) with domain \( \mathcal{L}((a,b)) \) (where \( a=(a_1,a_2), b=(b_1,b_2) \in \mathbb{R}^2 \) with \( a<b \) and \( (a,b) \) is the open
rectangle \((x,y) \in \mathbb{R}^2/ a_1 < x < b_1, a_2 < y < b_2\) is obtained. The conditions under which this Laplace transform considered as an analytic function of two variables splits as a product of two analytic functions, each of a single variable, are also studied.

In the fifth and final chapter the notion of a change of variable in the context of Boehmians is introduced. In [31] S.R. Yadava has obtained certain chain formulae in two dimensional Laplace transform. Motivated by the above results, in this chapter a new theory is presented to extend the above chain formula in the context of Laplace transform for Boehmians. A Boehmian space containing all right-sided Laplace transformable functions is constructed. A sufficient condition for Boehmians in this space to admit a change of variable is studied. The effect of this change of variable in the context of Laplace transform is also investigated. Examples of certain Boehmians (not representing functions) representing distributions which admit this change of variable and consequently satisfying the chain formula is presented.

The thesis ends with a detailed bibliography.

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